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On Some Characterisations of $\star^s-$ Fuzzy Subsemiring

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This paper aims to introduce and study the concept $\star^s-$ fuzzy subsemiring of a semiring. Given the idea of fuzzy subsemiring, we obtain the nature of the fundamental property $\star^s-$ fuzzy subsemiring and the property that states that every fuzzy subsemiring is $\star^s-$ fuzzy subsemiring, also we obtain the property that guarantees the existence of $\star^s-$ fuzzy subsemiring so that it can always be formed. In addition, we obtain the properties of two (more) intersections $\star^s-$ fuzzy subsemiring, and based on these intersection properties, we obtain a subsemiring generated by $\star^s-$ fuzzy subsemiring.

Keywords: semiring; fuzzy subset; fuzzy subsemiring; intersection

I. INTRODUCTION

Semiring is one of the generalisations of rings by eliminating the inverse axiom in the addition operation. Conditions for semirings that are not as stringent as rings are fascinating to study. It can be seen from the researchers who conducted research on semiring structures, including (Grillet, 1971; Goguadze, 2003; Bhuniya & Mondal, 2015; Vechtomov, Mikhalev, & Sidorov, 2019; Deldar, Ghalandarzadeh, & Namdari, 2022; Ganesh & Selvan, 2022; Vechtomov, 2022).

In addition, semiring research has been combined with other concepts, including the fuzzy set introduced by (Zadeh, 1965). The researchers who did this among them are (Kim & Park, 1996; Ghosh, 1998; Neggers, Jun & Kim, 1999; Akram & Dudek, 2008; Ahsan, Mordeson & Shabir, 2012a; Jagatap, 2014; Kar, Purkait & Shum, 2015; Massa'deh & Fellatah, 2019; Sardar, Goswami & Jun, 2019; Abdurrahman, 2020b, 2020a, 2021, 2023; Abdurrahman, Hira & Hanif Arif, 2022).

Based on the research facts above, we are motivated to research fuzzy semiring structures. In this research, we introduce a $\star^s-$ *fuzzy* subsemiring of a $\mathcal{K}$ semiring. In this research, we construct the definition and properties of the $\star^s-$ *fuzzy* subsemiring of the $\mathcal{K}$ semiring. The properties we studied included the properties associated with a subset of the semiring $\mathcal{K}$, which we associated with the membership value of the neutral element $0_{\mathcal{K}}$.

In addition, in this study, we will examine the characteristics of the slices of two (more) $\star^s-$ *fuzzy* subsemiring of the $\mathcal{K}$ semiring and a subset of $\mathcal{K}$ built from the results of these slices. Furthermore, the results of this study can be used as a basis for future researchers to carry out further research on the ideal fuzzy semiring of two (more) $\star^s-$ *fuzzy* subsemiring $\mathcal{K}$ associated with a homomorphism in the semiring $\mathcal{K}$.

II. MATERIALS AND METHOD

The research we did was a theoretical study. We studied the concepts of semirings, subsemirings, and fuzzy sets in the early stages. These concepts support understanding the idea of fuzzy semiring and fuzzy subsemiring. Furthermore, we define a $\star^s-$ *fuzzy* subsemiring. Next, we determine the assumptions in constructing the lemma or theorem so that a new lemma or theorem is formed. In the final step, using interrelated assumptions, we prove the lemma or theorem of the $\star^s-$ *fuzzy* subsemiring.

Here, we present the concepts that will support the discussion section, taken from (Zadeh, 1965; Dale, 1976;

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Golan, 1999, 2003; Guterman, 2009; Ahsan, Mordeson & Shabir, 2012a).

Definition 2.1. *A non-empty set $\mathcal{K}$ with two binary operations, addition " + " and multiplication " $\cdot$ ", such that it satisfies the condition:*

1. *$(\mathcal{K}, +)$ is an abelian semigroup with a neutral element of $\mathbf{0}_{\mathcal{K}}$.*
2. *$(\mathcal{K}, \cdot)$ is a semigroup with an identity element $\mathbf{1}_{\mathcal{K}}$.*
3. *The multiplication operation over the addition operation is left and right distributive.*
4. *$\boldsymbol{a} \cdot \mathbf{0}_{\mathcal{K}} = \mathbf{0}_{\mathcal{K}} \cdot \boldsymbol{a} = \mathbf{0}_{\mathcal{K}}$ For any $\boldsymbol{a} \in \mathcal{K}$.*

Next, a fuzzy subset is defined, an extension of the classical set concept.

Definition 2.2. A fuzzy subset of a non-empty set $\mathcal{A}$ is a function $\rho: \mathcal{A} \longrightarrow [0,1]$.

Definition 2.3. *Let ρ be a fuzzy subset of semiring $\mathcal{K}$. Then ρ is a fuzzy subsemiring of $\mathcal{K}$ if and only if*

$$\rho(a+z) \geq \rho(a) \wedge \rho(z) \text{ dan } \rho(a \cdot z) \geq \rho(a) \wedge \rho(z)$$

For any $a, z \in \mathcal{K}$.

III. RESULT AND DISCUSSION

Let ρ be a fuzzy subset of semiring $\mathcal{K}$ and $s \in [0,1]$. *We define the $\rho_{\star^s}$ the fuzzy subset of $\mathcal{K}$ such that*

$$\rho_{\star^s}(c) \stackrel{\text{def}}{=} (\rho(c) + s - 1) \vee 0$$

for any $c \in \mathcal{K}$. Furthermore, the ρ fuzzy subset of $\mathcal{K}$ is an $\star^s$*– fuzzy* subsemiring of $\mathcal{K}$ if and only if $\rho_{\star^s}$ It is a fuzzy subset of $\mathcal{K}$.

The following gives the properties of the $\star^s$*– fuzzy* subsemiring of the $\mathcal{K}$ semiring, induced from (Ghosh, 1998; Neggers, Jun & Kim, 1999; Ahsan, Mordeson & Shabir, 2012b; Abdurrahman, 2020b; 2020a; 2021; Abdurrahman, Hira & Hanif Arif, 2022).

Teorema 3.1. *Let ρ be a $\star^s$– fuzzy subsemiring of semiring $\mathcal{K}$. If $\rho_{\star^s}(0_{\mathcal{K}}) \geq \rho_{\star^s}(c)$ for any $c \in \mathcal{K}$, then*

$$\mathcal{K}_{\rho_{\star s}} \stackrel{\text{def}}{=} \{w \in \mathcal{K} \mid \rho_{\star^s}(w) = \rho_{\star^s}(0_{\mathcal{K}})\}$$

It is a subsemiring of $\mathcal{K}$.

Proof:

Based on the characteristics of membership $\mathcal{K}_{\rho_{\star s}}$, fulfilled $\mathcal{K}_{\rho_{\star s}} \subseteq \mathcal{K}$. Since $0_{\mathcal{K}} \in \mathcal{K}$, so $\rho_{\star^s}(0_{\mathcal{K}}) = \rho_{\star^s}(0_{\mathcal{K}})$. Thus $0_{\mathcal{K}} \in \mathcal{K}_{\rho_{\star s}}$. In other words, $\mathcal{K}_{\rho_{\star s}} \neq \emptyset$. Let $a, c \in \mathcal{K}_{\rho_{\star s}}$. Means $\rho_{\star^s}(a) = \rho_{\star^s}(0_{\mathcal{K}})$ and $\rho_{\star^s}(c) = \rho_{\star^s}(0_{\mathcal{K}})$. Because of $\rho_{\star^s}$ is a fuzzy subsemiring of $\mathcal{K}$, then

$$\rho_{\star^s}(c+d) \geq \rho_{\star^s}(c) \wedge \rho_{\star^s}(d) \geq \rho_{\star^s}(0_{\mathcal{K}}) \wedge \rho_{\star^s}(0_{\mathcal{K}}) = \rho_{\star^s}(0_{\mathcal{K}})$$

and

$$\rho_{\star^s}(cd) \geq \rho_{\star^s}(c) \wedge \rho_{\star^s}(d) \geq \rho_{\star^s}(0_{\mathcal{K}}) \wedge \rho_{\star^s}(0_{\mathcal{K}}) = \rho_{\star^s}(0_{\mathcal{K}}).$$

Thus,

$$c + d \in \mathcal{K}_{\rho_{\star s}} \text{ and } cd \in \mathcal{K}_{\rho_{\star s}}.$$

So, $\mathcal{K}_{\rho_{\star s}}$ It is a subsemiring of $\mathcal{K}$. ■

Furthermore, the $\star^s$*– fuzzy ρ* subsemiring of the $\mathcal{K}$ semiring referred to in this paper always fulfills the condition $\rho_{\star^s}(0_{\mathcal{K}}) \geq \rho_{\star^s}(c)$ For any $c \in \mathcal{K}$.

Theorem 3.2. *Let ρ and δ are $\star^s$– fuzzy subsemiring of semiring $\mathcal{K}$. If $\rho_{\star^s} \subseteq \delta_{\star^s}$ and $\rho_{\star^s}(0_{\mathcal{K}}) = \delta_{\star^s}(0_{\mathcal{K}})$ then $\mathcal{K}_{\rho_{\star s}} \subseteq \mathcal{K}_{\delta_{\star s}}$.*

Proof:

Let ρ and δ are $\star^s$– fuzzy subsemiring of semiring $\mathcal{K}$ where $\rho_{\star^s} \subseteq \delta_{\star^s}$ and $\rho_{\star^s}(0_{\mathcal{K}}) = \delta_{\star^s}(0_{\mathcal{K}})$. Let $c \in \mathcal{K}_{\rho_{\star s}}$, we have $\rho_{\star^s}(c) = \rho_{\star^s}(0_{\mathcal{K}})$. Thus

$$\delta_{\star^s}(c) \geq \rho_{\star^s}(c) = \rho_{\star^s}(0_{\mathcal{K}}) = \delta_{\star^s}(0_{\mathcal{K}}).$$

Therefore $c \in \mathcal{K}_{\delta_{\star s}}$ such that $\mathcal{K}_{\rho_{\star s}} \subseteq \mathcal{K}_{\delta_{\star s}}$. ■

Theorem 3.3 *Let H be a non-empty subset in the semiring $\mathcal{K}$ and $\rho_{H_{\star s}}$ Is a fuzzy subset of $\mathcal{K}$ defined by*

$$\rho_{H_{\star s}}(c) \stackrel{\text{def}}{=} \begin{cases} r, & c \in H \\ p, & c \in \mathcal{K} - H \end{cases}$$

for any $c \in \mathcal{K}$ and $r > p$. Fuzzy Subset ρ_H is a $\star^s$– fuzzy subsemiring of $\mathcal{K}$ *if and only if H is subsemiring* of $\mathcal{K}$. Moreover $\mathcal{K}_{\rho_{H_{\star s}}} = H$.

Proof:

Let ρ_H is a $\star^s$*– fuzzy* subsemiring of semiring $\mathcal{K}$. Then, for any $c, d \in H$ holds:

$$\rho_{H_{\star s}}(c+d) \geq \rho_{H_{\star s}}(c) \wedge \rho_{H_{\star s}}(d) = r$$

and

$$\rho_{H_{\star s}}(cd) \geq \rho_{H_{\star s}}(c) \wedge \rho_{H_{\star s}}(d) = r.$$

Since $Im\, \rho_{H_{\star s}} = \{r, p\}$ dan $r > p$. Then

$$\rho_{H_{\star s}}(c+d) = r \text{ and } \rho_{H_{\star s}}(cd) = r.$$

Thus,

$$c + d \in H \text{ and } cd \in H.$$

Therefore H is a subsemiring of $\mathcal{K}$. Conversely, assume that H is a subsemiring of $\mathcal{K}$. Let $a, c \in \mathcal{K}$. If $a, c \in H$, then $a + c \in H$ and $ac \in H$. We have

$$\rho_{H_{\star^s}}(a+c) = r = r \wedge r = \rho_{H_{\star^s}}(a) \wedge \rho_{H_{\star^s}}(c)$$

and

$$\rho_{H_{\star^s}}(ac) = r = r \wedge r = \rho_{H_{\star^s}}(a) \wedge \rho_{H_{\star^s}}(c).$$

If $a, c \notin H$ then

$$\rho_{H_{\star^s}}(a+c) \geq p = p \wedge p = \rho_{H_{\star^s}}(a) \wedge \rho_{H_{\star^s}}(c)$$

and

$$\rho_{H_{\star^s}}(ac) \geq p = p \wedge p = \rho_{H_{\star^s}}(a) \wedge \rho_{H_{\star^s}}(c).$$

If $a \in H$ or $c \notin H$ then

$$\rho_{H_{\star^s}}(a+c) \geq p = r \wedge p = \rho_{H_{\star^s}}(a) \wedge \rho_{H_{\star^s}}(c)$$

and

$$\rho_{H_{\star^s}}(ac) \geq p = r \wedge p = \rho_{H_{\star^s}}(a) \wedge \rho_{H_{\star^s}}(c).$$

Hence, based on the result of the analysis above, it is found that $\rho_{H_{\star^s}}$ It is a *fuzzy* subsemiring of $\mathcal{K}$. In other word, ρ_H is a $\star^s-$ *fuzzy* subsemiring of $\mathcal{K}$. Furthermore, since H is a subsemiring of $\mathcal{K}$ then H is a semiring such that $0_{\mathcal{K}} \in H$. Thus, $\rho_{H_{\star s}}(0_{\mathcal{K}}) = 1$. Moreover,

$$\begin{aligned}\mathcal{K}_{\rho_{H_{\star s}}} &= \{c \in \mathcal{K} \mid \rho_{H_{\star s}}(c) = \rho_{H_{\star s}}(0_{\mathcal{K}})\} \\ &= \{c \in \mathcal{K} \mid \rho_{H_{\star s}}(c) = r\} \\ &= H. \blacksquare\end{aligned}$$

Corollary 3.4. *Let ρ_H Be a characteristic function of the non-empty subset H of the semiring $\mathcal{K}$. The function ρ_H is a $\star^s-$fuzzy subsemiring of $\mathcal{K}$ if and only if H is a subsemiring of $\mathcal{K}$.*

Theorem 3.5 *Every fuzzy subsemiring of a semiring $\mathcal{K}$ is a $\star^s-$fuzzy subsemiring of K.*

Proof:

Suppose ρ is a fuzzy subsemiring of $\mathcal{K}$ and let $a, c \in \mathcal{K}$. Then

$$\begin{aligned}\rho_{\star^s}(a+c) &= (\rho(a+c) + s - 1) \vee 0 \\ &\geq \Big(\big(\rho(a) \wedge \rho(c)\big) + s - 1\Big) \vee 0 \\ &= \big((\rho(a) + s - 1) \wedge (\rho(c) + s - 1)\big) \vee 0 \\ &= \big((\rho(a) + s - 1) \vee 0\big) \wedge \big((\rho(c) + s - 1) \vee 0\big) \\ &= \rho_{\star^s}(a) \wedge \rho_{\star^s}(c)\end{aligned}$$

and

$$\begin{aligned}\rho_{\star^s}(ac) &= (\rho(ac) + s - 1) \vee 0 \\ &\geq \Big(\big(\rho(a) \wedge \rho(c)\big) + s - 1\Big) \vee 0 \\ &= \big((\rho(a) + s - 1) \wedge (\rho(c) + s - 1)\big) \vee 0 \\ &= \big((\rho(a) + s - 1) \vee 0\big) \wedge \big((\rho(c) + s - 1) \vee 0\big) \\ &= \rho_{\star^s}(a) \wedge \rho_{\star^s}(c).\end{aligned}$$

Hence $\rho_{\star^s}$ Is fuzzy subsemiring of $\mathcal{K}$ and so ρ is a $\star^s-$ fuzzy subsemiring *of K*. ■

The converse of theorem 3.5 is not always true. As an illustration, suppose $(\mathcal{K} = \mathbb{N} \cup \{0_{\mathbb{Z}}\}, +, \cdot)$ is semiring. Suppose ρ is a fuzzy subset of $\mathcal{K}$ where $\rho(2a) = 0{,}2$ and $\rho(2a+1) = 0{,}8$ for each $a \in \mathcal{K}$. Since there are $5{,}7 \in \mathcal{K}$ such that the terms of Definition 2.3 are not filled by ρ, i.e.:

$$\rho(5+7) = \rho(12) = 0{,}2 < 0{,}8 = \rho(5) \wedge \rho(7).$$

Therefore, the fuzzy subset ρ is not a fuzzy subsemiring of $\mathcal{K}$. Furthermore, taking $s = 0{,}1$, then for every $a \in \mathcal{K}$, we have $a = 2n$ or $a = 2n+1$ for some $n \in \mathcal{K}$. Therefore,

$$\rho_{\star^{0,1}}(a = 2n) = (0{,}2 + 0{,}1 - 1) \vee 0 = 0$$

or

$$\rho_{\star^{0,1}}(a = 2n+1) = (0{,}8 + 0{,}1 - 1) \vee 0 = 0$$

As a result, for every $a, c \in \mathcal{K}$ the following condition is obtained:

$$\rho_{\star^{0,1}}(a+c) \ = 0 = 0 \wedge 0 = \rho_{\star^{0,1}}(a) \wedge \rho_{\star^{0,1}}(c)$$

and

$$\rho_{\star^{0,1}}(ac) \ = 0 = 0 \wedge 0 = \rho_{\star^{0,1}}(a) \wedge \rho_{\star^{0,1}}(c).$$

Thus, $\rho_{\star^s}$ It is a fuzzy subsemiring of $\mathcal{K}$. In other words, ρ is a $\star^s-$*fuzzy* subsemiring of $\mathcal{K}$.

Furthermore, chosen $s = 0{,}9$ the following condition is obtained:

$$\rho_{\star^{0,9}}(a = 2n) = (0{,}2 + 0{,}9 - 1) \vee 0 = 0{,}1$$

or

$$\rho_{\star^{0,9}}(a = 2n+1) = (0{,}8 + 0{,}9 - 1) \vee 0 = 0{,}7.$$

Note that for $5{,}7 \in \mathcal{K}$ the following conditions are obtained:

$$\rho_{\star^{0,9}}(5+7) = \rho_{\star^{0,9}}(12) = 0{,}1 < 0{,}7 = \rho_{\star^{0,9}}(5) \wedge \rho_{\star^{0,6}}(7).$$

Therefore, $\rho_{\star^s}$ It is not a fuzzy subsemiring of $\mathcal{K}$. In other words, ρ is not a $\star^s-$*fuzzy* subsemiring of $\mathcal{K}$.

Based on the illustration of the above example, a theorem is obtained that guarantees the existence of ρ is always a $\star^s-$*fuzzy* subsemiring of $\mathcal{K}$, even though the condition ρ is only a fuzzy subset of $\mathcal{K}$.

Theorem 3.6 *Let ρ be a fuzzy subset of semiring $\mathcal{K}$ and $s \in [0,1]$. If $0 \leq \rho(c) + s \leq 1$ for any $c \in \mathcal{K}$ then ρ is a $\star^s-$fuzzy subsemiring of $\mathcal{K}$.*

Proof:

Since ρ is a fuzzy subset of semiring $\mathcal{K}$ and $s \in [0,1]$ where $0 \leq \rho(c) + s \leq 1$ *for any* $c \in \mathcal{K}$. Based on this condition, for every $c \in \mathcal{K}$, we have the following condition:

$$\rho_{\star^s}(c) = (\rho(a) + s - 1) \vee 0 = 0.$$

Therefore, for each $a, c \in \mathcal{K}$:

$$\rho_{\star^s}(a+c) = 0 = \rho_{\star^s}(a) \wedge \rho_{\star^s}(c)$$

and

$$\rho_{\star^s}(ac) = 0 = \rho_{\star^s}(a) \wedge \rho_{\star^s}(c).$$

Thus, $\rho_{\star^s}$ It is a fuzzy subsemiring of $\mathcal{K}$. In other word, ρ is a $\star^s-$*fuzzy* subsemiring of $\mathcal{K}$.■

Theorem 3.7 *The intersection of two $\star^s-$ fuzzy subsemirings of the semiring* of $\mathcal{K}$ *is a $\star^s-$ fuzzy subsemiring of $\mathcal{K}$.*

Proof:

Since σ and ξ are $\star^s-$*fuzzy* subsemiring of $\mathcal{K}$, it means that for any $a, c \in \mathcal{K}$, we have the following condition:

$(\sigma \cap \xi)_{\star^s}(a+c) = (\sigma \cap \xi(a+c) + s - 1) \vee 0$

$= \Big(\big(\sigma(a+c) \wedge \xi(a+c)\big) + s - 1\Big) \vee 0$

$= \big((\sigma(a+c) + s - 1) \wedge (\xi(a+c) + s - 1)\big) \vee 0$

$= \left(\left((\sigma(a+c) + s - 1) \vee 0 \right) \wedge \left((\xi(a+c) + s - 1) \vee 0 \right) \right)$
$= \sigma_{\star^s}(a+c) \wedge \xi_{\star^s}(a+c)$
$\geq (\sigma_{\star^s}(a) \wedge \sigma_{\star^s}(c)) \wedge (\xi_{\star^s}(a) \wedge \xi_{\star^s}(c))$
$= (\sigma_{\star^s}(a) \wedge \xi_{\star^s}(a)) \wedge (\sigma_{\star^s}(c) \wedge \xi_{\star^s}(c))$
$= (\sigma \cap \xi\ )_{\star^s}(a) \wedge (\sigma \cap \xi\ )_{\star^s}(c)$
and
$(\sigma \cap \xi\ )_{\star^s}(ac) = (\sigma \cap \xi(ac) + s - 1) \vee 0$
$= \left(\left(\sigma(ac) \wedge \xi(ac) \right) + s - 1 \right) \vee 0$
$= \left((\sigma(ac) + s - 1) \wedge (\xi(ac) + s - 1) \right) \vee 0$
$= \left(\left((\sigma(ac) + s - 1) \vee 0 \right) \wedge \left((\xi(ac) + s - 1) \vee 0 \right) \right)$
$= \sigma_{\star^s}(ac) \wedge \xi_{\star^s}(ac)$
$\geq (\sigma_{\star^s}(a) \wedge \sigma_{\star^s}(c)) \wedge (\xi_{\star^s}(a) \wedge \xi_{\star^s}(c))$
$= (\sigma_{\star^s}(a) \wedge \xi_{\star^s}(a)) \wedge (\sigma_{\star^s}(c) \wedge \xi_{\star^s}(c))$
$= (\sigma \cap \xi\ )_{\star^s}(a) \wedge (\sigma \cap \xi\ )_{\star^s}(c)$.

Therefore, $(\sigma \cap \xi\ )_{\star^s}$ It is a fuzzy subsemiring of $\mathcal{K}$. In the other word, $\sigma \cap \xi$ is $\star^s -$ *fuzzy* subsemiring of $\mathcal{K}$.∎

Based on the conditions of Theorem 3.1 and Theorem 3.7, it is obtained that the set $\mathcal{K}_{(\sigma \cap \xi\ )_{\star^s}}$ It is a subsemiring of $\mathcal{K}$, as presented in the following theorem.

Corollary 3.8 *Let* σ and ξ are $\star^s -$ *fuzzy subsemirings of the semiring* $\mathcal{K}$. *Then* $\mathcal{K}_{(\sigma \cap \xi\ )_{\star^s}}$ *It is a subsemiring of* $\mathcal{K}$.

Based on the conditions of Theorem 3.7, the intersection property of the finite $\star^s -$ *fuzzy* subsemiring of the semiring $\mathcal{K}$ is the $\star^s -$ *fuzzy* subsemiring of $\mathcal{K}$.

Corollary 3.9 *Let* $\sigma_1, \sigma_2, \cdots, \sigma_n$ and σ_n are $\star^s -$ *fuzzy semiring subsemiring of* $\mathcal{K}$. *Then* $\sigma_1 \cap \sigma_2 \cap \cdots \cap \sigma_n$ *is* $\star^s -$ *fuzzy semiring of* $\mathcal{K}$.

Analog to **Corollary 3.8**, we have that $\mathcal{K}_{(\sigma_1 \cap \sigma_2 \cap \cdots \cap \sigma_n\ )_{\star^s}}$ It is a subsemiring of $\mathcal{K}$.

Corollary 3.10. *Let* σ_1 , σ_2 , $\cdots$ *,and* σ_n *are* $\star^s -$ *fuzzy subsemiring of semiring* $\mathcal{K}$. *Then* $\mathcal{K}_{(\sigma_1 \cap \sigma_2 \cap \cdots \cap \sigma_n\ )_{\star^s}}$ *It is a subsemiring of* $\mathcal{K}$.

IV. CONCLUSION

Based on the results and discussion, it is obtained that the existence of ρ is a $\star^s -$ *fuzzy* subsemiring of semiring $\mathcal{K}$ determined by the condition of ρ, i.e.: (1) ρ is a fuzzy subsemiring of $\mathcal{K}$, or (2) ρ is a fuzzy subset where the condition $0 \leq \rho(c) \leq 1$ for every $c \in \mathcal{K}$. Furthermore, the intersection of $\star^s -$ *fuzzy* subsemiring $\sigma_1, \sigma_2, \cdots,$ and σ_n Of the semiring, $\mathcal{K}$ is the $\star^s -$ *fuzzy* subsemiring of $\mathcal{K}$ such that $\mathcal{K}_{(\sigma_1 \cap \sigma_2 \cap \cdots \cap \sigma_n\ )_{\star^s}}$ It is a subsemiring of $\mathcal{K}$.

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