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Bipolar Vague Almost α Generalised Continuous Mappings in Topological Spaces

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In this paper, we have introduced the notion of almost α generalised continuous mappings, almost contra α generalised continuous mappings and completely α generalised continuous mappings in bipolar vague topological spaces. Also, some of their properties are studied and analysed for almost α generalised continuous mappings, almost contra α generalised continuous mappings and completely α generalised continuous mappings in bipolar vague topological spaces with suitable illustrations.

Keywords: Bipolar vague topology; bipolar vague α generalised closed sets; bipolar vague α generalised continuous mappings; bipolar vague almost α generalised continuous mappings; bipolar vague almost contra α generalised continuous mappings; bipolar vague completely α generalised continuous mappings

I. INTRODUCTION

Fuzzy set was introduced by L.A. Zadeh (1965). The concept of fuzzy topology was introduced by C.L. Chang (1968). The generalised closed sets in general topology were first introduced by N. Levine (1970). K. Atanassov (1986) introduced the concept of intuitionistic fuzzy sets. The notion of vague set theory was introduced by W.L. Gau and D.J. Buehrer (1993). D. Coker (1997) introduced intuitionistic fuzzy topological spaces. Bipolar- valued fuzzy sets, which was introduced by K.M. Lee (2000) is an extension of fuzzy sets whose membership degree range is enlarged from the interval [0, 1] to [-1,1]. A new weaker form of continuity called as completely continuity was introduced by I.M. Hanafy (2003) in intuitionistic fuzzy topological spaces. E. Ekici and K. Etienne kerre (2006) introduced contra continuous mapping in fuzzy topological spaces. Intuitionistic fuzzy contra continuous mapping was introduced by B. Kresteska and E. Ekici (2007). Godwin Amechi Okeke and Johnson O Olaleru (2013) announced Convergence theorems.

Godwin Amechi Okeke and Johnson O Olaleru (2019) declared fixed points of Demi continuous φ-Nearly Lipschitzian. A new class of generalised bipolar vague sets was introduced by S. Cicily Flora and I. Arockiarani (2016). F. Prishka and L. Mariapresenti (2024) introduced bipolar vague α generalised continuous mappings in topological spaces. In this paper we have introduced almost α generalised continuous mappings, almost contra α generalised continuous mappings and completely α generalised continuous mappings in bipolar vague topological spaces. Also, we have studied some of their properties. We have provided some characterisations of almost α generalised continuous mappings, almost contra α generalised continuous mappings and completely α generalised continuous mappings in bipolar vague topological spaces.

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II. PRELIMINARIES

Here in this paper the bipolar vague topological spaces are denoted by (X, BV_τ). Also, the bipolar vague interior, bipolar vague closure of a bipolar vague set A are denoted by BVInt(A) and BVCl(A). The complement of a bipolar vague set A is denoted by A^c and the empty set and whole sets are denoted by $0_\sim$ and $1_\sim$ respectively.

Definition 2.1: (K.M. Lee, 2000) Let X be the universe. Then a bipolar valued fuzzy sets, A on X is defined by positive membership function μ_A^+, that is μ_A^+: X→ [0,1], and a negative membership function μ_A^-, that is μ_A^-: X→ [-1,0]. For the sake of simplicity, we shall use the symbol A = $\{\langle x, \mu_A^+(x), \mu_A^-(x)\rangle: x \in X\}$.

Definition 2.2: (K.M. Lee, 2000) Let A and B be two bipolar valued fuzzy sets then their union, intersection and complement are defined as follows:

(i) $\mu_{A\cup B}^+$= max $\{\mu_A^+(x), \mu_B^+ x)\}$

(ii) $\mu_{A\cup B}^-$= min $\{\mu_A^-(x), \mu_B^- x)\}$

(iii) $\mu_{A\cap B}^+$= min $\{\mu_A^+(x), \mu_B^+ x)\}$

(iv) $\mu_{A\cap B}^-$= max $\{\mu_A^-(x), \mu_B^- x)\}$

(v) $\mu_{A^c}^+$(x) = 1-$\mu_A^+(x)$ and $\mu_{A^c}^-$(x) = -1-$\mu_A^-(x)$ for all $x \in X$.

Definition 2.3: (W.L Gau & D.J. Buehrer, 1993) A vague set A in the universe of discourse U is a pair of (t_A, f_A) where t_A: U→[0,1], f_A: U→[0,1] are the mapping such that $t_A + f_A \leq 1$ for all $u \in U$. The function t_A and f_A are called true membership function and false membership function respectively. The interval $[t_A, 1 - f_A]$ is called the vague value of u in A and denoted by $v_A(u)$, that is $v_A(u) = [t_A(u), 1 - f(u)]$.

Definition 2.4: (W.L Gau & D.J. Buehrer, 1993) Let A be a non-empty set and the vague set A and B in the form A = $\{\langle x, t_A(x), 1 - f_A(x)\rangle: x \in X\}$, B = $\{\langle x, t_B(x), 1 - f_B(x)\rangle: x \in X\}$. Then

(i) A ⊆ B if and only if $t_A(x) \leq t_B(x)$ and $1 - f_A(x) \leq 1 - f_B(x)$

(ii) A ∪ B = $\left\{\langle \max(t_A(x), t_B(x)), \frac{\max(1-f_A(x), 1-f_B(x))\rangle}{x} \in X\right\}$.

(iii) A ∩ B = $\left\{\langle \min(t_A(x), t_B(x)), \frac{\min(1-f_A(x), 1-f_B(x))\rangle}{x} \in X\right\}$.

(iv) A^c = $\{\langle x, f_A(x), 1 - t_A(x)\rangle: x \in X\}$.

Definition 2.5: (I. Arockiarani & S. Cicily Flora, 2016) Let X be the universe of discourse. A bipolar-valued vague set A in X is an object having the form A = $\{\langle x, [t_A^+(x), 1 - f_A^+(x)], [-1 - f_A^-(x), t_A^-(x)]\rangle : x \in X\}$ where $[t_A^+, 1 - f_A^+]$: X→[0,1] and $[-1 - f_A^-, t_A^-]$: X→[-1,0] are the mapping such that $t_A^+(x) + f_A^+(x) \leq 1$ and $-1 \leq t_A^- + f_A^-$. The positive membership degree $[t_A^+(x), 1 - f_A^+(x)]$ denotes the satisfaction region of an element x to the property corresponding to a bipolar-valued set A and the negative membership degree $[-1 - f_A^-(x), t_A^-(x)]$ denotes the satisfaction region of x to some implicit counter property of A. For a sake of simplicity, we shall use the notion of bipolar vague set $v_A^+ = [t_A^+, 1 - f_A^+]$ and $v_A^- = [-1 - f_A^-, t_A^-]$.

Definition 2.6: (S. Cicily Flora & I. Arockiarani, 2017) A bipolar vague set A = $[v_A^+, v_A^-]$ of a set U with v_A^+= 0 implies that t_A^+ = 0, $1 - f_A^+$= 0 and v_A^-= 0 implies that t_A^- = 0, $-1 - f_A^-$ = 0 for all x ∈ U is called zero bipolar vague set and it is denoted by 0.

Definition 2.7: (S. Cicily Flora & I. Arockiarani, 2017) A bipolar vague set A = $[v_A^+, v_A^-]$ of a set U with v_A^+= 1 implies that t_A^+ = 1, $1 - f_A^+$= 1 and v_A^-= -1 implies that t_A^- = -1, $-1 - f_A^-$ = -1 for all x ∈ U is called unit bipolar vague set and it is denoted by 1.

Definition 2.8: (S. Cicily Flora & I. Arockiarani, 2016) Let A = $\langle x, [t_A^+, 1 - f_A^+], [-1 - f_A^-, t_A^-]\rangle$ and $\langle x, [t_B^+, 1 - f_B^+], [-1 - f_B^-, t_B^-]\rangle$ be two bipolar vague sets then their union, intersection and complement are defined as follows:

(i) A ∪ B = $\left\{\langle x, [t_{A\cup B}^+(x), 1 - f_{A\cup B}^+(x)], \frac{[-1-f_{A\cup B}^-(x), t_{A\cup B}^-(x)]\rangle}{x} \in X\right\}$ where

$t_{A\cup B}^+(x)$ = max $\{t_A^+(x), t_B^+(x)\}$, $t_{A\cup B}^-(x)$ = min $\{t_A^-(x), t_B^-(x)\}$ and

$1 - f_{A\cup B}^+(x)$ = max $\{1 - f_A^+(x), 1 - f_B^+(x)\}$,

$-1 - f_{A\cup B}^-(x)$ = min $\{-1 - f_A^-(x), -1 - f_B^-(x)\}$.

(ii) A ∩ B = $\left\{\langle x, [t_{A\cap B}^+(x), 1 - f_{A\cap B}^+(x)], \frac{[-1-f_{A\cap B}^-(x), t_{A\cap B}^-(x)]\rangle}{x} \in X\right\}$ where

$t_{A\cap B}^+(x)$ = min $\{t_A^+(x), t_B^+(x)\}$, $t_{A\cap B}^-(x)$ = max $\{t_A^-(x), t_B^-(x)\}$ and

$1 - f_{A\cap B}^+(x)$ = min $\{1 - f_A^+(x), 1 - f_B^+(x)\}$,

$-1 - f_{A\cup B}^-(x)$ = max $\{-1 - f_A^-(x), -1 - f_B^-(x)\}$.

(iii) A^c = { $\langle x, [f_A^+(x), 1 - t_A^+(x)], [-1 - t_A^-(x), f_A^-(x)]\rangle / x \in X\}$.

Definition 2.9: (S. Cicily Flora & I. Arockiarani, 2016) Let A and B be two bipolar vague sets defined over a universe of discourse X. We say that A ⊆ B if and only if $t_A^+(x) \leq t_B^+(x)$, $1 - f_A^+(x) \leq 1 - f_B^+(x)$ and $t_A^-(x) \geq t_B^-(x)$, $1 - f_A^-(x) \geq 1 - f_B^-(x)$ for all x ∈ X.

Definition 2.10: (S. Cicily Flora & I. Arockiarani, 2016) A bipolar vague topology (BVT) on a non-empty set X is a family BV_τ of bipolar vague set in X satisfying the following axioms:

(i) $0_\sim, 1_\sim \in$ BV_τ

(ii) $G_1 \cap G_2 \in$ BV_τ, for any $G_1, G_2 \in$ BV_τ

(iii) $\cup G_i \in$ BV_τ, for any arbitrary family $\{G_i : G_i \in$ BV_τ, I ∈ I}.

In this case the pair (X, BV_τ) is called a bipolar vague topological space and any bipolar vague set (BVS) in BV_τ is known as bipolar vague open set in X. The complement A^c of a bipolar vague open set (BVOS) A in a bipolar vague topological space (X, BV_τ) is called a bipolar vague closed set (BVCS) in X.

Definition 2.11: (S. Cicily Flora & I. Arockiarani, 2016) Let (X, BV_τ) be a bipolar vague topological space A = ⟨x, $[t_A^+, 1 - f_A^+], [-1 - f_A^-, t_A^-]$⟩ be a bipolar vague set in X. Then the bipolar vague interior and bipolar vague closure of A are defined by,

BVInt(A) = ∪ {G: G is a bipolar vague open set in X and G ⊆ A},

BVCl(A) = ∩ {K: K is a bipolar vague closed set in X and A⊆ K}.

Note that BVCl(A) is a bipolar vague closed set and BVInt(A) is a bipolar vague open set in X. Further,

(i) A is a bipolar vague closed set in X if and only if BVCl(A) = A,

(ii) A is a bipolar vague open set in X if and only if BVInt(A) = A.

Definition 2.12: (S. Cicily Flora & I. Arockiarani, 2016) Let (X, BV_τ) be a bipolar vague topological space. A bipolar vague set A in (X, BV_τ) is said to be a generalised bipolar vague closed set if BVCl(A) ⊆ G whenever A⊆ G and G is bipolar vague open. The complement of a generalised bipolar vague closed set is a generalised bipolar vague open set.

Definition 2.13: (S. Cicily Flora & I. Arockiarani, 2016) Let (X, BV_τ) be a bipolar vague topological space and A be a bipolar vague set in X. Then the generalised bipolar vague closure and generalised bipolar vague interior of A are defined by,

GBVCl(A) = ∩ {G: G is a generalised bipolar vague closed set in X and A⊆ G},

GBInt(A) = ∪ {G: G is a generalised bipolar vague open set in X and A ⊇ G}.

Definition 2.14: (F. Prishka & L. Mariapresenti, 2024) A bipolar vague set A of a bipolar vague topological space X, is said to be

(i) a bipolar vague α -open set if A ⊆ BVInt(BVCl(BVInt(A)))

(ii) a bipolar vague pre-open set if A ⊆ BVInt(BVCl(A))

(iii) a bipolar vague semi-open set if A ⊆ BVCl(BVInt(A))

(iv) a bipolar vague semi- α -open set if A ⊆ BVCl(αBVInt(A))

(v) a bipolar vague regular-open set BVInt(BVCl(A)) = A

(vi) a bipolar vague β -open set A ⊆ BVCl(BVInt(BVCl(A))).

Definition 2.15: (F. Prishka & L. Mariapresenti, 2024) A bipolar vague set A of a bipolar vague topological space X, is said to be

(i) a bipolar vague α -closed set if BVCl(BVInt(BVCl(A))) ⊆ A

(ii) a bipolar vague pre-closed set if BVCl(BVInt(A)) ⊆ A

(iii) a bipolar vague semi-closed set if BVInt(BVCl(A)) ⊆ A

(iv) a bipolar vague semi- α -closed set if BVInt(αBVCl(A)) ⊆ A

(v) a bipolar vague regular-closed set if BVCl(BVInt(A)) = A

(vi) a bipolar vague β -closed set if BVInt(BVCl(BVInt(A))) ⊆ A.

Definition 2.16: (F. Prishka & L. Mariapresenti, 2024) Let A be a bipolar vague set of a bipolar vague topological space (X, BV_τ). Then the bipolar vague α interior and bipolar vague α closure are defined as

BV_αInt(A) = ∪ {G: G is a bipolar vague α-open set in X and G ⊆ A},

$BV_\alpha Cl(A) = \cap \{K: K$ is a bipolar vague α-closed set in X and $A \subseteq K\}$.

Definition 2.17: (F. Prishka & L. Mariapresenti, 2024) A bipolar vague set A in a bipolar vague topological space X, is said to be a bipolar vague α generalised closed set if $BV_\alpha Cl(A) \subseteq U$ whenever $A \subseteq U$ and U is a bipolar vague open set in X. The complement A^c of a bipolar vague α generalised closed set A is a bipolar vague α generalised open set in X.

Definition 2.18: (S. Cicily Flora & I. Arockiarani, 2016) Let (X, BV_τ) and (Y, BV_σ) be two bipolar vague topological spaces and $\varphi : X \to Y$ be a function. Then φ is said to be bipolar vague continuous if and only if the preimage of each bipolar vague open set in Y is a bipolar vague open set in X.

Definition 2.19: (S. Cicily Flora & I. Arockiarani, 2016) A map $f : (X, BV_\tau) \to (Y, BV_\sigma)$ is said to be generalised bipolar vague continuous if the inverse image of every bipolar vague open set in (Y, BV_σ) is a generalised vague open set in (X, BV_τ).

Definition 2.20: (S. Cicily Flora & I. Arockiarani, 2016) Let f be a mapping from a bipolar vague topological space (X, BV_τ) into a bipolar vague topological space (Y, BV_σ). Then f is said to be a bipolar vague generalised irresolute mapping if the inverse image of every bipolar vague generalised closed set in (Y, BV_σ) is a bipolar vague generalised closed set in (X, BV_τ).

Definition 2.21: (F. Prishka & L. Mariapresenti, 2024) Let (X, BV_τ) and (Y, BV_σ) be two bipolar vague topological spaces. Then the mapping $f : (X, BV_\tau) \to (Y, BV_\sigma)$ is called

(i) a bipolar vague α continuous if the inverse image of every bipolar vague closed set in (Y, BV_σ) is a bipolar vague α-closed set in (X, BV_τ).

(ii) a bipolar vague pre continuous if the inverse image of every bipolar vague closed set in (Y, BV_σ) is a bipolar vague pre-closed set in (X, BV_τ).

(iii) a bipolar vague semi continuous if the inverse image of every bipolar vague closed set in (Y, BV_σ) is a bipolar vague semi-closed set in (X, BV_τ).

Definition 2.22: (F. Prishka & L. Mariapresenti, 2024) Let (X, BV_τ) and (Y, BV_σ) be two bipolar vague topological spaces. A mapping $f : (X, BV_\tau) \to (Y, BV_\sigma)$ is called a bipolar vague α generalised continuous mapping if $f^{-1}(B)$ is a bipolar vague α generalised closed set in (X, BV_τ) for every bipolar vague closed set B of (Y, BV_σ).

Definition 2.23: (F. Prishka & L. Mariapresenti, 2024) A mapping $f : (X, BV_\tau) \to (Y, BV_\sigma)$ is called a bipolar vague α generalised irresolute mapping if $f^{-1}(A)$ is a bipolar vague α generalised closed set in (X, BV_τ) for every bipolar vague α generalised closed set A of (Y, BV_σ).

Definition 2.24: (F. Prishka & L. Mariapresenti, 2024) A bipolar vague topological space (X, BV_τ) is said to be bipolar vague $\alpha a\ T_{1/2}$ $(BV_{\alpha a}T_{1/2})$ space if every bipolar vague α generalised closed set in X is a bipolar vague closed set in X.

Definition 2.25: (F. Prishka & L. Mariapresenti, 2024) A bipolar vague topological space (X, BV_τ) is said to be bipolar vague $\alpha b\ T_{1/2}$ $(BV_{\alpha b}T_{1/2})$ space if every bipolar vague α generalised closed set in X is a bipolar vague generalised closed set in X.

Definition 2.26: (F. Prishka & L. Mariapresenti, Accepted) Let (X, BV_τ) and (Y, BV_σ) be two bipolar vague topological spaces. Then the mapping $f : (X, BV_\tau) \to (Y, BV_\sigma)$ is called

(i) a bipolar vague contra continuous if the inverse image of every bipolar vague open set in (Y, BV_σ) is a bipolar vague closed set in (X, BV_τ).

(ii) a bipolar vague contra α continuous if the inverse image of every bipolar vague open set in (Y, BV_σ) is a bipolar vague α-closed set in (X, BV_τ).

(iii) a bipolar vague contra pre continuous if the inverse image of every bipolar vague open set in (Y, BV_σ) is a bipolar vague pre-closed set in (X, BV_τ).

(iv) A bipolar vague contra generalised continuous if the inverse image of every bipolar vague open set in (Y, BV_σ) is a bipolar vague generalised closed set in (X, BV_τ).

Definition 2.27: (F. Prishka & L. Mariapresenti, Accepted) Let (X, BV_τ) and (Y, BV_σ) be two bipolar vague topological spaces. A mapping $f : (X, BV_\tau) \to (Y, BV_\sigma)$ is called a bipolar vague contra α generalised continuous mapping if the inverse image of every bipolar vague open set in (Y, BV_σ) is a bipolar vague α generalised closed set in (X, BV_τ).

III. BIPOLAR VAGUE ALMOST α GENERALISED CONTINUOUS MAPPINGS IN TOPOLOGICAL SPACES

In this segment we have familiarised bipolar vague almost α generalised continuous mappings and examined some of their belongings.

Definition 3.1: Let f be a mapping from a bipolar vague topological space (X, BV_τ) into a bipolar vague topological space (Y, BV_σ). Then f is said to be bipolar vague almost continuous if the inverse image of every bipolar vague regular-closed set in (Y, BV_σ) is a bipolar vague closed set in (X, BV_τ).

Definition 3.2: A mapping f : (X, BV_τ) $\rightarrow$ (Y, BV_σ) is said to be a bipolar vague almost α generalised continuous mapping if f^{-1}(A) is a bipolar vague α generalised closed set in (X, BV_τ) for every bipolar vague regular-closed set A in (Y, BV_σ).

Example 3.3: Let X = {a,b} and Y = {u,v}. Then τ = {$0_\sim$, A, $1_\sim$} and σ = {$0_\sim$, B, $1_\sim$} are bipolar vague topologies on X and Y respectively, where A = ⟨x, [0.2, 0.3] [-0.5, -0.5], [0.3, 0.2] [-0.4, -0.4]⟩ and B = ⟨y, [0.4, 0.3] [-0.5, -0.5], [0.4, 0.4] [-0.5, -0.5]⟩. Define a mapping f : (X, BV_τ) $\rightarrow$ (Y, BV_σ) by f(a) = u and f(b) = v. Then f is a bipolar vague almost α generalised continuous mapping. For, consider the bipolar vague set B^c = ⟨y, [0.7, 0.6] [-0.5, -0.5], [0.6, 0.6][-0.5, -0.5]⟩ which is a bipolar vague regular-closed set in Y, since BVCl(BVInt(B^c)) = BVCl(B) = B^c. We have f^{-1} (B^c) = ⟨x, [0.7, 0.6] [-0.5, -0.5], [0.6, 0.6] [-0.5, -0.5]⟩ is a bipolar vague α generalised closed set in (X, BV_τ) as f^{-1} (B^c) ⊆ $1_\sim$ and BV_α Cl(f^{-1} (B^c)) = f^{-1} (B^c) ∪ BVCl(BVInt(BVCl(f^{-1} (B^c)))) = A^c ⊆ $1_\sim$.

Remark 3.4: Every bipolar vague regular-closed set A is a bipolar vague closed set in X.

Proof: Obvious.

Example 3.5: Let X = {a,b} and τ = {$0_\sim$, A, B, $1_\sim$} where A = ⟨x, [0.1, 0.2] [-0.3, -0.3], [0.4, 0.3][-0.3, -0.3]⟩ and B = ⟨y, [0.3, 0.3] [-0.5, -0.5], [0.5, 0.5] [-0.5, -0.5]⟩. Consider the bipolar vague closed set B^c = ⟨y, [0.7, 0.7] [-0.5, -0.5], [0.5, 0.5] [-0.5, -0.5]⟩ which is a bipolar vague regular-closed set in Y as BVCl(BVInt(B^c) = BVCl(B) = B^c.

Proposition 3.6: Every bipolar vague continuous is a bipolar vague almost α generalised continuous mapping but not conversely in general.

Proof: Let f : (X, BV_τ) → (Y, BV_σ) be a bipolar vague continuous mapping. Let A be a bipolar vague regular-closed set in Y. Since every bipolar vague regular-closed set is a bipolar vague closed set, A is a bipolar vague closed set in Y. Then f^{-1} (A) is a bipolar vague closed set in X, by hypothesis. Since every bipolar vague closed set is a bipolar vague α generalised closed set, f^{-1}(A) is a bipolar vague α generalised closed set in X. Hence f is a bipolar vague almost α generalised continuous mapping.

Example 3.7: Let X = {a,b} and Y = {u,v}. Then τ = {$0_\sim$, A, $1_\sim$} and σ = {$0_\sim$, B, $1_\sim$} are bipolar vague topologies on X and Y respectively, where A = ⟨x, [0.1, 0.2] [-0.3, -0.3], [0.4, 0.3] [-0.3, -0.3]⟩ and B = ⟨y, [0.3, 0.3] [-0.5, -0.5], [0.5, 0.5] [-0.5, -0.5]⟩. Define a mapping f : (X, BV_τ) $\rightarrow$ (Y, BV_σ) by f(a) = u and f(b) = v. Then f is a bipolar vague almost α generalised continuous mapping but not a bipolar vague continuous mapping. For consider the bipolar vague set B^c = ⟨y, [0.7, 0.7] [-0.5, -0.5], [0.5, 0.5] [-0.5, -0.5]⟩ which is a bipolar vague regular-closed set in Y as BVCl(BVInt(B^c) = BVCl(B) = B^c. We have f^{-1} (B^c) = ⟨x, [0.7, 0.7] [-0.5, -0.5], [0.5, 0.5] [-0.5, -0.5]⟩ is a bipolar vague α generalised closed set in (X, BV_τ) as f^{-1} (B^c) ⊆ $1_\sim$ and BV_αCl(f^{-1} (B^c)) = f^{-1} (B) ∪ BVCl(BVInt(BVCl(f^{-1} (B^c)))) = A^c ⊆ $1_\sim$.Therefore, f is a bipolar vague almost α generalised continuous mapping. Now consider the bipolar vague closed set B^c = ⟨y, [0.7, 0.7] [-0.5, -0.5], [0.5, 0.5] [-0.5, -0.5]⟩ in Y, where f^{-1} (B^c) is not a bipolar vague closed set in X as BVCl(f^{-1} (B^c)) = A^c ≠ f^{-1} (B^c). Therefore, f is not a bipolar vague continuous mapping.

Proposition 3.8: Every bipolar vague α continuous mapping is a bipolar vague almost α generalised continuous mapping but not conversely in general.

Proof: Let f : (X, BV_τ) → (Y, BV_σ) be a bipolar vague α continuous mapping. Let A be a bipolar vague regular-closed set in Y. Since every bipolar vague regular-closed set is a bipolar vague closed set, A is a bipolar vague closed set in Y. Then f^{-1} (A) is a bipolar vague α-closed set in X, by hypothesis. Since every bipolar vague α-closed set is a bipolar vague α generalised closed set, f^{-1}(A) is a bipolar vague α generalised closed set in X. Hence f is a bipolar vague almost α generalised continuous mapping.

Example 3.9: Let X = {a,b} and Y = {u,v}. Then $\tau = \{0_\sim, A, 1_\sim\}$ and $\sigma = \{0_\sim, B, 1_\sim\}$ are bipolar vague topologies on X and Y respectively, where A = ⟨x, [0.2, 0.3] [-0.4, -0.4], [0.3, 0.6] [-0.4, -0.4]⟩ and B = ⟨y, [0.3, 0.3] [-0.4, -0.4], [0.4, 0.6] [-0.4, -0.4]⟩. Define a mapping $f : (X, BV_\tau) \to (Y, BV_\sigma)$ by f(a) = u and f(b) = v. Then f is a bipolar vague almost α generalised continuous mapping but not a bipolar vague α continuous mapping. For consider the bipolar vague set B^c = ⟨y, [0.7, 0.7] [-0.6, -0.6], [0.4, 0.6] [-0.6, -0.6]⟩ which is a bipolar vague regular-closed set in Y as BVCl(BVInt(B^c)) = BVCl(B) = B^c. We have f^{-1} (B^c) = ⟨x, [0.7, 0.7] [-0.6, -0.6], [0.4, 0.6] [-0.6, -0.6]⟩ is a bipolar vague α generalised closed set in (X, BV_τ) as f^{-1} (B^c) ⊆ $1_\sim$ and BV_αCl(f^{-1} (B)) = f^{-1} (B^c) ∪ BVCl(BVInt(BVCl(f^{-1} (B^c)))) = $A^c \subseteq 1_\sim$.Therefore, f is a bipolar vague almost α generalised continuous mapping. Now consider the bipolar vague closed set B^c = ⟨y, [0.7, 0.7] [-0.6, -0.6], [0.4, 0.6] [-0.6, -0.6]⟩ in Y, where f^{-1} (B^c) is not a bipolar vague α -closed set in X as $\mathrm{BVCl}\left(\mathrm{BVInt}\left(\mathrm{BVCl}\left(f^{-1}\,(B^c)\right)\right)\right)$ = $A^c \not\subseteq f^{-1}$ (B^c). Therefore, f is not a bipolar vague α continuous mapping.

Proposition 3.10: Every bipolar vague almost continuous mapping is a bipolar vague almost α generalised continuous mapping but not conversely in general.

Proof: Let $f : (X, BV_\tau) \to (Y, BV_\sigma)$ be a bipolar vague almost continuous mapping. Let A be a bipolar vague regular-closed set in Y. Since f is a bipolar vague almost continuous mapping, f^{-1}(A) is a bipolar vague closed set in X. Since every bipolar vague closed set is a bipolar vague α generalised closed set, f^{-1} (A) is a bipolar vague α generalised closed set in X. Hence f is a bipolar vague almost α generalised continuous mapping.

Example 3.11: Let X = {a,b} and Y = {u,v}. Then $\tau = \{0_\sim, A, 1_\sim\}$ and $\sigma = \{0_\sim, B, 1_\sim\}$ are bipolar vague topologies on X and Y respectively, where A = ⟨x, [0.3, 0.2] [-0.2, -0.2], [0.6, 0.4] [-0.2, -0.2]⟩ and B = ⟨y, [0.4, 0.2] [-0.3, -0.3], [0.5, 0.4] [-0.3, -0.3]⟩. Define a mapping $f : (X, BV_\tau) \to (Y, BV_\sigma)$ by f(a) = u and f(b) = v. Then f is a bipolar vague almost α generalised continuous mapping but not a bipolar vague almost continuous mapping. For consider the bipolar vague set B^c = ⟨y, [0.8, 0.6] [-0.7, -0.7], [0.6, 0.4] [-0.7, -0.7]⟩ which is a bipolar vague regular-closed set in Y as BVCl(BVInt(B^c) = BVCl(B) = B^c. We have f^{-1} (B^c) = ⟨x, [0.8, 0.6][-0.7, -0.7], [0.6, 0.4] [-0.7, -0.7]⟩ is a bipolar vague α generalised closed set in (X, BV_τ) as f^{-1} (B^c) ⊆ $1_\sim$ and BV_αCl(f^{-1} (B^c)) = f^{-1} (B) ∪ BVCl(BVInt(BVCl(f^{-1} (B^c)))) = $A^c \subseteq 1_\sim$.Therefore, f is a bipolar vague almost α generalised continuous mapping. Now consider the bipolar vague closed set B^c = ⟨y, [0.8, 0.6] [-0.7, -0.7], [0.6, 0.4] [-0.7, -0.7]⟩ in Y, where f^{-1} (B^c) is not a bipolar vague closed set in X as BVCl(f^{-1} (B^c)) = $A^c \neq f^{-1}$ (B^c). Therefore, f is not a bipolar vague almost continuous mapping.

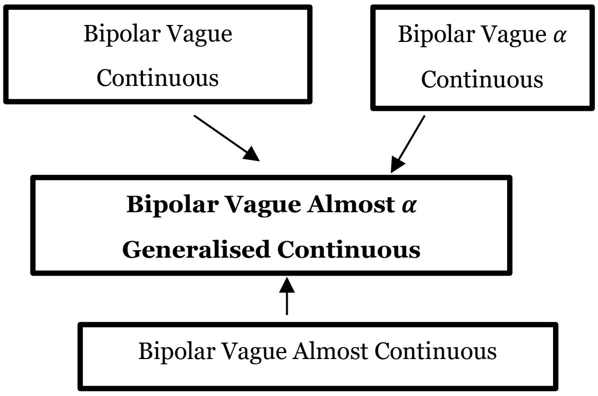

Figure 1. Bipolar Vague Almost α Generalised Continuous Mappings

Proposition 3.12: Let $f : (X, BV_\tau) \to (Y, BV_\sigma)$ be a bipolar vague α generalised irresolute mapping and $g : (Y, BV_\sigma) \to (Z, BV_\delta)$ be a bipolar vague almost α generalised continuous mapping, then $g \circ f : (X, BV_\tau) \to (Z, BV_\delta)$ is a bipolar vague almost α generalised continuous mapping.

Proof: Let A be a bipolar vague regular-closed set in Z. Since g is a bipolar vague almost α generalised continuous mapping, g^{-1}(A) be a bipolar vague α generalised closed set in Y. Since f is a bipolar vague α generalised irresolute mapping, f^{-1}(g^{-1}(A)) is a bipolar vague α generalised closed set in X. Since $(g \circ f)^{-1}$(A) = f^{-1}(g^{-1}(A)), $g \circ f$ is a bipolar vague almost α generalised continuous mapping.

Proposition 3.13: Let $f : (X, BV_\tau) \to (Y, BV_\sigma)$ be a bipolar vague α generalised continuous mapping and $g : (Y, BV_\sigma) \to (Z, BV_\delta)$ be a bipolar vague almost continuous mapping, then $g \circ f : (X, BV_\tau) \to (Z, BV_\delta)$ is a bipolar vague almost α generalised continuous mapping.

Proof: Let A be a bipolar vague regular-closed set in Z. Since g is a bipolar vague almost continuous mapping,

g^{-1}(A) is a bipolar vague closed set in Y. Since f is a bipolar vague α generalised continuous mapping, $f^{-1}(g^{-1}$(A)) is a bipolar vague α generalised closed set in X. Since $(g \circ f)^{-1}$(A) = $f^{-1}(g^{-1}$(A)), $g \circ f$ is a bipolar vague almost α generalised continuous mapping.

Proposition 3.14: Let f : (X, BV_τ) → (Y, BV_σ) be a bipolar vague continuous mapping and g : (Y, BV_σ) → (Z, BV_δ) be a bipolar vague almost continuous mapping, then $g \circ f$: (X, BV_τ) → (Z, BV_δ) is a bipolar vague almost α generalised continuous mapping.

Proof: Let A be a bipolar vague regular-closed set in Z. Since g is a bipolar vague almost continuous mapping, g^{-1}(A) is a bipolar vague closed set in Y. Since f is a bipolar vague continuous mapping, $f^{-1}(g^{-1}$(A)) is a bipolar vague closed set in X. Since every closed set is a bipolar vague α generalised closed set, $f^{-1}(g^{-1}$(A)) is a bipolar vague α generalised closed set in X. Since $(g \circ f)^{-1}$ (A) = $f^{-1}(g^{-1}$(A)), $g \circ f$ is a bipolar vague almost α generalised continuous mapping.

Proposition 3.15: Let f : (X, BV_τ) → (Y, BV_σ) be a bipolar vague α continuous mapping and g : (Y, BV_σ) → (Z, BV_δ) be a bipolar vague almost continuous mapping, then $g \circ f$: (X, BV_τ) → (Z, BV_δ) is a bipolar vague almost α generalised continuous mapping.

Proof: Let A be a bipolar vague regular-closed set in Z. Since g is a bipolar vague almost continuous mapping, g^{-1}(A) is a bipolar vague closed set in Y. Since f is a bipolar vague α continuous mapping, $f^{-1}(g^{-1}$(A)) is a bipolar vague α-closed set in X, $f^{-1}(g^{-1}$(A)) is a bipolar vague α generalised closed set in X. Since $(g \circ f)^{-1}$ (A) = $f^{-1}(g^{-1}$(A)), $g \circ f$ is a bipolar vague almost α generalised continuous mapping.

Proposition 3.16: A mapping f : (X, BV_τ) → (Y, BV_σ) is a bipolar vague almost α generalised continuous mapping if and only if the inverse image of each bipolar vague regular open set in Y is a bipolar vague α generalised open set in X.

Proof: Necessity: Let A be a bipolar vague regular-open set in Y. This implies A^c is a bipolar vague regular closed set in Y. Since f is a bipolar vague almost α generalised continuous mapping, f^{-1} (A^c) is a bipolar vague α generalised closed set in X. Since f^{-1}(A^c) = $(f^{-1}$(A))^c, f^{-1}(A) is a bipolar vague α generalised open set in X.

Sufficiency: Let A be a bipolar vague regular-closed set in Y. This implies A^c is a bipolar vague regular open set in Y. By hypothesis, f^{-1}(A^c) is a bipolar vague α generalised open set in X. Since f^{-1}(A^c) = $(f^{-1}$(A))^c , f^{-1}(A) is a bipolar vague α generalised closed set in X. Hence f is a bipolar vague almost α generalised continuous mapping.

Proposition 3.17: Let f : (X, BV_τ) → (Y, BV_σ) be a mapping from a bipolar vague topological space X into a bipolar vague topological space Y. Then the following conditions are equivalent if X is a B$V_{\alpha a}T_{1/2}$ space:

(i) f is a bipolar vague almost α generalised continuous mapping.

(ii) f^{-1}(B) is a bipolar vague α generalised open set in X for every bipolar vague regular open set B in Y.

(iii) f^{-1} (BVInt(B)) ⊆ BVInt(BVCl(BVInt(f^{-1} (B)))) for every bipolar vague regular open set B in Y.

Proof: (i) ⟹ (ii) is obviously true.

(ii) ⟹ (iii). Let B be any bipolar vague regular-open set in Y. Then BVInt(B) is a bipolar vague open set in Y. By hypothesis, f^{-1}(BVInt(B)) is a bipolar vague α generalised open set in X. Since X is a B$V_{\alpha a}T_{1/2}$ space, f^{-1}(BVInt(B)) is a bipolar vague open set in X. Therefore, f^{-1}(BVInt(B)) = BVInt(f^{-1}(BVInt(B))) ⊆ BVInt(BVCl(BVInt(f^{-1}(B)))).

(iii) ⟹ (i). Let B be a bipolar vague regular-closed set in Y. Then its complement B^c is a bipolar vague regular-open set in Y. By hypothesis, f^{-1} (BVInt(B^c)) ⊆ BVInt(BVCl(BVInt(f^{-1} (B^c)))). This implies f^{-1} (B^c) = f^{-1} (BVInt(B^c)) ⊆ BVInt(BVCl(BVInt(f^{-1} (B^c)))). Hence f^{-1}(B^c) is a bipolar vague α-open set in X. Since every bipolar vague α-open set is a bipolar vague α generalised open set, f^{-1}(B^c) is a bipolar vague α generalized open set in X. Therefore, f^{-1}(B) is a bipolar vague α generalised closed set in X as f^{-1}(B^c) = (f^{-1}(B))^c. Hence f is a bipolar vague almost α generalised continuous mapping.

Proposition 3.18: Let f : (X, BV_τ) → (Y, BV_σ) be a mapping from a bipolar vague topological space X into a bipolar vague topological space Y. Then the following conditions are equivalent if X is a B$V_{\alpha a}T_{1/2}$ space:

(i) f is a bipolar vague almost α generalised continuous mapping.

(ii) If f^{-1}(B) is a bipolar vague α generalised closed set in X, for every bipolar vague closed set B in Y.

(iii) BVCl(BVInt(BVCl(f^{-1} (A)))) ⊆ f^{-1} (BVCl(A)) for every bipolar vague regular closed set B in Y.

Proof: (i) ⟹ (ii) is obviously true.

(ii) ⟹ (iii). Let A be any bipolar vague regular-closed set in Y. Then BVCl(A) is a bipolar vague closed set in Y. By hypothesis, f^{-1}(BVCl(A)) is a bipolar vague α generalised closed set in X. Since X is a B$V_{\alpha a}T_{1/2}$ space, f^{-1}(BVCl(A)) is a bipolar vague closed set in X. Therefore, BVCl(f^{-1} (BVCl(A))) = f^{-1} (BVCl(A)). Now BVCl(BVInt(B f^{-1} (A)))) ⊆ BVCl(BVInt(BVCl((f^{-1}(BVCl(A)))) ⊆ f^{-1}(BVCl(A)).

(iii) ⟹ (i). Let A be a bipolar vague regular-closed set in Y. Then by hypothesis, BVCl(BVInt(BVCl(f^{-1}(BVCl(A))))) ⊆ f^{-1}(BVCl(A)) = f^{-1}(A). This implies f^{-1}(A) is a bipolar vague α-closed set in X and hence it is a bipolar vague α generalised closed set in X. Therefore, f is a bipolar vague almost α generalised continuous mapping.

Proposition 3.19: Let f : (X, BV_τ) → (Y, BV_σ) be a mapping. If f^{-1}(BV_α Int(B)) ⊆ BV_α Int(f^{-1}(B)) for every bipolar vague set B in Y, then f is a bipolar vague almost α generalised continuous mapping.

Proof: Let B be a bipolar vague regular-open set in Y. By hypothesis, f^{-1}(BV_αInt(B)) ⊆ BV_αInt(f^{-1}(B)). Since B is a bipolar vague regular-open set, it is a bipolar vague α-open set in Y. Therefore, BV_α Int(B) = B. Hence f^{-1}(B) = f^{-1}(BV_α Int(B)) ⊆ BV_α Int(f^{-1}(B)) ⊆ f^{-1}(B). Therefore f^{-1}(B) = BV_αInt(f^{-1}(B)). This implies f^{-1}(B) is a bipolar vague α-open set in X and hence f^{-1}(B) is a bipolar vague α generalised open set in X. Thus f is a bipolar vague almost α generalised continuous mapping.

Proposition 3.20: Let f : (X, BV_τ) → (Y, BV_σ) be a mapping. If BV_αCl(f^{-1}(B)) ⊆ f^{-1}(BV_αCl(B)) for every bipolar vague set B in Y, then f is a bipolar vague almost α generalised continuous mapping.

Proof: Let B be a bipolar vague regular-closed set in Y. By hypothesis, BV_αCl(f^{-1}(B)) ⊆ f^{-1}(BV_αCl(B)). Since B is a bipolar vague regular-closed set, it is a bipolar vague α-closed set in Y. Therefore, BV_αCl(B) = B. Hence f^{-1}(B) = f^{-1}(BV_αCl(B)) ⊇ BV_αCl(f^{-1}(B)) ⊇ f^{-1}(B). Therefore f^{-1}(B) = BV_αCl(f^{-1}(B)). This implies f^{-1}(B) is a bipolar vague α-closed set in X and hence f^{-1}(B) is a bipolar vague α generalised closed set in X. Thus f is a bipolar vague almost α generalised continuous mapping.

IV. BIPOLAR VAGUE ALMOST CONTRA α GENERALISED CONTINUOUS MAPPINGS IN TOPOLOGICAL SPACES

In this unit we have presented bipolar vague almost contra α generalised continuous mappings and deliberate some of their properties.

Definition 4.1: A mapping f : (X, BV_τ) → (Y, BV_σ) is called a bipolar vague almost contra continuous mapping if f^{-1}(A) is a bipolar vague closed set in (X, BV_τ) for every bipolar vague regular-open set A of (Y, BV_σ).

Definition 4.2: A mapping f : (X, BV_τ) → (Y, BV_σ) is called a bipolar vague almost contra generalised continuous mapping if f^{-1}(A) is a bipolar vague generalised closed set in (X, BV_τ) for every bipolar vague regular-open set A of (Y, BV_σ).

Definition 4.3: A mapping f : (X, BV_τ) → (Y, BV_σ) is called a bipolar vague almost contra α generalised continuous mapping if f^{-1}(A) is a bipolar vague α generalised closed set in (X, BV_τ) for every bipolar vague regular-open set A of (Y, BV_σ).

Example 4.4: Let X = {a,b} and Y = {u,v}. Then τ = {$0_\sim$, A, $1_\sim$} and σ = {$0_\sim$, B, $1_\sim$} are bipolar vague topologies on X and Y respectively, where A = ⟨x, [0.2, 0.3] [-0.4, -0.3], [0.5, 0.5] [-0.4, -0.2]⟩ and B = ⟨y, [0.4, 0.4] [-0.5, -0.5], [0.4, 0.4] [-0.3, -0.3]⟩. Define a mapping f : (X, BV_τ) → (Y, BV_σ) by f(a) = u and f(b) = v. Then f is a bipolar vague almost contra α generalised continuous mapping. For, consider the bipolar vague set B = ⟨y, [0.4, 0.4] [-0.5, -0.5], [0.4, 0.4] [-0.3, -0.3]⟩ which is a bipolar vague regular-open set in Y, since BVInt(BVCl(B)) = BVInt(B^c) = B. We have f^{-1}(B) = ⟨x, [0.4, 0.4] [-0.5, -0.5], [0.4, 0.4] [-0.3, -0.3]⟩ is a bipolar vague α generalised closed set in (X, BV_τ) as f^{-1}(B) ⊆ $1_\sim$ and BV_αCl(f^{-1}(B)) = f^{-1}(B) ∪ BVCl(BVInt(BVCl(f^{-1}(B)))) = A^c ⊆ $1_\sim$.

Proposition 4.5: A mapping f : (X, BV_τ) → (Y, BV_σ) is a bipolar vague almost contra α generalised continuous mapping if and only if the inverse image of each bipolar vague regular-closed set in Y is a bipolar vague α generalised open set in X.

Proof: Necessity: Let A be a bipolar vague regular-closed set in Y. This implies A^c is a bipolar vague regular-open set in Y. Since f is a bipolar vague almost contra α generalised continuous mapping, f^{-1} (A^c) is a bipolar vague α generalised closed set in X. Since $f^{-1}(A^c) = (f^{-1}(A))^c$, f^{-1}(A) is a bipolar vague α generalised open set in X.

Sufficiency: Let A be a bipolar vague regular-open set in Y. This implies A^c is a bipolar vague regular-closed set in Y. By hypothesis, $f^{-1}(A^c)$ is a bipolar vague α generalised open set in X. Since $f^{-1}(A^c) = (f^{-1}(A))^c$, f^{-1}(A) is a bipolar vague α generalised closed set in X. Hence f is a bipolar vague almost contra α generalised continuous mapping.

Proposition 4.6: Every bipolar vague contra continuous mapping is a bipolar vague almost contra α generalised continuous mapping but not conversely in general.

Proof: Let f : (X, BV_τ) → (Y, BV_σ) be a bipolar vague contra continuous mapping. Let A be a bipolar vague regular-open set in Y. This implies A is a bipolar vague open set in Y. Since f is a bipolar vague contra continuous mapping, f^{-1}(A) is a bipolar vague closed set in X. Since every bipolar vague closed set is a bipolar vague α generalised closed set in X [14]. Hence f is a bipolar vague almost contra α generalised continuous mapping.

Example 4.7: Let X = {a,b} and Y = {u,v}. Then τ = {$0_\sim$, A, $1_\sim$} and σ = {$0_\sim$, B, $1_\sim$} are bipolar vague topologies on X and Y respectively, where A = ⟨x, [0.2, 0.3] [-0.4, -0.2], [0.4, 0.6] [-0.5, -0.5]⟩ and B = ⟨y, [0.3, 0.3] [-0.3, -0.2], [0.4, 0.6] [-0.4, -0.3]⟩. Define a mapping f : (X, BV_τ) → (Y, BV_σ) by f(a) = u and f(b) = v. Then f is a bipolar vague almost contra α generalised continuous mapping but not a bipolar vague contra continuous mapping. For, consider the bipolar vague set B = ⟨y, [0.3, 0.3] [-0.3, -0.2], [0.4, 0.6] [-0.4, -0.3]⟩ which is a bipolar vague regular-open set in Y as BVInt(BVCl(B)) = BVInt(B^c) = B. We have f^{-1} (B) = ⟨x, [0.3, 0.3] [-0.3, -0.2], [0.4, 0.6] [-0.4, -0.3]⟩ is a bipolar vague α generalised closed set in (X, BV_τ) as f^{-1} (B) ⊆ $1_\sim$ and BV_αCl(f^{-1} (B)) = f^{-1} (B) ∪ BVCl(BVInt(BVCl(f^{-1} (B)))) = A^c ⊆ $1_\sim$.Therefore, f is a bipolar vague almost contra α generalised continuous mapping. Now consider the bipolar vague open set B = ⟨y, [0.3, 0.3] [-0.3, -0.2], [0.4, 0.6] [-0.4, -0.3]⟩ in Y, where f^{-1} (B) is not a bipolar vague closed set in X as BVCl(f^{-1} (B)) = A^c ≠ f^{-1} (B). Therefore, f is not a bipolar vague contra continuous mapping.

Proposition 4.8: Every bipolar vague contra α continuous mapping is a bipolar vague almost contra α generalised continuous mapping but not conversely in general.

Proof: Let f : (X, BV_τ) → (Y, BV_σ) be a bipolar vague contra α continuous mapping. Let A be a bipolar vague regular-open set in Y. This implies A is a bipolar vague open set in Y. Then by hypothesis, f^{-1}(A) is a bipolar vague α-closed set in X. Since every bipolar vague α-closed set is a bipolar vague α generalised closed set in X [14], f^{-1}(A) is a bipolar vague α generalised closed set in X. Hence f is a bipolar vague almost contra α generalised continuous mapping.

Example 4.9: Let X = {a,b} and Y = {u,v}. Then τ = {$0_\sim$, A, $1_\sim$} and σ = {$0_\sim$, B, $1_\sim$} are bipolar vague topologies on X and Y respectively, where A = ⟨x, [0.2, 0.3] [-0.3, -0.2], [0.3, 0.4] [-0.2, -0.3]⟩ and B = ⟨y, [0.3, 0.2] [-0.4, -0.5], [0.4, 0.5] [-0.2, -0.3]⟩. Define a mapping f : (X, BV_τ) → (Y, BV_σ) by f(a) = u and f(b) = v. Then f is a bipolar vague almost contra α generalised continuous mapping but not a bipolar vague contra α continuous mapping. For, consider the bipolar vague set B = ⟨y, [0.3, 0.2] [-0.4, -0.5], [0.4, 0.5] [-0.2, -0.3]⟩ which is a bipolar vague regular-open set in Y as BVInt(BVCl(B)) = BVInt(B^c) = B. We have f^{-1} (B) = ⟨x, [0.3, 0.2] [-0.4, -0.5], [0.4, 0.5] [-0.2, -0.3]⟩ is a bipolar vague α generalised closed set in (X, BV_τ) as f^{-1} (B) ⊆ $1_\sim$ and BV_αCl(f^{-1} (B)) = f^{-1} (B) ∪ BVCl(BVInt(BVCl(f^{-1} (B)))) = A^c ⊆ $1_\sim$.Therefore, f is a bipolar vague almost contra α generalised continuous mapping. Now consider the bipolar vague open set B = ⟨y, [0.3, 0.2] [-0.4, -0.5], [0.4, 0.5] [-0.2, -0.3]⟩ in Y, where f^{-1} (B) is not a bipolar vague α-closed set in X as BVCl(BVInt(BVCl((f^{-1} (B)))) = A^c ≠ f^{-1} (B). Therefore f is not a bipolar vague contra α continuous mapping.

Remark 4.10: Every bipolar contra pre continuous mapping and bipolar almost contra α generalised continuous mapping are independent to each other in general.

Example 4.11: Let X = {a,b} and Y = {u,v}. Then τ = {$0_\sim$, A, $1_\sim$} and σ = {$0_\sim$, B, $1_\sim$} are bipolar vague topologies on X and Y respectively, where A = ⟨x, [0.1, 0.1] [-0.5, -0.4], [0.7, 0.1] [-0.4, -0.3]⟩ and B = ⟨y, [0.2, 0.2] [-0.5, -0.5], [0.9, 0.1] [-0.5, -0.5]⟩. Define a mapping f : (X, BV_τ) → (Y, BV_σ) by f(a) = u and f(b) = v. Then f is a bipolar vague almost contra α generalised continuous mapping but not a bipolar vague contra pre continuous mapping. For, consider the bipolar vague set B = ⟨y, [0.2, 0.2] [-0.5, -0.5], [0.9, 0.1] [-0.5, -0.5]⟩ which is a bipolar vague regular-open set in Y as BVInt(BVCl(B)) = BVInt(B^c) = B. We have f^{-1} (B) = ⟨x, [0.2, 0.2] [-0.5, -0.5], [0.9, 0.1] [-0.5, -0.5]⟩ is a bipolar vague α generalised closed set in (X, BV_τ) as f^{-1} (B) ⊆ $1_\sim$ and BV_αCl(f^{-1} (B)) = f^{-1} (B) ∪ BVCl(BVInt(BVCl(f^{-1} (B)))) = A^c ⊆ $1_\sim$.Therefore, f is a bipolar vague almost contra α generalised continuous mapping. Now consider the bipolar vague open set B = ⟨y, [0.2, 0.2] [-0.5, -0.5], [0.9, 0.1] [-0.5, -0.5]⟩ in Y, where f^{-1} (B) is not a bipolar vague pre-closed set in X as BVCl(BVInt((f^{-1} (B))) = A^c ≠ f^{-1} (B). Therefore f is not a bipolar vague contra pre continuous mapping.

Example 4.12: Let X = {a,b} and Y = {u,v}. Then τ = {$0_\sim$, A, $1_\sim$} and σ = {$0_\sim$, B, $1_\sim$} are bipolar vague topologies on X and Y respectively, where A = ⟨x, [0.3, 0.4] [-0.6, -0.4], [0.5, 0.5] [-0.5, -0.5]⟩ and B = ⟨y, [0.3, 0.3] [-0.6, -0.4], [0.4, 0.3] [-0.5, -0.4]⟩. Define a mapping f : (X, BV_τ) → (Y, BV_σ) by f(a) = u and f(b) = v. Then f is a bipolar vague almost contra pre continuous mapping but not a bipolar vague almost contra α generalised continuous mapping. For, consider the bipolar vague set B = ⟨y, [0.3, 0.3] [-0.6, -0.4], [0.4, 0.3] [-0.5, -0.4]⟩ which is a bipolar vague regular-open set in Y as BVInt(BVCl(B)) = BVInt(B^c) = B. We have f^{-1} (B) = ⟨x, [0.3, 0.3] [-0.6, -0.4], [0.4, 0.3] [-0.5, -0.4]⟩ is a bipolar vague α generalised closed set in (X, BV_τ) as f^{-1} (B) ⊆ A and BV_αCl(f^{-1} (B)) = f^{-1} (B) ∪ BVCl(BVInt(BVCl(f^{-1} (B)))) = A^c ⊈ A. Therefore, f is not a bipolar vague almost contra α generalised continuous mapping. Now consider the bipolar vague open set B = ⟨y, [0.3, 0.3] [-0.6, -0.4], [0.4, 0.3] [-0.5, -0.4]⟩ in Y, where f^{-1} (B) is a bipolar vague pre-closed set in X as BVCl(BVInt((f^{-1} (B))) =BVCl($0_\sim$) = $0_\sim$ ⊆ f^{-1} (B). Therefore f is a bipolar vague contra pre continuous mapping.

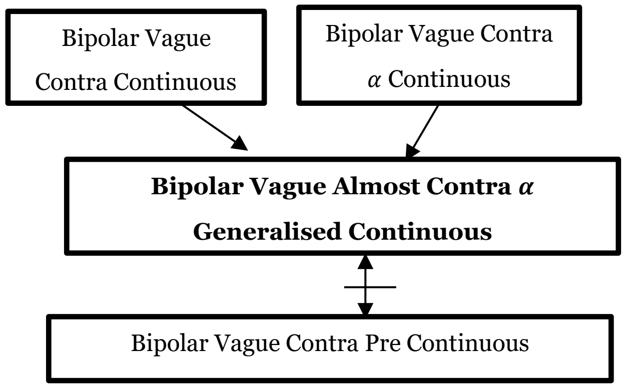

Figure 2. Bipolar Vague Almost Contra α Generalized Continuous Mappings

Proposition 4.13: Let f : (X, BV_τ) → (Y, BV_σ) be a bipolar vague almost contra α generalised continuous mapping, then f is a bipolar vague almost contra generalised continuous mapping, if X is a $BV_{\alpha b}T_{1/2}$ space.

Proof: Let A be a bipolar vague regular-open set in Y. Then f^{-1}(A) is a bipolar vague α generalised closed set in X, by hypothesis. Since X is a $BV_{\alpha b}T_{1/2}$ space, f^{-1}(A) is a bipolar vague generalised closed set in X. Hence f is a bipolar vague almost contra generalised continuous mapping.

Proposition 4.14: Let f : (X, BV_τ) → (Y, BV_σ) be a bipolar vague almost contra α generalised continuous mapping, then f is a bipolar vague almost contra continuous mapping, if X is a $BV_{\alpha a}T_{1/2}$ space.

Proof: Let A be a bipolar vague regular-open set in Y. Then f^{-1}(A) is a bipolar vague α generalised closed set in X, by hypothesis. Since X is a $BV_{\alpha a}T_{1/2}$ space, f^{-1}(A) is a bipolar vague closed set in X. Hence f is a bipolar vague almost contra continuous mapping.

Proposition 4.15: Let f : (X, BV_τ) → (Y, BV_σ) be a bipolar vague α generalised continuous mapping and g : (Y, BV_σ) → (Z, BV_δ) is a bipolar vague almost contra continuous mapping, then $g \circ f$: (X, BV_τ) → (Z, BV_δ) is a bipolar vague almost contra α generalised continuous mapping.

Proof: Let A be a bipolar vague regular-open set in Z. Then g^{-1}(A) is a bipolar vague closed set in Y, by hypothesis. Since f is a bipolar vague α generalised continuous mapping, f^{-1}(g^{-1}(A)) is a bipolar vague α generalised closed set in X. Hence $g \circ f$ is a bipolar vague almost contra α generalised continuous mapping.

V. BIPOLAR VAGUE COMPLETELY α GENERALISED CONTINUOUS MAPPINGS

In this sector we have presented bipolar vague completely α generalised continuous mappings and deliberate some of their properties.

Definition 5.1: A mapping f : (X, BV_τ) → (Y, BV_σ) is called a bipolar completely α generalised continuous mapping if f^{-1}(A) is a bipolar vague regular closed set in (X, BV_τ) for every bipolar vague α generalised closed set A of (Y, BV_σ).

Proposition 5.2: Every bipolar vague regular-closed set A is a bipolar vague α generalised closed set in X but not conversely in general.

Proof: Let A be a bipolar vague regular-closed set. Since every bipolar vague regular-closed set is a bipolar vague closed set. Then every bipolar vague closed set is a bipolar vague α generalised closed set in (X, BV_τ). Hence A is a bipolar vague α generalised closed set in X.

Example 5.3: Let X = {a,b} and τ = {0$_\sim$, A, B, 1$_\sim$} where A = ⟨x, [0.5, 0.6] [-0.6, -0.6], [0.6, 0.9] [-0.6, -0.5]⟩ and B = ⟨x, [0.4, 0.5] [-0.5, -0.5], [0.4, 0.6] [-0.5, -0.4]⟩. Then τ is a bipolar vague topology. Let M = ⟨x, [0.5, 0.6] [-0.5, -0.4], [0.4, 0.5] [-0.4, -0.3]⟩ be any bipolar vague set in X. Then M ⊆ A where A is a bipolar vague open set in X. Now BV_αCl(M) = M ∪ B^c = B^c ⊆ A. Therefore, M is a bipolar vague α generalised closed set in X but not a bipolar vague regular-closed set in X as BVCl(BVInt(M)) = 0$_\sim$ ≠ M.

Proposition 5.4: Every bipolar vague regular closed set A is a bipolar vague α-closed set in X but not conversely in general.

Proof: Let A be a bipolar vague regular-closed set. Since every bipolar vague regular-closed set is a bipolar vague closed set. Then every bipolar vague closed set is a bipolar vague α-closed set in (X, BV_τ). Hence A is a bipolar vague α-closed set in X.

Example 5.5: Let X = {a,b} and τ = {0$_\sim$, A, B, 1$_\sim$} where A = ⟨x, [0.4, 0.6] [-0.5, -0.5], [0.4, 0.3] [-0.5, -0.5]⟩ and B = ⟨x, [0.4, 0.6] [-0.5, -0.5], [0.7, 0.6] [-0.5, -0.5]⟩. Then τ is a bipolar vague topology. Let M = ⟨x, [0.5, 0.6] [-0.5, -0.5], [0.5, 0.5] [-0.5, -0.5]⟩ be any bipolar vague set in X. A is a bipolar vague open set in X. Now BV_αCl(M) = M ∪ B^c = B^c ⊆ M. Therefore, M is a bipolar vague α-closed set in X but not a bipolar vague regular-closed set in X as BVCl(BVInt(M)) = A^c≠ M.

Proposition 5.6: Every bipolar vague completely α generalised continuous mapping is a bipolar vague α generalised continuous mapping but not conversely in general.

Proof: Let f : (X, BV_τ) → (Y, BV_σ) be a bipolar vague completely α generalised continuous mapping. Let A be a bipolar vague closed set in Y. Since every bipolar vague closed set is a bipolar vague α generalised closed set, A is a bipolar vague α generalised closed set in Y. Then f^{-1}(A) is a bipolar vague regular-closed set in X. Since every bipolar vague regular-closed set is a bipolar vague α generalised closed set in X, f^{-1}(A) is a bipolar vague α generalised closed set in X. Hence f is a bipolar vague α generalised continuous mapping.

Example 5.7: Let X = {a,b} and Y = {u,v}. Then τ = {0$_\sim$, A, 1$_\sim$} and σ = {0$_\sim$, B, 1$_\sim$} are bipolar vague topologies on X and Y respectively, where A = ⟨x, [0.2, 0.3] [-0.5, -0.4], [0.4, 0.3] [-0.3, -0.2]⟩ and B = ⟨y, [0.4, 0.4] [-0.3, -0.2], [0.5, 0.6] [-0.3, -0.2]⟩. Define a mapping f : (X, BV_τ) → (Y, BV_σ) by f(a) = u and f(b) = v. Then f is a bipolar vague α generalised continuous mapping but not a bipolar vague completely α generalised continuous mapping. Since B^c is a bipolar α generalised closed set in Y, but f^{-1}(B^c) = ⟨x, [0.6, 0.6] [-0.8, -0.7], [0.4, 0.5] [-0.8, -0.7]⟩ is not a bipolar vague regular-closed set in X as BVCl(BVInt(f^{-1}(B^c))) = BVCl(A) = A^c ≠ f^{-1}(B^c).

Proposition 5.8: Every bipolar vague completely α generalised continuous mapping is a bipolar vague continuous mapping but not conversely in general.

Proof: Let f : (X, BV_τ) → (Y, BV_σ) be a bipolar vague completely α generalised continuous mapping. Let A be a bipolar vague closed set in Y. Since every bipolar vague closed set is a bipolar vague α generalised closed set, A is a bipolar vague α generalised closed set in Y. Then f^{-1}(A) is a bipolar vague regular-closed set in X. Since every bipolar vague regular-closed set is a bipolar vague closed set, f^{-1}(A) is a bipolar vague closed set in X. Hence f is a bipolar vague continuous mapping.

Example 5.9: Let X = {a,b} and Y = {u,v}. Then τ = {0$_\sim$, A, B 1$_\sim$} and σ = {0$_\sim$, C, 1$_\sim$} are bipolar vague topologies on X and Y respectively, where A = ⟨x, [0.5, 0.4] [-0.5, -0.4], [0.2, 0.1] [-0.3, -0.2]⟩, B = ⟨x, [0.2, 0.3] [-0.2, -0.1], [0.5, 0.5] [-0.5, -0.4]⟩ and C = ⟨y, [0.2, 0.3] [-0.2, -0.1], [0.5, 0.5] [-0.5, -0.4]⟩. Define a mapping f : (X, BV_τ) → (Y, BV_σ) by f(a) = u and f(b) = v. Then f is a bipolar vague continuous mapping but not a bipolar vague completely α generalised continuous mapping, Since C^c is a bipolar α generalised closed set in Y, but $f^{-1}(C^c)$ = ⟨x, [0.7, 0.8] [-0.9, -0.8], [0.5, 0.5] [-0.6, -0.5]⟩ is not a bipolar vague regular-closed set in X as BVCl(BVInt($f^{-1}(C^c)$)) = BVCl(A) = $A^c \neq f^{-1}(C^c)$.

Proposition 5.10: Every bipolar vague completely α generalised continuous mapping is a bipolar vague α continuous mapping but not conversely in general.

Proof: Let f : (X, BV_τ) → (Y, BV_σ) be a bipolar vague completely α generalised continuous mapping. Let A be a bipolar vague closed set in Y. Since every bipolar vague closed set is a bipolar vague α generalised closed set, A is a bipolar vague α generalised closed set in Y. Then f^{-1}(A) is a bipolar vague regular-closed set in X. Since every bipolar vague regular-closed set is a bipolar vague α-closed set, f^{-1}(A) is a bipolar vague α-closed set in X. Hence f is a bipolar vague α continuous mapping.

Example 5.11: Let X = {a,b} and Y = {u,v}. Then τ = {$0_\sim$, A, B $1_\sim$} and σ = {$0_\sim$, C, $1_\sim$} are bipolar vague topologies on X and Y respectively, where A = ⟨x, [0.5, 0.4] [-0.5, -0.4], [0.2, 0.1][-0.3, -0.2]⟩, B = ⟨x, [0.2, 0.3] [-0.2, -0.1], [0.5, 0.5] [-0.5, -0.4]⟩ and C = ⟨y, [0.2, 0.3] [-0.2, -0.1], [0.5, 0.5] [-0.5, -0.4]⟩. Define a mapping f : (X, BV_τ) → (Y, BV_σ) by f(a) = u and f(b) = v. Then f is a bipolar vague α continuous mapping but not a bipolar vague completely α generalised continuous mapping, Since C^c is a bipolar α generalised closed set in Y, but $f^{-1}(C^c)$ = ⟨x, [0.7, 0.8] [-0.9, -0.8], [0.5, 0.5] [-0.6, -0.5]⟩ is not a bipolar vague regular-closed set in X as BVCl(BVInt($f^{-1}(C^c)$)) = BVCl(A) = $A^c \neq f^{-1}(C^c)$.

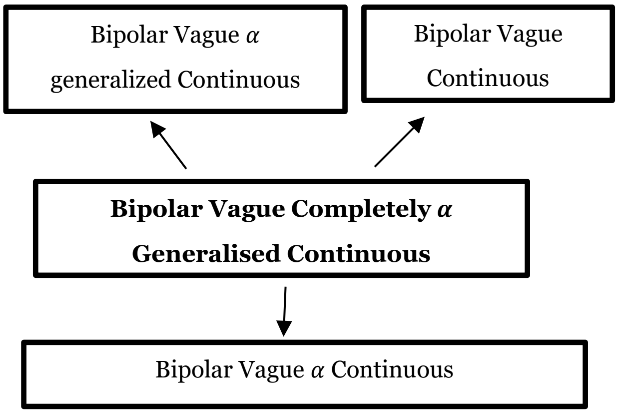

Figure 3. Bipolar Vague Completely α Generalised Continuous Mappings

Proposition 5.12: If f : (X, BV_τ) → (Y, BV_σ) is a bipolar vague completely α generalised continuous mapping then f is a bipolar vague α continuous mapping.

Proof: Let A be a bipolar vague closed set in Y. Since every closed set is a bipolar vague α generalised closed set, A is a bipolar vague α generalised closed set in Y. By hypothesis, f^{-1}(A) is a bipolar vague regular-closed set and hence f^{-1}(A) is a bipolar vague α-closed set in X. Then f is a bipolar vague α continuous mapping.

Proposition 5.13: A mapping f : (X, BV_τ) → (Y, BV_σ) is a bipolar vague completely α generalised continuous mapping if and only if the inverse image of each bipolar vague α generalised open set in Y is a bipolar vague regular-open set in X.

Proof: Necessity: Let A be a bipolar vague α generalised open set in Y. This implies A^c is a bipolar vague α generalised closed set in Y. Since f is a bipolar vague completely α generalised continuous mapping, $f^{-1}(A^c)$ is a bipolar vague regular-closed set in X. Since $f^{-1}(A^c) = (f^{-1}(A))^c$, f^{-1}(A) is a bipolar vague regular open set in X.

Sufficiency: Let A be a bipolar vague α generalised closed set in Y. This implies A^c is a bipolar vague α generalised open set in Y. By hypothesis, $f^{-1}(A^c)$ is a bipolar vague regular-open set in X. Since $f^{-1}(A^c) = (f^{-1}(A))^c$, f^{-1}(A) is a bipolar vague regular closed set in X. Hence f is a bipolar vague completely α generalised continuous mapping.

Proposition 5.14: Let f : (X, BV_τ) → (Y, BV_σ) be a bipolar vague completely α generalised continuous mapping and g : (Y, BV_σ) → (Z, BV_δ) is a bipolar vague α generalised irresolute mapping, then $g \circ f$: (X, BV_τ) → (Z, BV_δ) is a bipolar vague completely α generalised continuous mapping.

Proof: Let A be a bipolar vague α generalised open set in Z. Since g is a bipolar vague α generalised irresolute mapping, g^{-1}(A) is a bipolar vague α generalised open set in Y. Since f is a bipolar vague completely α generalised continuous mapping, f^{-1}(g^{-1}(A)) is a bipolar vague regular-open set in X. That is $(g \circ f)^{-1}$(A) = f^{-1}(g^{-1}(A)), $g \circ f$ is a bipolar vague completely α generalised continuous mapping.

Proposition 5.15: Let f : (X, BV_τ) → (Y, BV_σ) be a bipolar vague completely α generalised continuous mapping and g : (Y, B V_σ) → (Z, B V_δ) is a bipolar vague α generalised continuous mapping, then $g \circ f$: (X, BV_τ) → (Z, BV_δ) is a bipolar vague α generalised continuous mapping.

Proof: Let A be a bipolar vague α generalised open set in Z. Since g is a bipolar vague α generalised continuous mapping, g^{-1}(A) is a bipolar vague α generalised open set in Y. Since f is a bipolar vague completely α generalised continuous mapping, f^{-1} (g^{-1} (A)) is a bipolar vague regular-open set in X. Hence f^{-1}(g^{-1}(A)) is a bipolar vague α generalised open set in X as $(g \circ f)^{-1}$(A) = f^{-1}(g^{-1}(A)), $g \circ f$ is a bipolar vague α generalised continuous mapping.

Proposition 5.16: Let f : (X, BV_τ) → (Y, BV_σ) be a bipolar vague continuous mapping and g : (Y, BV_σ) → (Z, BV_δ) is a bipolar vague completely α generalised continuous mapping, then $g \circ f$: (X, BV_τ) → (Z, BV_δ) is a bipolar vague α generalised irresolute mapping.

Proof: Let A be a bipolar vague α generalised open set in Z. Since g is a bipolar vague completely α generalised continuous mapping, g^{-1}(A) is a bipolar vague regular-open set in Y. Since bipolar vague regular-open set is a bipolar vague open set, g^{-1}(A) is a bipolar vague open set in Y. Since f is a bipolar vague continuous mapping, f^{-1}(g^{-1}(A)) is a bipolar vague open set in X and hence f^{-1}(g^{-1}(A)) is a bipolar vague α generalised open set in X as $(g \circ f)^{-1}$(A) = f^{-1}(g^{-1}(A)). Then $g \circ f$ is a bipolar vague α generalised irresolute mapping.

VI. CONCLUSION

The outcomes of this paper are new opinions of bipolar vague almost α generalised continuous mapping, bipolar vague almost contra α generalised continuous mapping and bipolar vague completely α generalised continuous mapping have been considered with appropriate illustrations.

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