

# Hermeneutics in Teaching and Learning Mathematics: Revitalising Mathematics Education

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The paradigm that mathematics is a subject to be taught and learned is not correct. Mathematics is to be produced. However, without hermeneutics, it would be difficult to do so. Unfortunately, hermeneutics was all that have been neglected in mathematics education. Meanwhile, only with hermeneutics, teaching and learning mathematics and also research in mathematics could flourish and be fruitful. Without it, it would be extremely hard for students to come up with a new creation in mathematics. This paper aims to shift the paradigm of mathematics education from learning (only) of mathematics into producing mathematics. Some examples are presented to illustrate the power of hermeneutics in developing mathematical imagination, ideas, and creativity towards mathematical discovery or invention.

**Key words:** Aristotelian logic; axiomatic system; fuzzy logic; mathematical language; mathematical logic; scientific knowledge

Teaching and learning are part of scientific activity. According to German mathematician Gottfried Leibniz, the latter consists of proving process (*méthode de la certitude*) and discovering process (*méthode d'inventer*). It is about the process of verification and the process of signification (Popper 2002). The second process signifies *hermeneus* in Greek or hermeneutics in English. Therefore, there is no discovery process without hermeneutics. This shows the importance of hermeneutics in doing scientific research, and in organising students' activity during teaching and learning process. As Jean Piaget pointed out, teaching and learning process is dedicated towards students' preparation to become future discoverers or inventors (Bringuier 1980). This includes teaching and learning mathematics.

In western culture, mathematics is something to be invented and produced. Thus, learning mathematics is necessary, but not enough, in order for someone to be able to produce it. Scientific production is the ultimate result of intellectual adventure including in mathematics (Hadamard 1945) and, at the same time, shows the power of human intellect. Under this viewpoint, "long life

learning" paradigm is good but it is not good enough. "Long life producing" is enough. In teaching and learning mathematics, the ultimate goal is producing mathematics. In this regards, it is then a must to teach the student of mathematics how to develop a mathematical idea. For this purpose, Dryden and Vos (1999) remarked that the idea of development is important and helpful. They pointed out that new idea is a new combination of old elements.

On the contrary, in eastern countries such as Indonesia and most probably Malaysia, mathematics is considered as a subject to be taught and not to be invented nor produced. Unfortunately, most people teach mathematics without knowing what mathematics is all about. They do not even know any single description of mathematics. What can we expect from someone who teaches mathematics without knowing what he/she teaches? Analogically, what can we expect from someone who teaches music without knowing music? Or, what can we expect from someone who teaches swimming without knowing how to swim? This is really a philosophical question. Therefore, the answer must be searched in the philosophical field (Popper 2002).

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Eight years of experience as the Head of Task Force for empowering Indonesian mathematics school teachers and more than six years in teaching mathematics and statistics at Malaysian universities has strengthened my conjecture that hermeneutics is all that has been neglected by teachers and lecturers (and therefore students) as well as researchers in the mathematical field. How can we incorporate hermeneutics in mathematics education? It is not simple to answer but it is not impossible at all. This paper is to illustrate the power of hermeneutics in developing mathematical creativity towards mathematical discovery or invention. For this purpose, teaching and learning à la Socrates is employed and some simple examples are provided.

In the next section I would begin the discussion on the basic notion of hermeneutics followed by the viewpoints of Fields Medallists on mathematics. Section 3 would be focused on teaching and learn à la Socrates with some examples that will clarify how to practice hermeneutics and illustrate the power of hermeneutics in teaching and learning mathematics in high school. The fourth section would illustrate the university level of mathematics education. In the fifth section, a recommendation to improve continuously the quality of teaching and learning process would be delivered. This paper will conclude with some remarks in Section 6.

## HERMENEUTICS AND MATHEMATICS

Hermeneutics is the science of meaning. It is also called the theory and methodology of text interpretation. Every word that we say or write has its own meaning and it reflects our soul or mind. Traditionally, hermeneutics is developed in the field of theology when scholars try to understand the divine words. Ibnu Rusyd (also known in the West as *Averroes*) from the East in the twelfth century and Thomas Aquinas from the West in the thirteenth century were the two renowned scholars in this field during the medieval era. So, what is a word or text? Aristotle has remarked that “*Words spoken are symbols or signs of affections or impressions of the soul; written words are the signs of words spoken.*” This remark means that the role of hermeneutics is to see the unseen (soul or mind) hidden behind a given word or text. The following simple example, in statistics, will clarify that role.

Let us ask a high school or even university students to explain the median of a data sample and all about it. According to conversation with students, it is believed that they would give the following answer: “*median is the number separating the higher half of data from the lower half.*” Thus, the median of {3, 3, 5, 9, 11} is 5 and that of {3, 5, 7, 9} is the average of the two middle values  $(5 + 7) / 2 = 6$ . It is nothing wrong with these examples. However, for those who were familiar with hermeneutics, would immediately realise that this notion of the median is not well defined; it is not applicable for all cases of data. Look at the case where data are vectors. What can we say about the median? The answer needs hermeneutics and, with a huge degrees of freedom, students are free to define.

Now, what is mathematics? Roundtable discussion of seven Fields Medallists in 1991 came up with the conclusion that there is no definition that can cover the wide aspects of mathematics (Casacuberta & Castellet 1991). In that book, the editors have remarked that mathematics is what mathematicians say that mathematics is. This remark reflects how difficult to construct an axiomatic system of mathematics. However, we can easily distinguish it from the other sciences particularly those included in normal science (Kuhn 1996). If, for example, biology, chemistry and physics are about the study of matter, mathematics has nothing to do with matter except when it is considered as a language. A detailed explanation is given by Alain Connes, a Fields Medallist in 1982 (see Casacuberta & Castellet 1991). He pointed out that mathematics has two aspects which make it difficult to explain and different from other sciences. The first aspect is that mathematics is, in many ways, used as a language in other sciences. More precisely, mathematics is reduced to a language in the sense that, if scientists use mathematics (say, in modelling), they do not use mathematics as mathematicians do.

Mathematics as a language is actually in accordance with what Galileo (Suzuki 2002) has remarked that “*The universe is written in mathematical language.*” For applied mathematicians and scientists, including mathematical scientists and statisticians, mathematics is the only language that can be used effectively to communicate with nature. In this regards, what we call mathematical formula is a tangible or visible result

of human intellect activity in expressing a natural phenomenon in the form of a mathematical sentence. If scientists just use mathematics as a language, mathematicians go beyond it. They go to the second aspect. In this aspect, mathematics has nothing to do with the matter. Mathematicians are also studying an aspect of reality (say, mathematical reality) which is not material. The theory of sets is an example of that study (Casacuberta & Castellet, 1991).

The two aspects, mathematics as a language and as the study of some kind of reality, make it different from other sciences. Nevertheless, according to our experience, the best way of teaching and learning mathematics at the very beginning is by using mathematics as a language. Then, slowly but surely, teachers introduce hermeneutics before going to work in the so-called mathematical reality. In this regards, Jean Piaget has pointed out that student can start learning, and thus can be taught, mathematics in the sense of the second aspect at age 15. But, remember that his research was conducted by using French students as respondent. How about Malaysian students? We need to conduct similar research as Piaget did.

Since it is difficult to define, scholars from Plato until Paul Valéry have tried to describe what mathematics is. Here are some of them. Plato said: "... *that the reality which scientific thought is seeking must be expressible in mathematical terms, mathematics being the most precise and definite kind of thinking of which we are capable.*" This description is the most fundamental among others which is reduced as a language by Galileo. In-line with the spirit of Galileo, Lord Kelvin indirectly related mathematics with knowledge. He remarked: "*When you can measure what you are speaking about and express it in numbers, you know something about it, but when you cannot express it in numbers, your knowledge is of a meagre and unsatisfactory kind.*" Another scholar, Indian writer Rabindranath Tagore, describes mathematics from philosophical point of view. He wrote the following beautiful words: "*We believe any fact to be true because of a harmony, a rhythm in reason, the process of which is analysable by the logic of mathematics.*" This statement is written as introductory words in the first edition of *Sankhya* in 1935. Among the famous descriptions, the most stunning one was addressed by Paul Valéry who lived in 1871–1945, a French poet, essayist, and

philosopher who was nominated for the Nobel Prize in Literature in 12 different years. He said: "*Aucune investigation humaine ne se peut appeler vraie science si elle ne passe pas par démonstration mathématique* (No human investigation can be called true science if it is not passing through mathematical demonstration)."

Therefore, it was not shocking when in 1977 we found that Partial Differential Equation was a subject for a degree programme at the Faculty of Literature, Université Paul Valéry, Montpellier, France. In France, mathematics is a way of life. Look at the statement of their Minister of Education in *Science* of June 1998 edition. He said: "*La France est la terre des mathématiciens* (France is the land of mathematicians)." There, we can hear student, teacher, and lecturer say that mathematics is the science of all possible worlds. In India, students consider mathematics as the mother of technology. What is our vision? Actually, whatever our vision is or our description of mathematics is, it reflects our soul, mindset and spirit.

### TEACHING AND LEARNING À LA SOCRATES

The method of Socrates (died in 399 BC) in teaching and learning is very simple. He went to meet young people (students) and then opened and developed their mind. For that purpose, once he met them, he conducted the following three activities. First, question and answer session. Second, he leads a discussion with and among students. Third, he manages the debate among students to prepare their skill in rhetoric. Here are five examples of question that we prepared to trigger high school students' mind towards their ability in hermeneutics.

1. Could you draw a circle?
2. Is carrot a member of the set of all green vegetables?
3. Is it true that two parallel lines never intersect to each other?
4. Is it true that the sum of all angles in a triangle =  $180^\circ$ ?; and
5. What is the distance between Kuala Lumpur and Johor Bahru?

To clarify these triggering questions, let us see one by one using the following four perspectives.

## Perspective 1: Circle

Most probably the students drew a circle as the circumference of a disk illustrated in Figure 1(a).

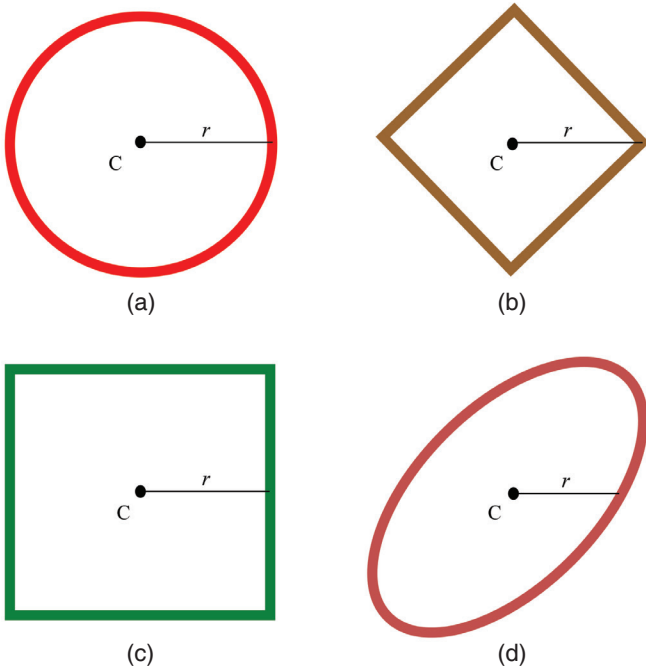


Figure 1. Four illustrations for a circle of radius  $r$  with centre  $C$ .

However, if we remind them on the definition of a circle, they could easily recognise that their drawing might not be true. By definition, a circle is a set of points having the same distance from a certain point  $C$  called centre. Once we fixed  $C$ , then a circle depended solely on what we meant by distance. So, what is a distance? Mathematically, it is a non-negative real-valued function which relates two objects (also called points in mathematics)  $A$  and  $B$  satisfying these three properties:

1. The distance between two points is always non-negative. It is 0 if and only if the two points coincide.
2. The distance from  $A$  to  $B$  is equal to that from  $B$  to  $A$ ; and
3. The distance from  $A$  to  $B$  does not exceed the sum of the distance from  $A$  to any point  $P$  and that from  $P$  to  $B$ .

According to this definition, there are infinitely many distances that we could define. One of them is the so-called Euclidean distance, say  $d$ , the distance between two points  $A(x_1, y_1)$  and  $B(x_2, y_2)$  are measured as:

$$d(A, B) = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} \quad (1)$$

It is based on this distance that Figure 1(a) was represented in the circle of radius  $r$  with centre  $C$ .

Now, imagine the points  $A$  and  $B$  represent two different houses in a housing complex as shown in Figure 2. This figure was cropped from Google Map (see Reference 8).



Figure 2. A housing complex.

In real daily activities, it is unrealistic to measure the distance between the two houses  $A$  and  $B$  using Euclidean distance  $d(A, B)$ . The realistic one is the sum of the distance between  $A$  and  $D$  and that between  $D$  and  $B$ . In mathematical sentence, we write:

$$\delta(A, B) = |x_1 - x_2| + |y_1 - y_2| \quad (2)$$

If we use this distance in the definition of a circle, then a circle of radius  $r$  with centre  $C$  is geometrically represented in Figure 1(b) and not Figure 1(a).

Another stunning form is when we measure the distance between two points  $A$  to  $B$  by using:

$$\partial(A, B) = \max\{|x_1 - x_2|, |y_1 - y_2|\} \quad (3)$$

the largest among  $|x_1 - x_2|$  and  $|y_1 - y_2|$ . In this case, the geometrical form of a circle of radius  $r$  with centre  $C$  is shown in Figure 1(c). This distance is usually used in computer programming, for example, when we stop the convergence of an iteration process. People call "square" the geometric form in Figure 1(c) but for mathematicians, it might represent a circle.

The last form in Figure 1(d) is also a circle of radius  $r$  with centre  $C$  if the distance between  $A$  and  $B$  is measured by using Mahalanobis distance:

$$\Delta(A, B) = (x - y)^t M^{-1}(x - y) \quad (4)$$



Here,  $M$  is a symmetric and positive definite matrix,  $M^{-1}$  is its inverse, and  $x$  and  $y$  are vector representations of the points  $A$  and  $B$ . The symbol " $t$ " refers to the transpose operator. This distance is usually used in statistics and in the manufacturing industry to monitor and control the quality of production process (Montgomery 2009; Masson & Young 2014).

The above simple example of distance and circle will open students' mind. They can see that every object in mathematics is subject to appropriate interpretation. At the same time, without speaking mathematics in terms of axiomatic system, that example will effectively introduce the students what is mathematics. The following three examples will clarify how our mental activity is working during the process of teaching and learning mathematics.

### Perspective 2: Aristotelian Logic

According to Aristotelian logic, the carrot is not a member of the set of green vegetables. However, if we practice the dialectic of Socrates, we will come up with a contradictory theory. Ask a little girl to collect some green vegetables from the refrigerator. Suppose she has collected asparagus, broccoli, spinach, green bean, fresh water salad, .... etc. Now, put those vegetables on the table and then ask her to show whether the collected vegetables have the same degree of greenness. She would be surprised and realises that the degrees of greenness were different. If we assign a number between 0 and 1 to represent the degree of greenness, then she could easily realise that carrot is a member of the set of green vegetables (with 0 degrees of greenness).

All that mathematical creativity starts with the hermeneutics of the word "member". This creativity leads to a new invention in mathematics which, since 1999, has become an indispensable tool in financial industry especially in econophysics. See, for example, Mantegna (1999), Mantegna and Stanley (2000), Djauhari (2012), Djauhari and Gan (2014), and Gan and Djauhari (2015).

### Perspective 3: Euclidean Geometry

"Is it true that two parallel lines never intersect to each other?" and "Is it true that the sum of all angles in a triangle =  $180^\circ$ ?" The answer to these questions is TRUE if we are working on a plane or, more generally, in Euclidean geometry. It becomes FALSE if we work on the surface of a ball (Non-Euclidean geometry), for example. In the surface of a ball, two lines perpendicular

to the "equator" will intersect each other at the "poles". Consequently, the sum of all angles in a triangle on that surface could be greater than  $180^\circ$ .

This example showed the vital importance of mindset development in the mathematical invention process. Actually, one of the most important jobs of teachers and lecturers is to open and then develop students' mind.

### Perspective 4: Distance from Kuala Lumpur to Johor Bahru

Regarding the last question, "What is the distance between Kuala Lumpur and Johor Bahru?", do not be surprised if the answer given by your students is in "kilometre" unit. Why? Because their mindset has been seemingly formatted (perhaps since they were at the primary school). According to our experience, each time we ask the students, no one even at degree level of the university has answered in other unit such as time unit. This showed that students were not well trained to think freely in the other way around. This is evident that teaching and learning process might become dangerous; once students' minds are formatted, their creativity is killed.

### TRIGGERING UNIVERSITY STUDENTS

We also prepared the following five examples of question that could trigger students' mind at the university level. The first three are common but the last two were difficult and needed special experience.

1. How do you measure the variability of data?
2. Given 10 vectors of 2 dimensions. How do you order those vectors?
3. Let  $M$  be a matrix of size  $(4 \times 4)$ . If the determinant of  $M$  is 27, what does this number mean?
4. If  $M$  is a non-singular matrix and  $N$  is its inverse, what is the meaning of each element in  $N$ ?
5. Look at the most famous determinantal inequality; a monumental work of Hadamard in 1893 where, until the present, it has hundreds of different proofs (Maz'ya & Shaposhnikova 1998). Could you find your own proof?

It is difficult to answer these questions if students do not understand the meaning or, in other words, the hermeneutics of the primitive terms such as "data variability" in the first question, "order", "determinant", "inverse", and again "determinant" in the second

until the last questions. When it was introduced, the determinantal inequality in the last question is nothing to do with the application. It was created, like many other mathematical creations, just to entertain the mind while exploring the beauty of mathematical reality. It is similar to classical music which was created to entertain the mood.

The message behind those questions is to make students at any level (degree, Masters or PhD) dare to criticise someone's mathematical work published in the book or journal paper. In this regards, they have to be willing to ask themselves about what they read and to be proactive to explore possible alternative solution of any problem, conduct their intellectual adventure, and discuss with lecturers. Lecturers are responsible for guiding students not just to be the user of mathematics but most importantly to be inventor or discoverer.

One of the great breakthrough in mathematics in the last century is when in 1960s Aristotelian logic was developed into fuzzy logic just by changing the *Law of Excluded Middle*. During 24 centuries, since Aristotle in the fourth century BC until Zadeh in the twentieth century, humans have been the witness of the success of Aristotelian logic in bringing the world into the present state of civilisation. Actually, the axiomatic system of Aristotelian logic is very simple. It consists of three laws:

1. Law of identity; every object is equal to itself.
2. Law of contradiction; impossible for two objects to be equal to each other and at the same time they differ to each other; and
3. Law of excluded middle; two objects are equal to each other or differ to each other.

In 1960s, the third law was developed into the following law:

3\*. Two objects  $x$  and  $y$  could be  $x = y$  or  $x = 0.9y$  or  $x = 0.75y$  or ... or  $x = 0.01y$  or  $x = 0$  ( $x \neq y$ )

This is a brilliant idea that changed the way we work in mathematics; from crisp logic, to fuzzy logic, as discussed in Perspective 2.

## RECOMMENDATION

Teaching is entertaining. This will ensure students' enjoyment during teaching and learning process. This is also a necessary condition to make sure that the students would have 100% freedom to use their soul

and mind and conduct their intellectual adventure. The 10 questions mentioned in the previous two sections were typical examples to illustrate the essence of teaching and learning mathematics; from question to idea development to innovation and, at the same time, to show the power of hermeneutics in doing mathematics.

Our predecessors taught us that the key success factor in doing mathematics, and science in general, is the ability to "break the rule" and develop a new one which is more advantageous. The existing rule that governs science and mathematics, usually represented by an axiomatic system, is subject to be developed or improved. Look, for example, at the history of:

1. Heliocentric theory of Copernicus which replaced Ptolemy's geocentric theory in astronomy,
2. Relativity theory of Einstein in physics,
3. Econophysics in Economy,
4. Functional analysis in mathematics,
5. Student  $t$ -distribution in statistics, and
6. Gondran algebra in data analysis, etc.

All those discoveries were manifested after scientists/mathematicians had broken the existing rule. It is important to note, especially for young students, that those pioneers have the same character. They were curious, the true observers, the brave explorers, love making the intellectual journey, sharp in looking for an alternative way, patient and full of passion. However, "break the rule" is still difficult to do without hermeneutics. To achieve that level of ability, we recommend to beginning the process of teaching and learning with "how to develop new idea". Remember that new idea is "a new combination of old elements". Moreover, when we conduct the process of teaching and learning, remember always what Jean Piaget pointed out about the meaning of education.

*"For me, education means making creators, even if there aren't many of them, even if the creations of one are limited by comparison with those of another. But, you have to make inventors, innovators, not conformists."* That is what Piaget remarked about education in Bringuier (1980). Education is synonym with making inventors and innovators. In accordance with Piaget vision, about innovator, Prime Minister of Malaysia, Mohd Najib Tun Abdul Razak, has remarked in the *New Straight Times*, 1 November, 2011, edition. In that edition, it was written: *"Malaysians must take a risk in efforts to become innovators, said Datuk Seri Najib Razak. The PM said an innovator would explore*

*new avenues rather than follow the tracks of others.*" PM Najib's message "Do not just be the follower of others" is in-line with Piaget vision about education. In this regards, mathematics education is to make creators, inventors, and innovators in the field of mathematics; not to make users of mathematics.

Actually, hermeneutics is not only needed by teachers, lecturers, researchers and students but also by all members of creative and innovative knowledge-based society. Therefore, a national campaign on hermeneutics needs support from government and society at large.

### CONCLUSION

Mathematics is not only a subject to be taught and learned. Most importantly, it is to be produced. However, without hermeneutics, it would be difficult to develop our own idea and produce mathematics. As hermeneutics is not separable from the history of inventions or discoveries, history of mathematics should be an integral part of teaching and learning mathematics. This was the best opportunity for students to learn from the pioneers how to develop a new idea and create something new. Only with hermeneutics, teaching and learning mathematics and also research in mathematics could be flourishing and fruitful. In a simple word, according to The Qur'anic terminology, without the ability to do *iqra* (which covers all aspects of hermeneutics) it is extremely hard for the students to come up with a new creation.

How to make hermeneutics a habit of the students? First, they could always be reminded to understand the meaning of every single word that they write or speak; they should be trained to be responsible for every word they use. Second, as mathematics deals with all possible worlds, a necessary condition that would enable students to develop their hermeneutical ability was to make sure that they have the freedom to use their soul and mind during their intellectual adventure. Third, students need conducive and comfortable environment. This would facilitate them in developing their imagination. In teaching and learning mathematics, students' imagination development is far more important than logic. If logic could bring them from one place to another, imagination could bring them everywhere. However, imagination is required to think freely.

As a general remark, the paradigm of teaching and learning mathematics must be shifted; stop only learning or only applying mathematics; should have to go beyond and start producing it.

### Recognition

The author would like to express his sincere gratitude to all teachers and lecturers who had guided him during his studentship. My admiration towards their noble deed is expressed in this poem entitled *Great Teacher* borrowed from *The Emperor's Club*, although I understand that it is inadequate.

### GREAT TEACHER

A great teacher has little external history to record... His life goes over into other lives... These men are pillars in the intimate structure of our schools... They are essential than its stones and beams... And they will continue to be a kindling force... And a revealing power in our lives

### ACKNOWLEDGEMENT

The author gratefully acknowledges the editor and the anonymous referees for their valuable comments and suggestions. Special thanks go to the Institute for Mathematical Research, University Putra Malaysia for providing the facilities to conduct this research.

*Date of Submission: 1 November 2015*

*Date of Acceptance: 21 April 2016*

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