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The Effect of Conversion Rate on Prey-predator Model Incorporating Disease in the Prey Population

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The spread of infectious diseases poses a threat to the interaction between prey and predator species. This research aims to understand how these diseases significantly influence both populations. By investigating the stability of the prey-predator system, the impact of varying the conversion rate is analysed through one-parameter bifurcation analysis. The conversion rate is specifically chosen as the bifurcation parameter, revealing the occurrence of transcritical bifurcation points. Visual representations, including bifurcation diagrams, phase planes, and time series plots, are generated using mathematical computing software such as XPPAUT, Maple, and MATLAB. The results demonstrate that variations in the conversion rate can induce shifts in stability, transitioning between stable and unstable states. This research emphasises the significance of considering the conversion rate when investigating the effects of disease on prey-predator populations. It can be useful for understanding the complex interactions between prey and predator species and for developing strategies to prevent the spread of infectious diseases among these populations.

Keywords: prey-predator model; prey disease; bifurcation analysis; stability analysis

I. INTRODUCTION

Interactions between two species and their impact on one another make up the prey-predator relationship. Both the prey and the predator engage in hunting and attacking behaviours. Predators hunt other creatures, while the prey are those whom other animal's attack. The predator's surroundings include prey, and the predator will die if it does not catch its prey, as the predator is entirely reliant on prey for survival. The prey-predator relationship maintains the earth's ecological balance because if predators are not present, the prey population increases, causing overgrazing and thus directly affecting the natural plant life cycle. If the prey population in an environment increases, predator numbers will increase to meet the increased food supply. The food supply will eventually be depleted to the point that the predator population will not be able to support itself. Disease can be spread among predators, leading to a strong Allee effect and cooperation (Hilker *et al.,* 2017). The disease prevalence in the prey population affects the prey-predator relationship since infected individuals become more vulnerable to predation (Banerjee *et al.*, 2017; Jang & Wei, 2020; Silva, 2017). Therefore, there has been a surge of interest in studying disease effects in prey-predator systems in recent decades.

II. LITERATURE REVIEW

Several recent studies have focused on prey-predator systems whereby an infection spreads amongst predator populations (Rihan & Rajivganthi, 2020; Huang *et al.,* 2019; Shaikh *et al.,* 2018). In a study conducted by Juneja and Agnihotri (2018), they investigated a prey-predator system where the predator species was afflicted with the disease. They concluded that diseased predators have a lower predation rate than healthy predators because they are less

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mobile. As a result, the lower predation rate of the diseased predator contributes to the system's disease-free condition. Banerjee *et al.* (2017), in contrast, investigated the prey-predator system with disease in prey, in which predators feed on both healthy and infected prey indiscriminately. Jang and Wei (2020) also analysed two prey-predator systems in which the disease in the prey population has no effect on population growth. More research on disease in prey populations can be found here (Wuhaib & Abd, 2020; Lu *et al.,* 2018; Saha & Samanta, 2020).

Recent research has focused on investigating various prey-predator systems where disease transmission occurs within both the prey and predator populations (Das, 2016; Kant & Kumar, 2017; Mandal *et al.,* 2018; Ghasemabadi & Rahmani Doust, 2021). Kant and Kumar (2017) conducted a study that considered a prey-predator system involving migrating prey and disease infection in both populations. Building upon the findings of Mandal *et al.* (2018), they further examined a predator-prey model featuring an infectious disease capable of spreading among both predators and prey, but not between them. Considering the fact that many diseases are not vertically transmitted, they assumed that the disease in the predator population was not genetic in nature. Additionally, Bera *et al.* (2015) explored the dynamic effects of a prey-predator species where disease impacts both the prey and predator populations.

The utilisation of bifurcation analysis in studying population interactions and infectious rates has been observed in various research studies (Manaf & Mohd, 2019; Manaf & Mohd, 2021; Kadhim & Majeed, 2022; Santra *et al.,* 2021; Suryanto *et al.,* 2018). However, the conversion rate of the prey-predator model has rarely been studied. The conversion rate parameters for both populations are therefore selected for investigation in this study to understand the influence of shifting predator populations on prey with infectious diseases. In conclusion, there is a growing interest in studying the prey-predator model with infectious disease to improve its reliability. Nevertheless, previous models have had limitations, and new research is being conducted to improve upon them. The impact of the conversion rate of prey and predator populations on disease spread is an area that requires further investigation.

III. METHODOLOGY

This section discusses mathematical modelling formulation and stability analysis. Bera *et al.* (2015) presented a prey-predator system with four differential equations, where both prey and predators are impacted by diseases. However, our research mainly focuses on the impact of diseases on prey populations, so we reduced the system from four to three differential equations by excluding the density of infected predators. The system is then represented as follows:

$$\begin{aligned}
\frac{dS}{dT} &= rS\left(1-\frac{S+I}{K}\right)-a_1SI-b_1SX \\
\frac{dI}{dT} &= -a_1SI-d_1I-f_1IX \\
\frac{dX}{dT} &= c_1SX+g_1IX-\delta_1X.
\end{aligned} \tag{1}$$

S, I, and X represent the densities of susceptible prey, infected prey, and susceptible predators at time T, respectively. All the parameters $r, K, a_1, b_1, c_1, d_1, f_1, g_1$ and δ_1 are considered as positive constants. The parameters r is the susceptible prey's growth rate; K is the carrying capacity of susceptible prey; a_1 is the infection rate of susceptible prey; b_1 is the rate of predation of susceptible prey by susceptible predators; d_1 is the disease-induced death which affects the infected prey; f_1 is the predation rates of infected prey by susceptible predators; c_1 is the conversion rate of susceptible prey to infected preys; g_1 is the conversion rate of infected prey to susceptible predators and δ_1 is the predator's death rate.

The analysis of the prey-predator system (1) is complicated due to the presence of multiple parameters. To simplify the analysis, a technique called non-dimensionalisation is employed. In this technique, the parameters are transformed into dimensionless variables to effectively reduce the total number of parameters involved. By reducing the complexity of the system through this process, it becomes more manageable to study and analyse the stability of the prey-predator dynamics. Therefore, the non-dimensionalised system followed the scalar.

$$s=\frac{S}{K},i=\frac{I}{K},x=\frac{X}{K},t=rT.$$

Thus, the prey-predator system (1) forms the following:

$$\begin{aligned}\frac{ds}{dt}&=s(1-s-i)-asi-bsx\\ \frac{di}{dt}&=asi-di-fxi\\ \frac{dx}{dt}&=csx+gix-\delta x\end{aligned}\qquad(2)$$

where,

$$a=\frac{a_1K}{r},b=\frac{b_1K}{r},c=\frac{c_1K}{r},d=\frac{d_1K}{r},f=\frac{f_1K}{r},g=\frac{g_1K}{r},\delta=\frac{\delta_1K}{r}.$$

To calculate the steady states, the prey-predator system (2) is set equal to zero. Therefore, this prey-predator system can have a maximum of six steady states:

(i) The trivial steady state: $E_0=(0,0,0)$. The steady state represents the scenario where susceptible prey, infected prey, and susceptible predators all go extinct.

(ii) The susceptible-prey-free steady state: $E_1=\left(0,\frac{\delta}{g},-\frac{d}{f}\right)$. This steady state indicates the extinction of susceptible prey.

(iii) The disease-free steady state: $E_2=\left(\frac{\delta}{c},0,\frac{c-\delta}{bc}\right)$. This steady state implies that there is no disease in the population.

(iv) The axial steady state: $E_3=(1,0,0)$. This steady state indicates that susceptible prey survives in the absence of both disease and predators.

(v) The predator-free steady state: $E_4=\left(\frac{d}{a},\frac{a-d}{a(a+1)},0\right)$.

This steady state describes prey survival and the absence of predators.

(vi) The interior steady state: $E_5=\left(-\frac{af\delta-bdg-fg+f\delta}{abg-acf-cf+fg},\frac{ab\delta-bcd-cf+f\delta}{abg-acf-cf+fg},\frac{a^2\delta-acd-ag-cd+dg}{abg-acf-cf+fg}\right)$.

This steady state demonstrates the existence of all species.

Subsequently, each of these steady states needs to be classified by applying the stability analysis, and the results will be presented in the upcoming section. By using the Maple software, the Jacobian matrix is then formed and used to analyse the stability of these six steady states:

$$J_{(s,i,x)}=\begin{bmatrix}1-2s-i-ai-bx & s(-1-a) & -bs\\ ai & as-d-fx & -fi\\ cx & gx & cs+gi-h\end{bmatrix}.$$

In this research, the stability analysis of the system heavily relies on the Jacobian matrix. Its role is vital in determining the stability characteristics of the system. The stability assessment involves substituting the steady states of the system into the Jacobian matrix, which then calculates the eigenvalues associated with each steady state. The signs of these eigenvalues serve as indicators of the stability of the system. If all the eigenvalues are negative or less than zero, the system is considered to be asymptotically stable, meaning that small perturbations will eventually die out and the system will tend towards a particular state. On the other hand, if there is at least one positive eigenvalue, the system is deemed unstable, meaning that small perturbations will grow over time and the system will not tend towards a particular state. This analytical approach provides valuable understanding of the system's long-term dynamics and assists in predicting the stability of prey-predator populations when disease is present.

IV. RESULT AND DISCUSSION

This section illustrates the results of stability and numerical analysis for prey-predator system (2).

A. Stability Analysis

The stability analysis of the prey-predator system (2) is analysed in this section. Stability analysis is a crucial tool in studying dynamical systems, allowing us to understand how small disturbances affect the long-term behaviour of a system. An asymptotically stable node is an equilibrium point where trajectories converge towards it over time, indicating a predictable and robust behaviour. This occurs when the eigenvalues of the Jacobian matrix have negative real parts. On the other hand, an unstable node is an equilibrium point where trajectories move away from it over time, showing sensitivity to initial conditions. This happens when the eigenvalues have positive real parts. Understanding the stability properties of different equilibrium points, such as asymptotically stable nodes and unstable nodes, is crucial for predicting and analysing the behaviour of dynamical systems.

Table 1 lists the parameter values used in the system. The parameter values were mostly taken from Bera *et al.* (2015), except for one specific parameter d which represents the rate of disease-induced death in infected prey. To determine a suitable value for this parameter, a technique called parameter variation was employed using the numerical bifurcation software called XPPAUT. Through this process, a value of $d = 0.3$ was selected. This particular value allows for the tracking of bifurcation points and facilitates numerical bifurcation analysis in the next section.

Table 1. The Parameter Value Used in The Stability Analysis

Parameter	Definition	Value	Source
a	Infection rate of prey	0.4	(Bera et al., 2015)
b	Predation rate of susceptible prey by susceptible predators	1.0	
c	Conversion rate of susceptible prey to susceptible predators	0.1	
d	Disease-induced death in infected prey	0.3	Assumed
f	Predation rate of infected prey by susceptible predators	0.2	(Bera et al., 2015)
g	Conversion rate of infected prey to susceptible predators	0.15	
δ	The natural death rate of predator	0.2	

The stability analysis results shown in Table 2 provide information about the behaviour of a system over time. The table lists six different steady states, each with its own set of eigenvalues. The eigenvalues, which are mathematical representations of how a system changes over time, are essential in determining the stability of a steady state. Steady states E_0 and E_3, are considered unstable because their eigenvalues have conflicting signs, meaning that the system will not remain at a constant state, but will continue to change over time. On the other hand, steady state E_4 is asymptotically stable because all of its eigenvalues are negative real distinct roots. This means that the system will tend towards this steady state over time, and that any small fluctuations away from the steady state will eventually decay back towards it. Steady states E_1, E_2 and E_5, however, have negative populations, which are not considered biologically meaningful. This is because, to ensure biological accuracy, a minimum population size of zero was established for each species, meaning that steady states with negative populations are disregarded. In conclusion, the stability analysis results presented in Table 2 provide valuable insights into the behaviour of a system over time and help to determine which steady states are biologically meaningful and can be used to make predictions about the system's behaviour in the future.

Table 2. The Summary of Stability Analysis

Steady states	Eigenvalues	Results
$E_0=(0,0,0)$	$\lambda_1=1$ $\lambda_2=-2$ $\lambda_3=-0.3$	Unstable node
$E_1=(0,1.3333,-1.5)$	$\lambda_1=0.2449$ $\lambda_2=0.6333$ $\lambda_3=-0.2449$	Not biologically significant
$E_2=(2,0,-1)$	$\lambda_1=0.7$ $\lambda_2=-2.0954$ $\lambda_3=0.0954$	Not biologically significant
$E_3=(1,0,0)$	$\lambda_1=-0.1$ $\lambda_2=0.1$ $\lambda_3=-1$	Unstable node
$E_4=(0.75,0.1786,0)$	$\lambda_1=-0.0982$ $\lambda_2=-0.1188$ $\lambda_3=-0.6311$	Asymptotically stable node
$E_5=(0.3064,1.129,-0.8871)$	$\lambda_1=0.1057$ $\lambda_2=-0.2061+0.371i$ $\lambda_3=-0.2061+0.371i$	Not biologically significant

The phase plane for the stable steady state E_4 was plotted using MATLAB with five different initial values: $(4,0.3,0.3),(4,0.6,0.6),(4,0.9,0.9),(4,1.2,1.2)$ and $(4,1.5,1.5)$. As illustrated in Figure 1, the plot shows that all the solution trajectories converge towards the steady state E_4. This finding indicates that, in this steady state, the prey species are able to exist, but the predator species have become extinct.

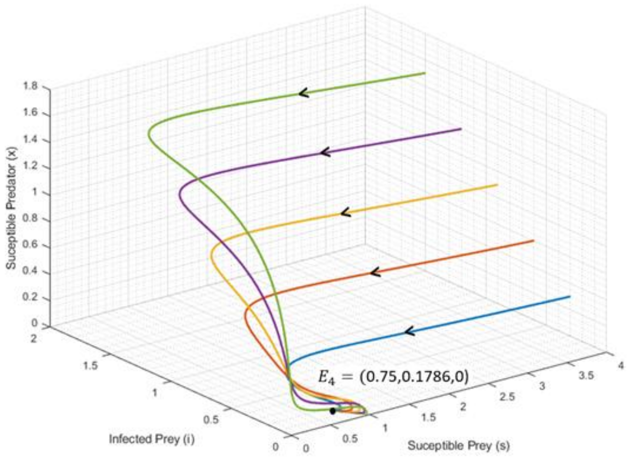


Figure 1. 3-D Phase Plane for Prey-Predator System (2)

B. Numerical Bifurcation Analysis

This section delves into the impact of converting infected prey into susceptible predators. The one-parameter bifurcation is the focus of this analysis, which employs the parameter variation technique with the assistance of the XPPAUT numerical tools. The conversion rate, g was selected as the bifurcation parameter to investigate the presence of transcritical bifurcation. The aim of this study is to understand the dynamics of the system and how it changes with the conversion rate, which could potentially provide valuable insights for future research in this field.

According to Strogatz (2018), a transcritical bifurcation occurs when one steady state "crosses" another, leading to a shift in the stability of the system. Figure 2 demonstrates the presence of transcritical bifurcation, where the Q_1 steady state branches interchange with each other after crossing the bifurcation point at $g = 0.7$. This is a result of a slight change in the conversion rate parameter of infected prey to susceptible predators, which impacted the stability and equilibrium of the system. As shown in the figure, the steady state Q_1 was originally an asymptotically stable node, but after the transcritical bifurcation event, it became an unstable node. The summary of stability and bifurcation analysis provided in Table 3 further supports the findings and helps to illustrate the dynamics of the system.

The phase plane provides a clear understanding of the behaviour of the dynamic. The diagrams illustrate the system's motion direction and the stability of the steady states. The direction field is represented by arrows, with the direction of the arrow indicating the direction of motion for the system. The representation of each value of the bifurcation parameter in the phase plane is presented in Figure 3. The different colours in the phase plane diagrams represent different initial values used in the analysis. In the case of an asymptotically stable node, as shown in Figure 3 (a), the direction field moves towards the steady state Q_1 , indicating that the system is converging towards that point. Conversely, for an unstable node, as depicted in Figure 3 (b), the direction field diverges from the steady state Q_1 , implying that the system is drifting away from that particular point.

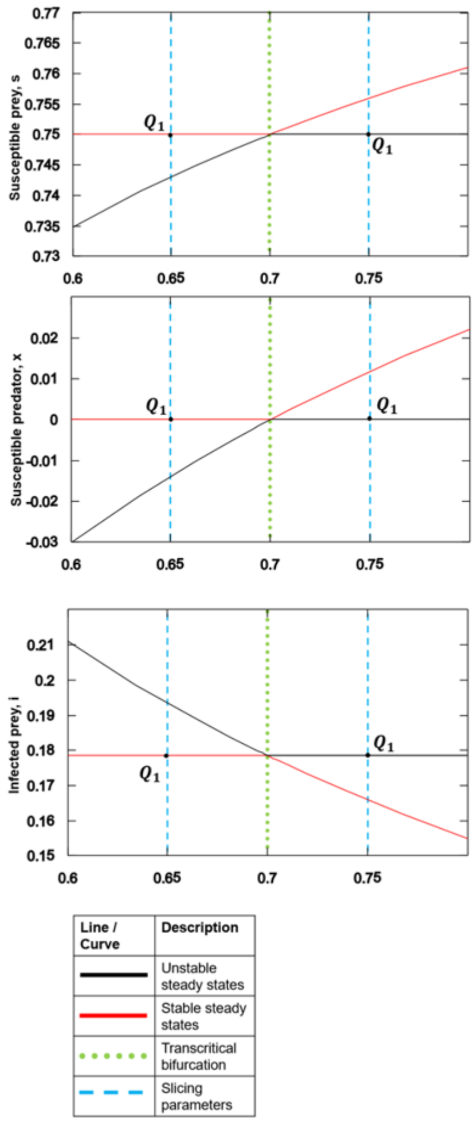
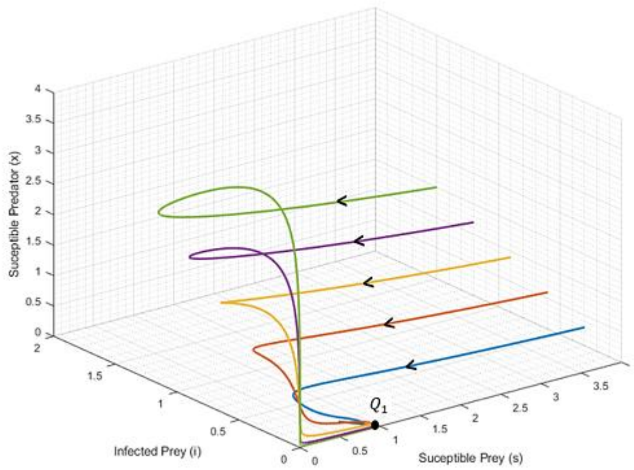


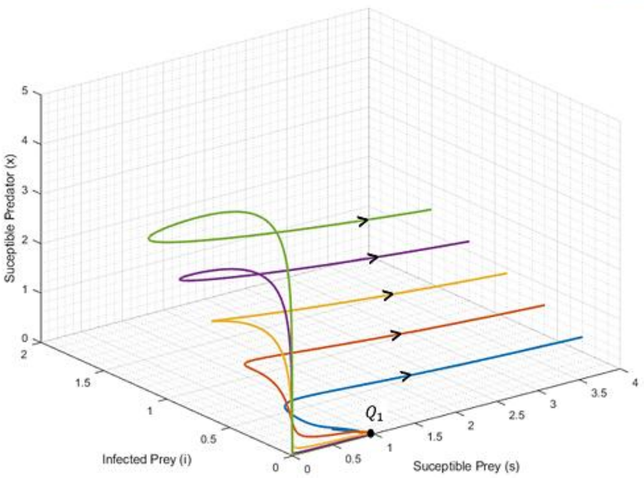
Figure 2. The slicing bifurcation diagram for bifurcation parameter, g

Table 3. Summary of Stability and Bifurcation Analysis

Parameter values	Critical Points	Eigenvalues	Results
$g = 0.65$	$Q_1 = (0.75, 0.1786, 0)$	$\lambda_1 = -0.0089$ $\lambda_2 = -0.1188$ $\lambda_3 = -0.6312$	Asymptotically stable node
$g = 0.70$	Transcritical bifurcation point		
$g = 0.75$	$Q_1 = (0.75, 0.1786, 0)$	$\lambda_1 = 0.0007$ $\lambda_2 = -0.1188$ $\lambda_3 = -0.6312$	Unstable node



(a) $g = 0.65$



(b) $g = 0.75$

Figure 3: 3-D phase plane for system (2) with different bifurcation parameters, g

C. Dynamics of Prey-predator Interactions

This section examines the system's dynamic behaviour over time by analysing the time-series graph created using MATLAB software. Figure 4 illustrates the interplay between the populations of susceptible prey, infected prey, and susceptible predators. The initial conditions for the population are $(4, 0.05, 0.05)$ and the conversion rate parameter, g, is set at 0.65. As time progresses and reaches 1000 days, the susceptible prey population stabilises at a steady state value, while the infected prey population experiences a slight increase before also reaching a stable state. Conversely, the susceptible predator population gradually decreases and becomes extinct. This behaviour is a result of the increasing disease rates among the susceptible prey population, which has a profound effect on the predator population. As the predator species share the same ecosystem with the infected prey, their numbers decline severely, highlighting the significant impact that infected prey can have on predator populations. Hence, the purpose of conducting this time series analysis is to investigate how the conversion rate influences the populations of prey and predators. By doing so, valuable insights into the dynamics of the system and its potential reactions to parameter variations can be obtained.

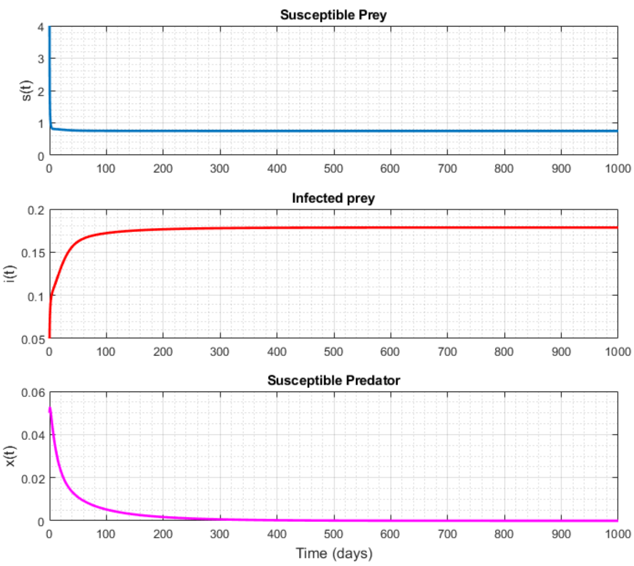


Figure 4: Time series graph of system (2) with conversion rate parameter $g = 0.65$

V. CONCLUSION

This research aimed to comprehensively investigate a prey-predator interactions considering the existence of disease within the prey population. The primary objective was to gain insights into the effects of conversion rates, specifically those involving susceptible predators and the transmission of disease among the prey, on the coexistence dynamics of both species. To achieve this objective, a one-parameter bifurcation analysis was employed, which is a mathematical tool used to study the effects of parameter changes on the behaviour of a system.

The findings revealed that different conversion rates have a meaningful influence on the behaviour of both species. The stability of the steady states changed as the bifurcation parameter crossed the transcritical bifurcation point. Moreover, it is found that as the bifurcation parameter g varied, the predators faced extinction as the prey populations struggled with the spread of disease. The density of susceptible predator species decreased dramatically, indicating the significant impact of infected prey on the predator population. This occurs as the infected preys cannot grow due to a lack of energy, which is essential for growth and survival.

The information gathered from this research can be used to better understand the behaviour of similar dynamic systems and how they may be influenced by changes in conversion rates. It could also provide valuable insights for future research in the field of transcritical bifurcations and their impact on dynamic systems. This study highlights the importance of conducting bifurcation analysis as a tool for understanding the behaviour of dynamic systems and their response to changes in parameters.

It is worth mentioning that the model presented in this study is limited to exploring the conversion rate's impact on where disease affects only the prey. Consequently, it is essential to conduct additional research on how conversion rates impact a prey-predator system where disease affects the predator population. Furthermore, future investigations should consider studying the prey-predator system incorporating disease in both populations to gain a more comprehensive understanding of how the disease influences the coexistence of both species.

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