ASM Sc. J., 12, Special Issue 1, 2019 for IQRAC2018, 108-116

Onset of Convection in a Dielectric Nanofluid Saturated Anisotropic Porous Medium

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The onset of thermal convection in a horizontal layer of a dielectric nanofluid saturated an anisotropic porous medium with vertical AC(alternate current) electric field has been studied. We considered Darcy model for porous medium while for nanofluid model used, it incorporates the effects of thermophoresis, electrophoresis and Brownian motion. A linear stability analysis based upon a normal mode has been performed, and the expression of thermal Rayleigh number is obtained using the Galerkin method. The results show that an increase value of AC electric Rayleigh number, Re and mechanical anisotropy parameter, ξ is to destabilize the system of nanofluid layer while the thermal anisotropy parameter, η has stabilizing effect on the onset of electroconvection.

Keywords: AC electric field, anisotropic, nanofluid, porous medium, Galerkin method.

I. Introduction

Electrothermal instability is a convection of dielectric fluid with the effect of the vertical AC electric field. Studies reveal the important role of electric force in driving the motion of poor electrically conducting fluid such as dielectric fluid. The thermal instability problems of fluid layer in the presence of applied electric field have been studied by Takashima and Aldridge (1976) and Stiles (1991). Many researchers attracted to work on electrothermal convection due to its growing applications in electrical equipments and electronic devices since heat transfer in high-voltage devices enhanced by applying electric force. Recently, Yadav et al. (2016) started to investigate electro thermal instability in the presence of nanoparticles in fluid layer.

The fluid is dispersed by the solid particles in dimension less than 100 nanometers known as nanofluids (Choi, 1995). Nanoparticles suspended stably in the base fluids (water, oil, bio-fluids) increase the thermal conductivity of the fluids thus it enhanced the heat-transfer characteristics. This fact is due to several factors such as the size of particles, volume fraction of particles and the thermal properties of fluid. The applications, of nanofluid went through the rapid development for over last few years due to the enhancement of heat transfer. Nanofluids have been used widely in various applications i.e. cooling system in electronic devices, automotive, and cooling industrial. The detail of characteristics and applications of nanofluid has been explained clearly in a few review papers (Wong and De Leon, 2010; Yu and Xie, 2012; Mukherjee and Paria, 2013).

The combined effects of thermophoresis and Brownian motion are considered in Buongiorno's model of nanofluids (Buongiorno, 2006). Tzou (2008) applied this model to investigate the thermal convection problem in nanofluid layer, and he noted that the stability of nanofluids is less compared to regular fluids. A lot of literatures employed Buongiorno's model in solving the problem of thermal instability (Kuznetsov and Nield, 2010; Yadav et al., 2011; Yadav et al., 2012).

Owing to the contribution of heat transfer enhancement, dielectric nanofluid has attracted attention of researchers to scrutinize its thermal stability. Yadav et al. (2016) have considered the nanoparticles in dielectric fluid to examine the effect of electric field on the onset of convection. The problem has been extendend by Wakif et al. (2016) by considering the effect of Coriolis force on the stability of the nanofluid system. Very recently, Yadav (2017) investigated the influence of AC electric field on the thermal instability of dielectric nanofluid porous layer with internal heating.

Porous media are often anisotropic since anisotropy effect is a consequence of orientation of asymmetric grains making up the solid matrix of porous media. Anisotropy is defined as the property of exhibiting the different values of permeability along different direction. For example, the permeability of porous media is an anisotropy when it differs with the variation of pressure or the direction of fluid flow. Anisotropy effect has been treated on the onset of convective instablity in nanofluid by Chand et al. (2013) as well as Shivakumara and Dhananjaya (2015). They found that the mechanical anisotropy accelerates the onset of thermal instability whereas the effect of thermal anisotropy is to delay. More recently, Swamy (2017) considered the effect of anisotropy on the electrothermal instability in a dielectric fluid saturated porous medium under the influence of thermal modulation.

As far as we concern, there is no such study consider the effect of anisotropy on electroconvection in a dielectric nanofluid since latest literature by Swamy (2017) considers the dielectric fluid. Such study seems to be significance in geological context since the strafication of sedimentary rocks are anisotropy; vertical permeability being much less than horizontal permeability. This motivates us to scrutinize the effect of anisotropy on the electro thermal instability in a porous medium saturated by nanofluid in the presence of a vertical AC electric field.

II. Methodology

A. Mathematical Formulation

An infinite horizontal of incompressible dielectric nanofluid of thickness,d with a vertical uniform AC electric field applied across the anisotropic porous layer heated from below is considered. The temperatures are denoted T_0 at the lower boundary and T_1 at the upper boundary where ($T_0 > T_1$). The boundary values nanoparticle volume fractions are ϕ_0 and ϕ_1 at $z = 0$ and $z = d$, respectively. We chose a Cartesian coordinate system (x, y, z) where z axis is vertically upward. The gravitational force $\mathbf{g}$ is acting in the negative vertical z-direction. According to Chand et al. (2014) and Yadav et al. (2016), the governing equations including conservation of mass, momentum, nanoparticles and energy were given as follows:

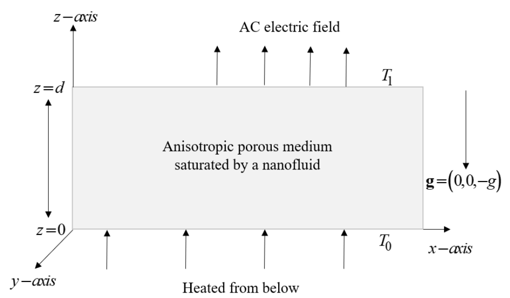

Figure 1:

$$\nabla \cdot \mathbf{q} = 0 \tag{1}$$

$$\nabla P + \mu K^{-1}\mathbf{q} = \big[\phi\rho_p + \rho_0(1-\phi) \times \big\{1 - \beta(T - T_1)\big\}\big]\mathbf{g} + \mathbf{f_e} \tag{2}$$

$$\left[\frac{\partial}{\partial t} + \frac{1}{\varepsilon}(\mathbf{q}\cdot\nabla)\right]\phi = D_B\nabla^2\phi + \frac{D_T}{T_0}\nabla^2 T \tag{3}$$

$$\left[(\rho c)_m \frac{\partial}{\partial t} + (\rho c)(\mathbf{q}\cdot\nabla)\right]T = k_m \nabla^2 T + \varepsilon(\rho c)_p \quad (4)$$
$$\nabla T \cdot \left[D_B \nabla\phi + \frac{D_T}{T_0}\nabla T\right]$$

where $\nabla = \frac{\partial}{\partial u}\hat{i} + \frac{\partial}{\partial v}\hat{j} + \frac{\partial}{\partial w}\hat{k}$, $\mathbf{q} = (u, v, w)$ is the velocity vector, P is the pressure, ρ_p is the nanoparticle density, ρ_0 is the nanofluid density at the lower boundary, ρ_p is the porous medium density, ϕ is the nanoparticle volume fraction, T is the temperature, $\mathbf{g}$ is the gravitational force, $\mathbf{f_e}$ is the force of electrical origin, μ is the viscosity, ε is the porosity of the porous medium, β is the coefficient of thermal expansion, c is the specific heat and c_p is the specific heat of nanoparticles. The other quantities are the thermophoretic diffusion coefficient, D_T and the Brownian diffusion coefficient D_B. The anisotropic permeability tensor and thermal conductivity tensor are denoted by K and k_m, respectively such that:

$$K^{-1} = K_x^{-1}(\hat{i}\hat{i} + \hat{j}\hat{j}) + K_z^{-1}(\hat{k}\hat{k}) \qquad (5)$$

$$k_m = k_{mx}^{-1}(\hat{i}\hat{i} + \hat{j}\hat{j}) + k_{mz}^{-1}(\hat{k}\hat{k}) \qquad (6)$$

where K_x and K_z are the permeability, k_{mx} and k_{mz} are the thermal conductivity in the x and z direction, respectively. Obviously, mechanical and thermal anisotropy have been assumed. $\vec{\mathbf{f}}_e$ is the force of electrical origin which can be stated as:

$$\mathbf{f}_e = \rho_e \mathbf{E} - \frac{1}{2}(\mathbf{E}\cdot\mathbf{E})\nabla\epsilon + \frac{1}{2}\nabla\left(\rho\frac{\partial\varepsilon}{\partial p}\mathbf{E}\cdot\mathbf{E}\right), \quad (7)$$

where $\mathbf{E}$ is the root mean square value of electric field, ρ_e is the charge density and ϵ is the dielectric constant. Since free charge density is infinitesimally small , the Maxwell equations are

$$\nabla \times \mathbf{E} = 0; \qquad (8)$$
$$\nabla \cdot (\epsilon\, \mathbf{E}) = 0, \qquad (9)$$

Employing equation (8), the electric potential can be represented in equation below

$$\mathbf{E} = -\nabla\psi, \qquad (10)$$

where ψ is the root mean square value of the electric potential. The electrical conductivity is considered to vary linearly function with temperature in the form

$$\epsilon = \epsilon_0\left[1 - \delta(T - T_1)\right] = 0, \qquad (11)$$

where $\delta(> 0)$ is the thermal coefficient of electrical conductivity.

Assuming that the temperature is fixed and there is no vertical nanoparticle flux on the boundaries and the boundary conditions can be written under the Darcy model at the lower boundary at $z = 0$

$$w = 0, T = T_0, D_B\frac{\partial\phi}{\partial z} + \frac{D_T}{T_1}\frac{\partial T}{\partial z} = 0 \qquad (12)$$

and at the upper boundary $z = d$,

$$w = 0, T = T_1, D_B\frac{\partial\phi}{dz} + \frac{D_T}{T_1}\frac{\partial T}{\partial z} = 0 \qquad (13)$$

The basic state is taken to be quiescent layer;

$$\vec{\mathbf{q}} = 0, \quad p = p_b(z), \quad T = T_b(z), \quad \phi = \phi_b(z),$$
$$\epsilon = \epsilon_b(z), \quad E = E_b(z), \quad \psi = \psi_b(z), \qquad (14)$$

basic state is denoted by the subscript b. The solutions of basic state are given by

$$T_b = 1 - z, \qquad \phi_b = \phi_0 + N_A z,$$
$$E_b = \frac{E_0}{1 + \delta(T_0 - T_1)z/d},$$
$$\psi_b(z) = -\frac{E_0 d}{\delta(T_0 - T_1)}\log\left(1 + \frac{\delta(T_0 - T_1)}{d}\right),$$
$$\epsilon_b = \epsilon_0\left(1 + \frac{\delta(T_0 - T_1)}{d}z\right). \qquad (15)$$

where ϕ_0 is a reference nanoparticle volume fraction and $E_0 = -\frac{\psi_1\delta(T_0 - T_1)/d}{\log(1 + \delta(T_0 - T_1))}$ is the root mean square value of the electric field at $z = 0$.

The basic solution is slightly being purturbed in the form

$$\vec{\mathbf{q}} = \vec{\mathbf{q}}', \quad p = p_b(z) + p', \quad T = T_b(z) + T',$$
$$\phi = \phi_b(z) + \phi', \quad \epsilon = \epsilon_b + \epsilon',$$
$$\mathbf{E} = \mathbf{E}_b + \mathbf{E}', \quad \psi = \psi_b + \psi', \qquad (16)$$

By substituting Eq. (16) into Eqs. (1)-(4),(7)-(13), utilizing Eqs. (5)-(6), operating curl twice to eliminate the pressure from the momentum equation and retaining the vertical component. In order to non-dimentionalizing the governing equations, we scale variables as follows:

$$(u^*, v^*, w^*) = \left(\frac{d}{\kappa_z}\right)(u', v', w'),$$
$$(x^*, y^*, z^*) = \left(\frac{1}{d}\right)(x', y', z'),$$
$$t^* = \left(\frac{\kappa_z}{\sigma d^2}\right)t',$$
$$p^* = \left(\frac{K_z}{\mu \kappa_z}\right)p',$$
$$T^* = \frac{T' - T_1}{T_0 - T_1},$$
$$\phi^* = \frac{\phi' - \phi_0}{\phi_0},$$
$$\psi^* = \frac{\psi'}{\delta E_0 (T_0 - T_1) d}, \qquad (17)$$

where $\kappa_z = \dfrac{k_{mz}}{(\rho c)_0}$ and $\sigma = \dfrac{(\rho c)_m}{(\rho c)}$. We obtain the non dimesional form as follows (for simplicity, vanishing the asterisk $(*)$):

$$\left(\frac{1}{\xi}\frac{\partial}{\partial z} + \nabla_H^2\right)w - \big(Rt + Re\big)\nabla_H^2 T \qquad (18)$$
$$+Rn\nabla_H^2\phi - Re\nabla_H^2\frac{\partial \psi}{\partial z} = 0,$$
$$w + \left(\eta\nabla_H^2 - \frac{\partial}{\partial t} - \frac{N_A N_B}{Le}\frac{\partial}{\partial z}\right)T \qquad (19)$$
$$-\frac{N_B}{Le}\frac{\partial \phi}{\partial z} = 0,$$
$$\frac{N_A}{\varepsilon}w - \frac{N_A}{Le}\nabla^2 T + \left(\frac{1}{\sigma}\frac{\partial}{\partial z} - \frac{1}{Le}\nabla^2\right)\phi = 0 \qquad (20)$$
$$\frac{\partial T}{\partial z} - \nabla^2\psi = 0, \qquad (21)$$

where $\nabla_H^2 = \dfrac{\partial^2}{\partial x^2} + \dfrac{\partial^2}{\partial y^2}$ is the two-dimensional horizontal Laplacian operator. The dimensionless boundary conditions can be written as:

$$w = \frac{\partial \psi}{\partial z} = T = \frac{\partial \phi}{\partial z} + N_A\frac{\partial T}{\partial z} = 0 \qquad (22)$$

at $z = 0, 1$.

Where the dimesionless parameters are:
$Rt = \dfrac{\rho_0 g \beta (T_0 - T_1) d K_z}{\mu \kappa_z}$ is the thermal Rayleigh number,
$Rn = \dfrac{(\rho_p - \rho_0)\phi_0 g K d}{\mu \kappa_z}$ is the concentration nanoparticle Rayleigh number
$Re = \dfrac{\gamma^2 \epsilon E_0^2 (T_0 - T_1)^2 d^2}{\mu \kappa_z}$ is the AC electric field Rayleigh number,
$\xi = \dfrac{K_x}{K_z}$ is the mechanical anisotropy parameter,
$\eta = \dfrac{k_{mx}}{k_{mz}}$ is the thermal anisotropy parameter
$L_e = \dfrac{\kappa_z}{\varepsilon D_B}$ is the Lewis number,
$N_A = \dfrac{(T_0 - T_1)}{(\phi_1 - \phi_0)}\dfrac{D_T}{D_B T_1}$ is the modified diffusivity ratio,
$N_B = \varepsilon \phi_0 \dfrac{(\rho c)_p}{(\rho c)}$ is the modified diffusivity ratioparticle density increment.

B. Normal modes and stability analysis

According to normal mode analysis, the perturbation quantities are assumed in the form

$$\begin{bmatrix} w \\ T \\ \phi \\ \psi \end{bmatrix} = \begin{bmatrix} W(z) \\ \Theta(z) \\ \Phi(z) \\ \Psi(z) \end{bmatrix} \times e^{(ia_x x + ia_y y + nt)} \qquad (23)$$

where $a = \sqrt{a_x^2 + a_y^2}$ is the resultant dimensionless wave number (subscript x and y denote the direction in the plane).

Substituting Eq. (23) into Eqs (18)-(22) yields the following stability equations:

$$\left(\tfrac{1}{\xi}D^2 - a^2\right)W + R_t a^2\Theta - R_n a^2\Phi \quad (24)$$

$$+R_e a^2\left(\Theta - \tfrac{\partial\Psi}{\partial z}\right) = 0,$$

$$W + \left[D^2 - \eta a^2 - n + \tfrac{N_A N_B}{L_e}\right]\Theta \quad (25)$$

$$+\tfrac{N_B}{L_e}D\Phi = 0,$$

$$-\tfrac{N_A}{\varepsilon}W + \tfrac{N_A}{L_e}\left(D^2 - a^2\right)\Theta \quad (26)$$

$$+\left[\tfrac{1}{Le}\left(D^2 - a^2 - \tfrac{n}{\sigma}\right)\right]\Phi = 0,$$

$$\left(D^2 - a^2\right)\Psi - D\Theta = 0. \quad (27)$$

$$W = 0, \quad \Theta = 0, \quad (28)$$

$$N_A\ D\ \Theta + D\ \Phi = 0 \quad D\ \Psi = 0,$$

at $z = 0, 1$.

where $D = \dfrac{d}{dz}$.

Employing the Galerkin technique in order to get an analytical solution to the system of equations. The series of base (trial) functions are as follows:

$$W = \sum_{s=1}^{N} A_s W_s, \quad \Theta = \sum_{s=1}^{N} B_s \Theta_s,$$

$$\Phi = \sum_{s=1}^{N} C_s \Phi_s, \quad \Psi = \sum_{s=1}^{N} D_s \Psi_s, \quad (29)$$

where A_s, B_s, C_s and D_s are unknown coefficients such that $s = 1, 2, 3, \ldots, N$. We assume the solutions W, Θ, Φ and Ψ in the form of:

$$W_i = \sin(i\pi z)\ , \Theta_i = \sin(i\pi z)\ ,$$

$$\Phi_i = -N_A \sin(i\pi z)\ , \Psi_i = \cos(i\pi z)\ , \quad (30)$$

which satisfy the boundary conditions Eq.(28).

Using the Eq. (30) into equations (24-27) and multiplying the equation (24) by W_i, equation (25) by Θ_i, equation (26) by Φ_i and equation (27) by Ψ_i and then performing integration by parts in the limit from $z = 0$ to $z = 1$. This gives a system of linear algebraic equations in 4N unkwowns which admits a nontrivial solution only if its determinant is equal to zero. The matrix is as follows:

$$\begin{vmatrix} -\frac{1}{2}\left(a^2 + \frac{\pi^2}{\xi}\right), & \frac{1}{2}a^2\left(R_t + R_e\right), & \frac{1}{2}\left(a^2 N_A R_n\right), & \frac{1}{2}\left(a^2 \pi R_e\right) \\ \frac{1}{2}, & -\frac{1}{2}\left(\eta a^2 + \pi^2 + n\right), & 0, & 0 \\ \frac{1}{2}\left(\frac{N_A^2}{\varepsilon}\right), & \frac{1}{2}\left(\frac{N_A^2 J}{L_e}\right), & -\frac{1}{2}\left[\frac{N_A^2(n + J\sigma)}{L_e\sigma}\right], & 0 \\ 0, & -\frac{1}{2}\pi, & 0, & -\frac{1}{2}J \end{vmatrix} = 0 \quad (31)$$

Setting $n = i\omega$(complex growth rate), since the real part of n is zero for neutral stability (ω is real and is a dimensionless frequency). As we consider the case of stationary convection, n is equal to zero since $\omega = 0$. Thus, the thermal Rayleigh number Rt as the definition of the characteristics equation is given by equation below

$$R_t = \left(a^2 + \tfrac{\pi^2}{\xi}\right)\tfrac{(\eta a^2 + \pi^2)}{a^2} - \tfrac{a^2 R_e}{J} \quad (32)$$

$$-N_A Rn\left[\tfrac{(\eta a^2 + \pi^2)L_e}{\varepsilon J} + 1\right].$$

where $J = a^2 + \pi^2$. It is interesting to note that Eq. (32) is independent of modified particle density increment, N_B. It indicates nanoparticles flux has no significant effect on the conservation of thermal energy in nanofluid system.

III. Results and Discussion

The onset of electroconvection of the dielectric nanofluid layer is shown graphically in Figures 1-5 for several effects with fixed parameters such that $\xi = 0.8$, $\eta = 0.6$, $Le = 5$, $Rn = 0.5$, $Re = 10$, $N_A = 2$ and $\varepsilon = 0.7$.

Figure 2 illustrates the thermal Rayleigh number, Rt varies with a wave number, a for various value of Re. The graph shows an increasing of AC electric Rayleigh number Re is to decrease the stability region under the curve indicating that electric has a destabilizing effect upon the stability of the system. This fact implies the temperature varies under the consequence of electric field causing the variation of electric constant which induce destabilizing electrical energy to increase. Hence, the system of nanofluid is less stable. This finding is well agreed with the paper of Yadav et al. (2016) and Yadav (2017).

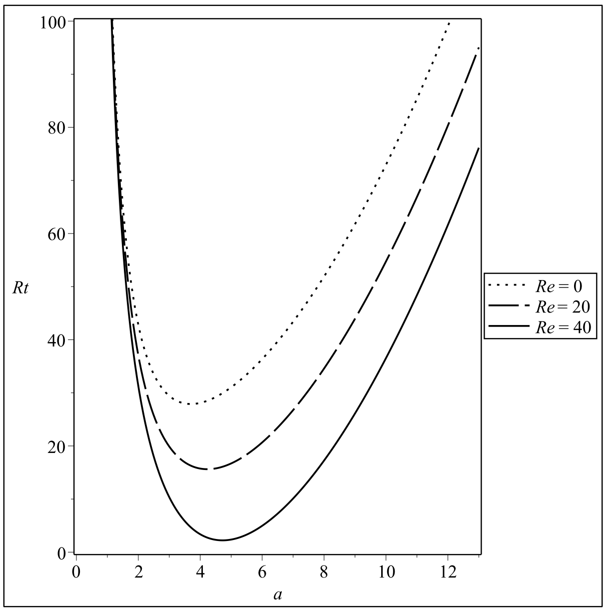

Figure 2: Rt vs a for different value of Re

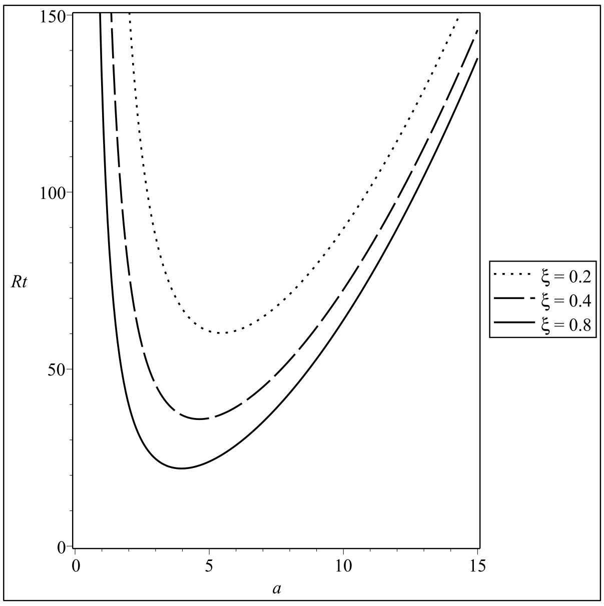

Figure 3: Rt vs a for different value of ξ.

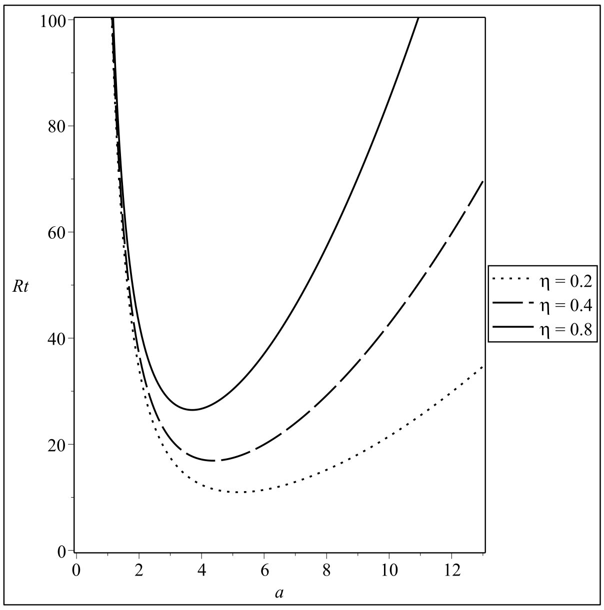

Figure 4: Rt vs a for different value of η.

The response of thermal Rayleigh number with different value of mechanical anisotropy parameter ξ on the onset of convection is shown in Figure 3. In Figure 3, the value of thermal Rayleigh number decrease with increasing value of mechanical anisotropy parameter ξ and thus accelerates the electro thermal instability. This result implies larger horizontal permeability which in turn facilitates the velocity of the fluid horizontally causing the conduction state in impermeable layer becomes less stable thus the system is destablized (Malashetty and Heera, 2008).

Figure 4 depicts the instability of the nanofluid is influenced by the effect of the thermal anisotropy. The graph shows the thermal Rayleigh number, Rt increase as the value of the thermal anisotropy parameter, η increase. This implies the thermal anisotropy effect inhibits the stationary convection.

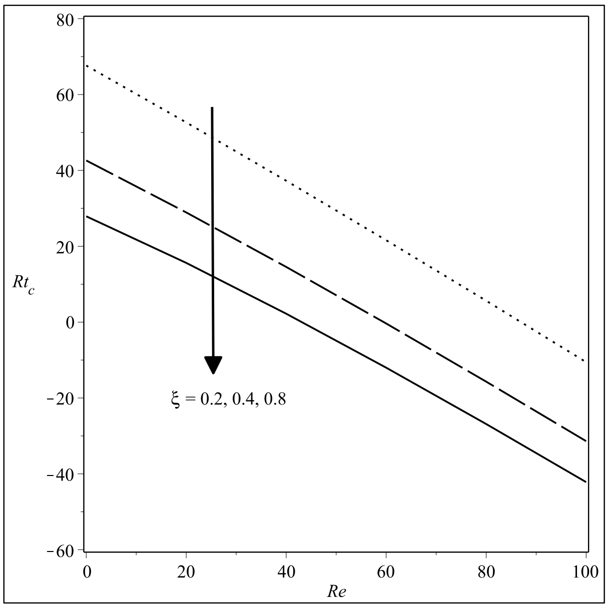

Figure 5: Rt_c vs Re for different value of ξ.

The influences of the mechanical anisotropy parameter ξ and thermal anisotropy parameter η on the convection of dielectric nanofluid are displayed in Figure 5 and 6, respectively. In Figure 5, it is observed Rt_c decreases with increasing ξ, thus it hastens the convection. However, the thermal anisotropy parameter seems to show contra effect in Figure 6 since Rt_c increases with increasing η. Therefore, the onset of convection in nanofluid is delayed. From the both graphs, it can be seen clearly that the elevating the AC electric Rayleigh number Re is to increase the thermal instability thus the nanofluid system is destabilized. These findings agree well with the results that we obtained in Figures 2-4.

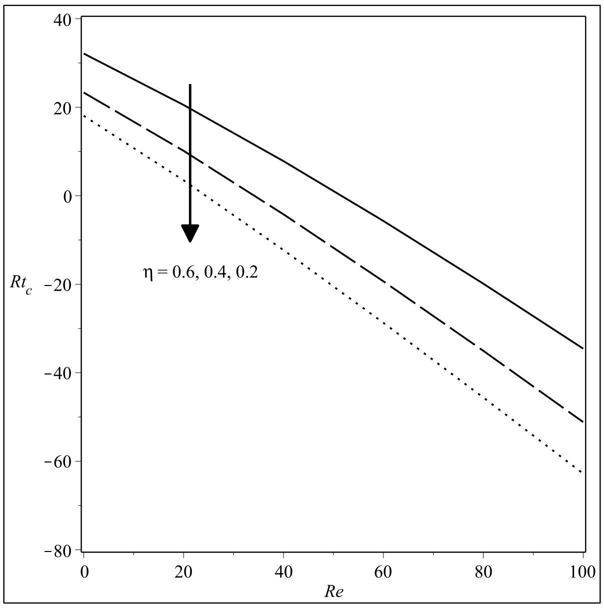

Figure 6: Rt_c vs Re for different value of η.

IV. Conclusion

The electrothermal convection in a nanofluid saturated by the porous medium with the presence of AC electric field and anisotropy effect has been examined. The Galerkin method is employed to solve the eigenvalue problem numerically. It is observed that the effect of AC electric field Rayleigh number Re and the mechanical anisotropy parameter ξ are to reinforce the stationary convection of dielectric nanofluid layer. However, the thermal anisotropy parameter, η delays the electrothermal instability of fluid layer.

Acknowledgements

Authors gratefully acknowledge the financial assistance from Ministry of Higher Education (MOHE) under FRGS/1/2015/SG04/UPM/02/2.

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