ASM Sc. J., 12, Special Issue 1, 2019 for IQRAC2018, 126-136

Effect of Nonlinear Temperature Profile and Magnetic Field on Thermal Convection in a Binary Fluid Saturated an Anisotropic Porous Medium

Nur Zarifah Abdul Hamid[1], Nor Fadzillah Mohd Mokhtar*[2], Norihan Md Arifin[3], and Mohammad Hasan Abdul Sathar[4]

[1] *Department of Mathematics, Faculty of Science, University Putra Malaysia, 43400 Serdang, Selangor, Malaysia.*
[2,3,4] *Laboratory of Computational Sciences and Mathematical Physics, Institute for Mathematical Research(INSPEM), University Putra Malaysia, 43400 Serdang, Selangor, Malaysia.*

* *Corresponding author: norfadzillah.mokhtar@gmail.com*

A linear stability analysis is applied to study the stationary thermal convection in a horizontal system consist of binary fluid saturated an anisotropic porous medium in the presence of nonlinear temperature profile and vertical magnetic field. The problem is solved numerically using the method of Galerkin with respect to rigid-rigid isothermal boundary condition. The effects of magnetic field, mechanical and thermal anisotropic parameter, solute Rayleigh number and Lewis number on the onset of stationary convection in the system for six models of basic state temperature profile are shown graphically. We found that the system can be stabilize by the effect of magnetic field, thermal anisotropic parameter, solute Rayleigh number and Lewis number and destabilize by mechanical anisotropic parameter.

I. Introduction

The studies of thermal convection based on microscopic point of view in a fluid saturated a porous medium has attracted the attention of scientist and researchers due to its innumerable applications in the real fields such as geophysical system, petroleum reservoir, storage and recovery of thermal energy system and etc. A comprehensive studies based on the convection problem in a fluid saturated an isotropic porous medium has been considered by (Knoblach, 1986), (Goyeau et al., 1996), (Hill, 2005), (Shivakumara et al., 2006) and (Mokhtar et al., 2009).

Most of the studies has assumed the porous medium to hold an isotropic properties but this assumption is rather unphysical. This is because natural phenomena such as sedimentation, frost action, compaction and reorientation of solid matrix or artificial porous like pelleting used in chemical engineering will become the main reason for the porous medium to hold an anisotropic properties. The experimental and theoretical studies of thermal convection in an anisotropic porous medium was first introduced by (Castinel and Combarnous, 1974). (Epherre, 1975) studied the instability problem in a porous layer with thermal anisotropy properties. (Malashetty, 1993) examined the onset of double diffusive convection in a binary fluid saturated a porous medium in the presence of anisotropic thermo-convective current, found that the anisotropic parameter can affect the stability of the system under small amplitude of the convection. Later, (Degan et al., 1995) explored the thermal convection in a fluid saturated an anisotropic porous medium bounded vertically and horizontally. He found that the permeability ratio and thermal conductivity ratio can have a powerful influence on the stability of the system. (Malashetty and Swamy, 2010) investigated the threshold of double diffusive convection in a binary fluid saturated an anisotropic porous medium with respect to free-free conducting boundary by considering a uniform temperature profile. Their results showed that the effect of Lewis number is to destabilize the oscillatory convection and stabilize the stationary convection. (Capone et al., 2012) studied the effect of linear and nonlinear permeability and thermal diffusivity on the onset of convection in a fluid saturated an anisotropic porous medium. (Bhadauria, 2012) studied the effect of internal heat generation on the onset of double diffusive convection in a binary fluid saturated an anisotropic porous medium. He showed that the combined effect of increasing mechanical anisotropic and internal heat parameter can enhance the onset of convection. (Bhadauria and Kiran, 2013) have explored the threshold of convection in a temperature dependent viscous fluid saturated an anisotropic porous medium in the presence of temperature modulation. They revealed that the presence of mechanical anisotropic parameter is to delay the onset of convection for the modulated cases. In other work, (Altawallbeh et al., 2012) investigated the effect of internal heating and soret effect on the double diffusive convection in a binary fluid saturated anisotropic porous medium.

The presence of magnetic field in studying the convection problem both in a fluid an porous medium can give many fundamental importance in many fields of science, chemical engineering, and technology such as mobile, satellite and microwave communication. (Rudraiah, 1986) concerned about the effect of magnetic field on the double diffusive convection in a binary fluid layer. (Alchaar et al., 1995) investigated the effect of magnetic field on the threshold of convection in a fluid saturated an isotropic porous medium subject to a uniform temperature gradient. (Bhadauria and Srivastava, 2010) have carried out a linear stability analysis on the double diffusive convection in an electrically conducting binary fluid saturated porous medium in the presence of magnetic field and temperature modulation. (Srivastava et al., 2012) studied the effect of magnetic field on the arrival of steady and oscillatory convection in a binary fluid saturated an anisotropic porous medium in the presence of Soret effect. They showed that the increase in magnetic field, thermal anisotropic and Soret parameter can stabilize the stationary convection in the system. (Sekar et al., 2013) examined the effect of magnetic field on the onset of convection in a ferrofluid saturated an anisotropic porous medium in the presence of soret effect. (Khalid et al., 2013) develop the linear stability analysis on the convection problem in a micropolar fluid in the presence of magnetic field and internal heat generation.

Nowadays, the studies of non-uniform temperature gradient on the onset of convection has been consider more by some researchers in the last few years. (Siddheshwar and Paranesh, 1998) had made a numerical studies on the stationary thermal convection in a micropolar fluid with nonlinear temperature profile subjected to various boundaries conditions. Their results revealed that the stability of system can be controlled using appropriate nonlinear temperature profile. (Char and Chen, 2003) have carried an analytical study on the onset of oscillatory Benard-Marangoni convection under the influence of magnetic field and electric subjected to nonlinear temperature profile. (Idris et al., 2009) investigated the onset of Benard Marangoni convection in a micropolar fluid subjected to nonlinear temperature profile. (Mokhtar et al., 2009) have observed the onset of thermal convection in porous medium by considering the presence of non-uniform temperature profile and magnetic field with respect to free-free and rigid-free adiabatic boundaries. They conclude that the presence of magnetic field together with cubic temperature profile plays an important role in stabilizing the system. The effect of cubic temperature profile on the threshold of Benard Marangoni convection in a ferrofluid with the presence of feedback control has been investigated by (Idris and Hashim, 2010). (Shivakumara et al., 2012) investigated the effect of nonlinear temperature profile on the threshold of thermal convection in a couple stress fluid saturated porous medium. (Paranesh and Baby, 2012) further the studies of (Siddheshwar and Paranesh, 1998) with the presence of electric field. (Nanjundappa et al., 2014) consider the studies of (Idris and Hashim, 2010) under the influence of magnetic field dependent viscosity. (Azmi and Idris, 2014) studied the effect of nonuniform temperature gradient and controller on the Benard Marangoni electroconvection in a micropolar fluid.

The purpose of this paper is to investigate the effect of magnetic field on the onset of thermal convection in a binary fluid saturated an anisotropic porous medium subjected to nonlinear temperature profile with respect to upper rigid conducting and lower rigid conducting boundary condition. The linear stability analysis is used to solve the system and the resulting eigenvalue obtained is solved numerically by Galerkin method.

II. Methodology

We consider a binary fluid layer saturated an anisotropic porous medium with depth, d which are bounded vertically between two horizontal plate at $z = 0$ and $z = 1$ and unbounded in $x-$direction. The system is heated from below and subjected to a uniform vertical magnetic field, $H_b = |\vec{H}_b|$ with gravity force, $g = (0, 0, -g)$ acting vertically downward on it. Let ΔT and ΔC be the temperature and concentration difference between the lower and upper layer of the system respectively. We assume that the mechanical and thermal properties are anisotropy in the vertical direction and isotropy in the horizontal direction. The governing equation based on the Boussinesq approximation in the presence of magnetic field are given by

$$\nabla \cdot \vec{u} = 0, \tag{1}$$

$$\nabla \cdot H = 0, \tag{2}$$

$$\frac{\rho_0}{\phi}\frac{\partial \vec{u}}{\partial t} + \nabla p + \frac{\mu}{\vec{K}} \cdot \vec{u} - \rho\vec{g} - \mu_m H \cdot \nabla H = 0, \tag{3}$$

$$\gamma\frac{\partial T}{\partial t} + (\vec{u} \cdot \nabla)T = \kappa_T(\nabla^2 T), \tag{4}$$

$$\phi\frac{\partial C}{\partial t} + (\vec{u} \cdot \nabla)C = \kappa_c(\nabla^2 C), \tag{5}$$

$$\frac{\partial H}{\partial t} + (\vec{u} \cdot \nabla)H = (H \cdot \nabla)\vec{u} + \gamma_m(\nabla^2 H), \tag{6}$$

$$\rho = \rho_0[1 - \alpha(T - T_0) + \beta(C - C_0)], \tag{7}$$

where $\vec{u} = (u, v, w)$ is the velocity vector, p is the pressure, ϕ is the porosity, $\vec{K} = K_x(\vec{i}\vec{i} + \vec{j}\vec{j}) + K_z(\vec{k}\vec{k})$ is the permeability tensor, μ is the dynamic viscosity, μ_m is the magnetic permeability, γ is the ratio of heat capacity, T is the temperature, $\kappa_T = \kappa_{T_x}(\hat{i}\hat{i} + \hat{j}\hat{j}) + \kappa_{T_z}(\hat{k}\hat{k})$ is the anisotropic thermal diffusivity tensor, S is the solute concentration, κ_c is the solute diffusivity, γ_m is the magnetic viscosity, ρ_0 and T_0 is the reference density and temperature respectively and α and β is the coefficient of thermal and solute expansion respectively.

The basic state of the fluid is assumed to be motionless which takes the form

$$\vec{u}_b = (0, 0, 0), p = p_b(z), \rho = \rho_b(z), T = T_b(z),$$
$$\frac{-d}{\Delta T}\frac{dT_b}{dz} = f(z), H = H_b(z), C = C_b(z), \tag{8}$$

where $f(z)$ is a non-dimensional temperature gradient which hold the following condition

$$\int_0^1 f(z)dz = 1. \tag{9}$$

Using Eq.(8) in Eqs.(1)-(7) to get

$$\frac{dp_b}{dz} = -\rho g, \frac{d^2 T_b}{dz^2} = 0, \frac{d^2 C_b}{dz^2} = 0,$$
$$\rho_b = \rho_0[1 - \alpha(T_b - T_0) + \beta(C_b - C_0)], \tag{10}$$

where subscript b represent the basic state. The conduction state solution are given by

$$T_b = T_0 + \Delta T(1 - \frac{z}{d}), \quad C_b = C_0 + \Delta C(1 - \frac{z}{d}). \tag{11}$$

We superpose the basic state of the fluid subjected to an infinitesimal perturbation by

$$\vec{u} = \vec{u}_b + \vec{u}', T = T_b + T', H = H_b + H',$$
$$C = C_b + C', p = p_b + p', \rho = \rho_b + \rho', \tag{12}$$

where primes represent the perturbation quantities. Introducing Eq.(12) into Eqs.(1)-(7) and using the basic state solution, we obtain

$$\nabla \cdot \vec{u}' = 0, \tag{13}$$

$$\frac{\rho_0}{\phi}\frac{\partial \vec{u}'}{\partial t} + \nabla p' + \frac{\mu}{\vec{K}} \cdot \vec{u}' + (\alpha T' - \beta C')g - \mu_m H_b \nabla H' = 0, \tag{14}$$

$$\gamma \frac{\partial T'}{\partial t} + (\vec{u}' \cdot \nabla)T' - w'\frac{\Delta T}{d} f(z) = \kappa_T(\nabla^2 T'), \tag{15}$$

$$\phi \frac{\partial C'}{\partial t} + (\vec{u}' \cdot \nabla)C' + w'\frac{dC_b}{dz} = \kappa_c(\nabla^2 C'), \tag{16}$$

$$\rho' = \rho_0[-\alpha T' + \beta C']. \tag{17}$$

$$\frac{\partial H'}{\partial t} - H_b \frac{\partial w'}{\partial z} = H' \nabla \vec{u}' + \gamma_m(\nabla^2 H'), \tag{18}$$

The pressure term is eliminate by applying curl twice on Eq.(14) and then the resulting equation together with Eqs.(15)-(18) by the following transformations

$$(x', y', z') = (x^* d, y^* d, z^* d), t = \frac{\gamma d^2 t^*}{\kappa_{T_z}}, \quad (u', v', w') = \left(\frac{\kappa_{T_z} u^*}{d}, \frac{\kappa_{T_z} v^*}{d}, \frac{\kappa_{T_z} w^*}{d}\right), \quad T' = (\Delta T)T^*, H' = H_b H^*, C' = (\Delta C)C^*, \tag{19}$$

to obtain the non-dimensional equations of the form

$$\left[\frac{1}{\gamma Pr}\frac{\partial}{\partial t^*}\nabla^2 + \nabla_h^2 + \frac{1}{\xi}\frac{\partial^2}{\partial z^{*2}}\right] w^* - Ra \nabla_h^2 T^* + Ra_s \nabla_h^2 S^* + H \frac{Pr}{Pm} \nabla_h^2 \frac{\partial H^*}{\partial z} = 0, \tag{20}$$

$$\left[\frac{\partial}{\partial t^*} - \eta \nabla_h^2 - \frac{\partial^2}{\partial z^{*2}} + \vec{u}^* \cdot \nabla\right] T^* - f(z) w^* = 0, \tag{21}$$

$$\left[\epsilon_n \frac{\partial}{\partial t^*} - \frac{1}{Le}\nabla^2 + \vec{u}^* \cdot \nabla\right] S^* - w^* = 0, \tag{22}$$

$$\left[\frac{\partial}{\partial t^*} - \nabla^2\right] H^* - \frac{Pm}{Pr}\frac{\partial w*}{\partial z} = 0, \tag{23}$$

where $Pr = \frac{\mu}{\rho_0 \kappa_{T_z}}$ is the Prandtl number, $Pm = \frac{\gamma_m}{\kappa_{T_z}}$ is the magnetic Prandtl number, $Ra = \frac{\alpha g \Delta T d K_z}{\nu \kappa_{T_z}}$ is the thermal Rayleigh number, $Ra_s = \frac{\beta g \Delta C d K_z}{\nu \kappa_{T_z}}$ is the solute Rayleigh number, $\nu = \frac{\mu}{p_0}$ is the kinematic viscosity, $H = \frac{\mu_m H_b^2 d^2}{\mu \gamma_m}$ is the Chandrasekhar number, $Le = \frac{\kappa_{T_z}}{\kappa_s}$ is the Lewis number, $\xi = \frac{K_x}{K_z}$ and $\eta = \frac{\kappa_{T_x}}{\kappa_{T_z}}$ is the mechanical and thermal anisotropy parameter respectively, $\epsilon_n = \frac{\phi}{\gamma}$ is the normalized porosity and $\nabla_h^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$.

We applied the linear stability analysis on Eqs.(20)-(23) in order to predict the onset of stationary convection in the system. We solved the linearized version of Eqs.(20)-(23) by normal mode expansion which defined as

$$(w, T, C) = (W(z), \Theta(z), \Phi(z)) \, exp[i(mx + ny) + \sigma t], \tag{24}$$

and obtain

$$\left[\frac{\sigma}{Pr}(D^2 - a^2) + \left(\frac{D^2}{\xi} - a^2\right)\right] W + a^2 Ra \Theta - a^2 Ra_s \Phi - H \frac{Pr}{Pm}(D^2 - a^2) DH = 0, \tag{25}$$

$$\left[\sigma - (D^2 - a^2)\right] H - \frac{Pm}{Pr} DW = 0, \tag{26}$$

$$\left[\sigma - (D^2 - \eta a^2)\right] \Theta - f(z) W = 0, \tag{27}$$

$$\left[\sigma - \frac{1}{Le}(D^2 - a^2)\right] \Phi - W = 0, \tag{28}$$

where m, n represent the horizontal wave number in $x-$ and $y-$ direction respectively, σ is the growth rate parameter, $D = d/dz$ and $a^2 = l^2 + m^2$.

Substituting Eq.(26) into Eq.(25) and yield

$$\left[\frac{\sigma}{Pr}(D^2 - a^2) + \left(\frac{D^2}{\xi} - a^2\right) + HD^2\right] W + a^2 Ra \Theta - a^2 Ra_s \Phi = 0, \tag{29}$$

Equations (27)-(29) are to be solved subjected to upper rigid conducting and lower rigid conducting boundary condition which are given by

$$W = DW = \Theta = \Phi = 0 \quad \text{at} \quad z = 0, 1. \quad (30)$$

The resulting eigenvalue obtained from Eqs.(27)-(29) with respect to boundary conditions (30) are solved numerically using the single term Galerkin technique. The basis functions of the variables are given by

$$W = \sum_{n=1}^{N} A_n W_n, \Theta = \sum_{n=1}^{N} B_n \Theta_n, \Phi = \sum_{n=1}^{N} C_n \Phi_n, \quad (31)$$

where A_n, B_n, and C_n are constants and W_n, Θ_n and Φ_n are the trial functions that satisfies the respective boundary conditions. Eq.(27), Eq.(28) and Eq.(29) are multiplied by Θ_m, Φ_m and W_m respectively. The resulting equations are integrate by part with respect to z between $z = 0$ and $z = 1$, we obtained the following equation

$$Ra = \frac{C_4[C_1 C_6 - C_2 C_5]}{a^2 < \Theta \cdot W > C_3 C_6}, \quad (32)$$

where

$$\begin{aligned} C_1 = &-\frac{\sigma}{Pr} < (DW)^2 > -\frac{\sigma a^2}{Pr} < W^2 > \\ &-\frac{1}{\xi} < (DW)^2 > -a^2 < W^2 > \\ &-H < (DW)^2 > \end{aligned} \quad (33)$$

$$C_2 = -a^2 Ra_s < \Phi \cdot W > \quad (34)$$

$$C_3 = - < f(z) W \cdot \Theta > \quad (35)$$

$$C_4 = \sigma < \Theta^2 > + < (D\Theta)^2 > +\eta a^2 < \Theta^2 > \quad (36)$$

$$C_5 = - < W \cdot \Phi > \quad (37)$$

$$C_6 = \sigma < \Phi^2 > +\frac{1}{Le} < (D\Phi)^2 > +\frac{a^2}{Le} < \Phi^2 > \quad (38)$$

where $< ... >$ represent the integration with respect to z from 0 to 1. We set $\sigma = 0$ in order to observe the onset of stationary convection in the system.

We consider six models of $f(z)$ as shown in Table 1.

We solved Eq.(32) using a MAPLE software and obtained the critical Rayleigh number, Ra_c with respect to rigid-rigid conducting boundary conditions.

III. Results and Discussion

The aim of this paper is to study the thermal instability in the binary fluid saturated an anisotropic porous medium in the presence of nonlinear temperature profile and magnetic field. The problem is solved analytically using linear stability analysis and numerically using Galerkin method. We have obtained the function of Ra in terms Prandtl number, solute Rayleigh number, Lewis number, mechanical anisotropy parameter, thermal anisotropy parameter, Chandrasekhar number and $f(z)$. The critical value of thermal depth which depend on the parameters of the problem, ϵ for model 5 and 6 is 0.71 for $Rs = 10$, $Pr = 10$, $Le = 5$, $\xi = 0.5$, $\eta = 0.3$ and $H = 5$. We showed graphically in Figs. 1 - 5 the effects of various parameter on the critical Rayleigh number, Ra_c for all the six model of basic state temperature profiles.

For the validity of the present studies, we have done the comparison studies between the present work and the result obtained by (Paranesh and Baby, 2012). We observed that the pattern of critical Rayleigh number, Ra_c are $Ra_{c6} = Ra_{c5} < Ra_{c1} = Ra_{c2}$, which showed a good agreement with the result obtained by (Paranesh and Baby, 2012). In this studies, we also consider the effect of Cubic 1 and Cubic 2 temperature profile on the onset of convection. We identified that the pattern of critical Rayleigh number, Ra_c obtained in this studies are in the form of $Ra_{c6} = Ra_{c5} < Ra_{c1} = Ra_{c2} < Ra_{c4} < Ra_{c3}$.

Figure 1 indicates the effect of solute Rayleigh number,Ra_s for the six models of basic state temperature profile on the value of Ra_c by keeping others parameter fixed $H = 5$, $Pr = 10$, $Le = 5$, $\xi = 0.5$ and $\eta = 0.3$. We have observed that as Ra_s increase, Ra_c also increases for all the six models of basic state

Table 1: Six models of basic state temperature profiles

Model	Basic state temperature profile		$f(z)$
1	Linear	(Ra_{c1})	$f1 = 1$
2	Inverted parabola	(Ra_{c2})	$f2 = 2(1 - z)$
3	Cubic 1	(Ra_{c3})	$f3 = 3(z - 1)^2$
4	Cubic 2	(Ra_{c4})	$f4 = 0.6 + 1.02(z - 1)^2$
5	Heating from below	(Ra_{c5})	$f5 = \epsilon^{-1}$ for $0 \leq z < \epsilon$
			$f5 = 0$ for $\epsilon < z \leq 1$
6	Cooling from above	(Ra_{c6})	$f6 = 0$ for $0 \leq z < 1 - \epsilon$
			$f6 = \epsilon^{-1}$ for $1 - \epsilon < z \leq 1$

temperature profile. This is because, increase in Ra_s correspond to increase in the concentration difference between the upper and lower layer of the fluid. As $\Delta S > \Delta T$, the fluid density will become greater causes the fluid at the bottom layer to be heavy. Therefore, the movement of warmer fluid vertically upward due to density difference is hindered, thus slow down the transfer of heat in the system or in other word convection is delayed. We found that, $Ra_{c5} = Ra_{c6}$ and $Ra_{c1} = Ra_{c2}$, which means the effect of heating from below temperature profile is similar to the effect of cooling from above temperature profile on the system and the effect of linear temperature profile is similar to the effect of inverted parabola temperature profile on the system. $Ra_{c5} = Ra_{c6}$ is the most unstable and Ra_{c3} is the most stable for every fixed value of Ra_s in the system. Ra_{c3} and Ra_{c4} are more stable as compared to the linear temperature profile,Ra_{c1}.

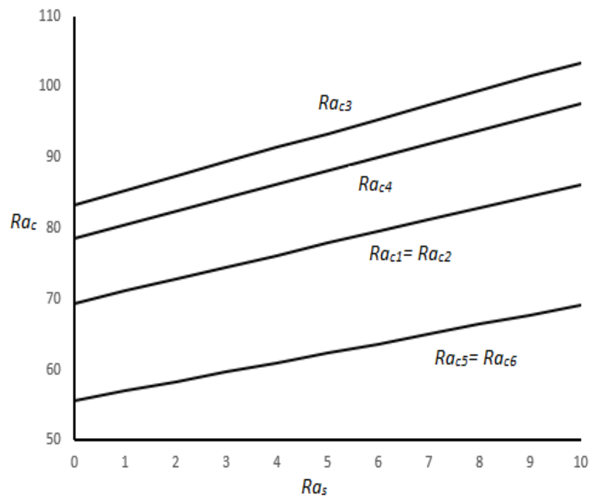

Figure 1: Variation of Ra_c with Ra_s for different temperature profile.

Figure 2 gives the Ra_c versus Lewis number,Le for the different models of basic state temperature profile. The value of other parameters are kept at $Ra_s = 10$, $H = 5$, $Pr = 10$, $\zeta = 0.5$ and $\eta = 0.3$. Increasing the Le causes the Ra_c to increase, thus delay the onset of steady thermal convection for all the basic state temperature profile models, which show a good agreement with (Malashetty and Swamy, 2010) for the case of convection in a binary fluid saturated an anisotropic porous medium subjected to a uniform temperature profile with respect to free-free conducting boundary condition. This is because increase in Le correspond to decrease in solute diffusivity,κ_c. Thus, slow down the heat transfer on the system. Along with the increasing value of Le, it is found that Ra_{c3} is the most stable and $Ra_{c5} = Ra_{c6}$ is the most unstable as compared with other models in the system. When $Le = 0$, it is found that the system are at the most unstable condition for all the six models of basic state temperature profile.

Figure 3 presents the effect of increasing mechanical anisotropy parameter,ξ with respect to six models of basic state temperature profile for fixed value of $H = 5$, $Ra_s = 10$, $Pr = 10$, $Le = 5$ and $\eta = 0.3$. The value of Ra_c decrease with increasing value of ξ. Since ξ is

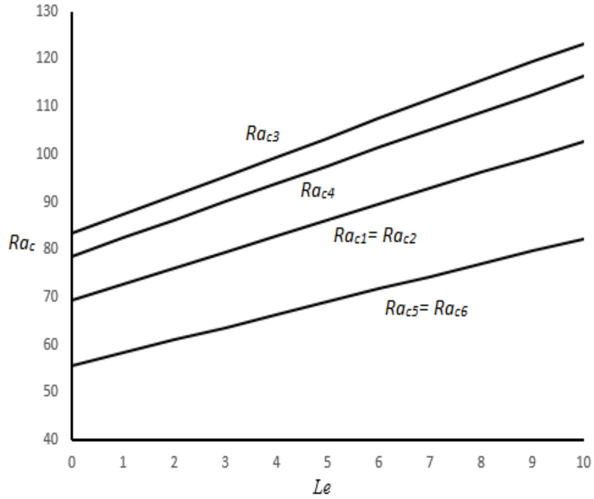

Figure 2: Variation of Ra_c with Le for different temperature profile.

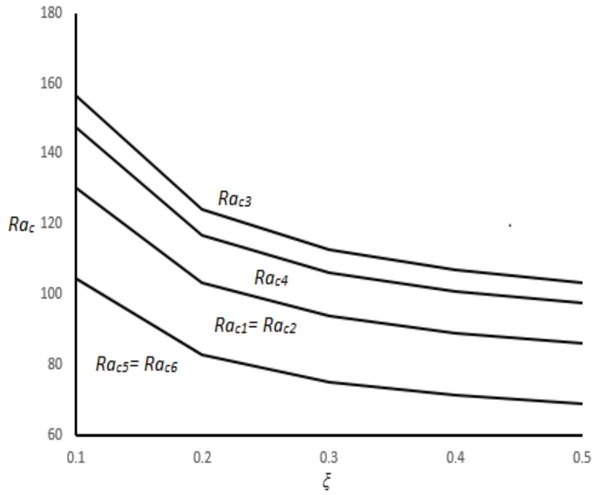

Figure 3: Variation of Ra_c with ξ for different temperature profile.

directly depend on K_x, increasing the value of K_x, larger the cell size of the porous medium and advance the heat transfer vertically upward through the porous medium. Hence the system become destabilize with increasing ξ. It is found that, $Ra_{c1} = Ra_{c2}$ and $Ra_{c5} = Ra_{c6}$ at the fixed value of ξ. This indicate that, heating from below and cooling from above temperature profiles have the same effect on the onset of convection and same with the case of linear and inverted parabola temperature profile. The combination of Ra_{c5} or Ra_{c6} with the increasing value of ξ hasten the onset of stationary convection in the system. The value of Ra_c fall drastically from ξ=0.1 to $\xi = 0.2$ and gradually from ξ=0.2 to $\xi = 0.5$ for all the six models of basic state temperature profile.

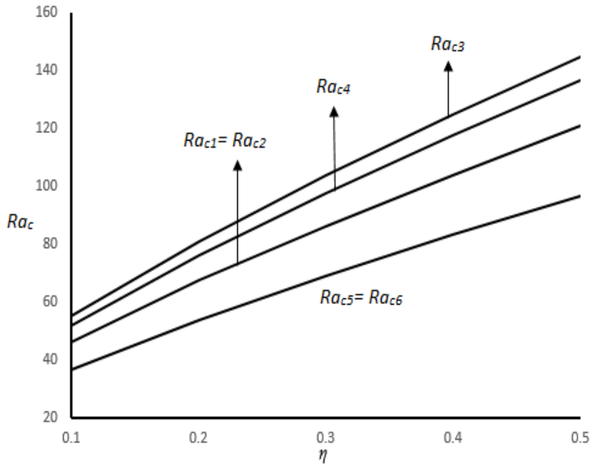

Figure 4: Variation of Ra_c with η for different temperature profile.

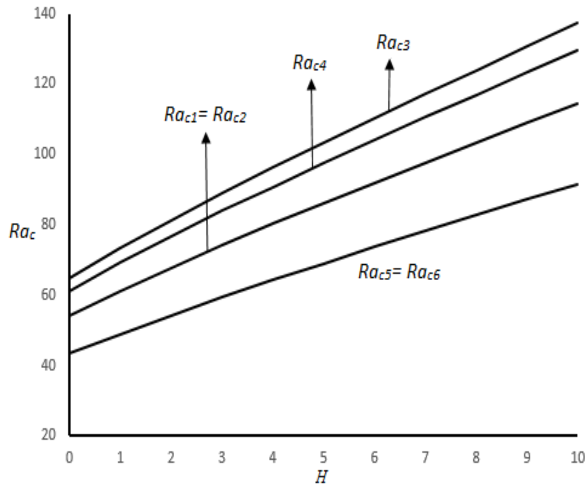

Figure 5: Variation of Ra_c with H for different temperature profile.

The influence of thermal anisotropy parameter,η on the Ra_c for various models of basic state temperature profile at fixed value of $H = 5$, $Ra_s = 10$, $Pr = 10$, $Le = 5$

and $\xi = 0.5$ is revealed in Figure 4. Increasing η will increase the value of Ra_c. Since η is inversely proportional to vertical thermal diffusivity κ_{Tz}, which means decrease in κ_{Tz} correspond to increase in η. When κ_{Tz} decreases, it make the system to conduct heat slowly in the vertical direction and thus convection is delayed. The combination of model 3 and the increasing η can stabilize the system. The system for $Ra_{c5} = Ra_{c6}$ are the most unstable as compared to other models. For every fixed value of η, it is observe that $Ra_{c6} = Ra_{c5} < Ra_{c1} = Ra_{c2} < Ra_{c4} < Ra_{c3}$.

Figure 5 depicted the influence of Chandrasekhar number,H on the Ra_c for different models of basic state temperature profile at fixed value of $Ra_s = 10$, $Pr = 10$, $Le = 5$, $\xi = 0.5$ and $\eta = 0.3$. It is observe that, increases H will lead to increase in Ra_c. Since H is directly proportional to the strength of magnetic field,H_b. Therefore, increase in H_b will delayed the onset of stationary thermal convection in the system. At the every fixed value of H, it is observe that $Ra_{c6} = Ra_{c5} < Ra_{c1} = Ra_{c2} < Ra_{c4} < Ra_{c3}$. That is, the onset of convection is delayed for model 3 and advance for model 5 and model 6. The combination of model 3 and increasing H can makes the system become more stable. Ra_{c3} and Ra_{c4} are more stable as compared to the linear temperature profile,Ra_{c1} and $Ra_{c1} = Ra_{c2}$ for every fixed value of H.

IV. Conclusion

The linear stability analysis has been used to investigate the effect of nonlinear temperature profile on the threshold of steady thermal convection in a binary fluid saturated an anisotropic porous medium in the presence of magnetic field. The problem is solved numerically using single-term Galerkin method with respect to upper and lower rigid isothermal boundary condition. The effect of solute Rayleigh number and Lewis number is to delay the onset of convection in the system. The mechanical anisotropy parameter act as destabilizer while thermal anisotropy parameter act as stabilizer in the system. The increasing value of magnetic field can make the system become more stable. We can summarize that for each varying value of H, Ra_s, Le, ξ and η, $Ra_{c6} = Ra_{c5} < Ra_{c1} = Ra_{c2} < Ra_{c4} < Ra_{c3}$.

(The contribution of the work to the overall knowledge of the subject could be shown. Relevant conclusions should be drawn, and the potential for further work indicated where appropriate)

Acknowledgements

The authors acknowledge the financial support received from University Putra Malaysia under FRGS and GRF.

References

[1] S. Alchaar, P. Vasseur, and E. Bilgen. Effects of a magnetic field on the onset of convection in a porous medium. *Heat and Mass Transfer*, 30:259–267, 1995.

[2] A. A. Altawallbeh, B. S. Bhasauria, and I. Hashim. Convection problems in anisotropic porous media with nonhomogeneous porosity and thermal diffusivity. *Acta Appl Math*, 122:85–91, 2012.

[3] H. M. Azmi and R. Idris. Effects of controller and nonuniform temperature profile on the onset of rayleigh-benard-marangoni electroconvection in a micropolar fluid. In *Journal of Applied Mathematics*, page 571437. HINDAWI Publishing Corporation, 2014.

[4] B. S. Bhadauria. Double diffusive convection in a saturated anisotropic porous layer with internal heat source. *Transp Porous Med*, 92:299–320, 2012.

[5] B. S. Bhadauria and P. Kiran. Heat transport in an anisotropic porous medium saturated with variable viscosity liquid under temperature modulation. *Transp Porous Med*, 100:279–295, 2013.

[6] B. S. Bhadauria and A. K. Srivastava. Magneto-double diffusive convection in an electrically conducting-fluid-saturated porous medium with temperature modulation of the boundaries. *International Journal of Heat and Mass Transfer*, 53: 2530–2538, 2010.

[7] F. Capone, M. Gentile, and A. A. Hill. Convection problems in anisotropic porous media with nonhomogeneous porosity and thermal diffusivity. *Acta Appl Math*, 122:85–91, 2012.

[8] G. Castinel and M. Combarnous. Critere d' apparitition de la convection naturelle dans une couche poreuse anisotrope horizontale. *C.R. Acad. Sci. B*, 278:701–704, 1974.

[9] M. I. Char and C. C. Chen. Effect of a non-uniform basic temperature gradient on the onset of oscillatory benard-marangoni convection of an electrically conducting liquid in an magnetic field. *International Journal of Engineering Science*, 41:1711–1727, 2003.

[10] G. Degan, P. Vasseur, and E. Bilgen. Convective heat transfer in a vertical anisotropic porous layer. *International Journal of Heat and Mass Transfer*, 38: 1975–1987, 1995.

[11] J. F. Epherre. Critere d' apparitition de la convection naturelle dans une couche poreuse anisotrope. *Reveu Générale de Thermique*, 168:949–950, 1975.

[12] B. Goyeau, J.-P. Songbe, and D. Gobin. Numerical study of double-diffusive natural convection in a porous cavity using the darcy-brinkman formulation. *International Journal of Heat and Mass Transfer*, 39(7):1363–1378, 1996.

[13] A. A. Hill. Double-diffusive convection in a porous medium saturated with a concentration based internal heat source. *Proc. R. Soc. A.*, 461:561–574, 2005.

[14] R. Idris and I. Hashim. Effects of controller and cubic temperature profile on onset of benard-marangoni convection in ferrofluid. *International Communications in Heat and Mass Transfer*, 37:624–628, 2010.

[15] R. Idris, H. Othman, and I. Hashim. On effect of non-uniform basic temperature gradient on benard-marangoni convection in micropolar fluid. *International Communications in Heat and Mass Transfer*, 36:255–258, 2009.

[16] I. K. Khalid, N. F. M. Mokhtar, and N. M. Arifin. Uniform solution on the combined effect of magnetic field and internal heat generation on rayleigh-bénard convection in micropolar fluid. In *Journal of Heat Transfer*, volume 135, page 102502. ASME, 2013.

[17] E. Knoblach. Oscillatory convection in binary mixture. *Phys. Rev. A*, 34:1538–1549, 1986.

[18] M. S. Malashetty. Anisotropic thermoconvective effects on the onset of double diffusive convection in a porous medium. *International Journal of Heat and Mass Transfer*, 36(9):2397–2401, 1993.

[19] M. S. Malashetty and M. Swamy. The onset of convection in a binary fluid saturated an anisotropic porous layer. *International Journal of Thermal Sciences*, 49:867–878, 2010.

[20] N. F. M. Mokhtar, N. M. Arifin, R. Nazar, F. Ismail, and M. Suleiman. Effects of non-uniform temperature gradient and magnetic field on benard convection in saturated porous medium. *European Journal of Scientific Research*, 34: 365–371, 2009.

[21] N. F. M. Mokhtar, N. M. Arifin, R. Nazar, F. Ismail, and M. Suleiman. Marangoni convection in a fluid saturated porous layer with a deformable free

surface. *International Journal of Computational and Mathematical Sciences*, 3: 251–256, 2009.

[22] C. E. Nanjundappa, I. S. Shivakumara, and B. Savitha. Onset of benard-marangoni ferroconvection with a convective surface boundary condition: The effects of cubic temperature profile and mfd viscosity. *International Communications in Heat and Mass Transfer*, 51: 39–44, 2014.

[23] S. Paranesh and R. Baby. Effect of a non-uniform temperature gradient on the onset of rayleigh-benard electro convection in a micropolar fluid. *Applied Mathematics*, 3:442–450, 2012.

[24] N. Rudraiah. Double-diffusive magneto-convection. *J. Phys.*, 27(1 and 2):233–266, 1986.

[25] R. Sekar, K. Raju, and R. Vasanthakumari. A linear analytical study of soret-driven ferrothermohaline convection in an anisotropic porous medium. *Journal of Magnetism and Magnetic Materials*, 331:122–128, 2013.

[26] I. S. Shivakumara, S. P. Suma, and K. B. Chavaraddi. Onset of surface-tension-driven convection in superposed layers of fluid and saturated porous medium. *Warszawa Arch. Mech*, 58:71–92, 2006.

[27] I. S. Shivakumara, S. Sureshkumar, and N. Devaraju. Effect of non-uniform temperature gradient on the onset of convection in a couple-stress fluid-saturated porous medium. *Journal of Applied Fluid Mechanics*, 5(1):49–55, 2012.

[28] P. G. Siddheshwar and S. Paranesh. Effect of a non-uniform basic temperature gradient on rayleigh-benard convection in a micropolar fluid. *International Journal of Engineering Science*, 36:1183–1196, 1998.

[29] A. K. Srivastava, B. S. Bhaudaria, and V. K. Gupta. Magneto-convection in an anisotropic porous layer with soret effect. *International Journal of Non-Linear Mechanics*, 47:426–438, 2012.

Footnotes, spelling and measurement units

d	depth
$f(z)$	non-dimensional temperature gradient
g	gravity force, $(0, 0, -g)$
H	Chandrasekhar number, $\frac{\mu_m H_b^2 d^2}{\mu \gamma_m}$
H_b	magnetic field strength, $\lvert \vec{H_b} \rvert$
$\vec{K}$	permeability tensor,
Le	Lewis number, $\frac{\kappa_{T_z}}{\kappa_s}$
m, n	horizontal wave number
p	pressure
Pm	magnetic Prandtl number, $\frac{\gamma_m}{\kappa_{T_z}}$
Pr	Prandtl number, $\frac{\mu}{\rho_0 \kappa_{T_z}}$
Ra	thermal Rayleigh number, $\frac{\alpha g \Delta T d K_z}{\nu \kappa_{T_z}}$
Ra_s	solute Rayleigh number, $\frac{\beta g \Delta S d K_z}{\nu \kappa_{T_z}}$
S	solute concentration
T	temperature
T_0	reference temperature
$\vec{u}$	velocity vector, $\vec{u} = (u, v, w)$
α	coefficient of thermal expansion
β	coefficient of solute expansion
σ	growth rate parameter
γ	ratio of heat capacity
γ_m	magnetic viscosity
ϵ_n	normalized porosity, $\frac{\phi}{\gamma}$
η	thermal anisotropy parameter, $\frac{\kappa_{T_x}}{\kappa_{T_z}}$
κ_c	solute diffusivity
κ_T	anisotropic thermal diffusivity tensor
κ_{T_x}	horizontal thermal diffusivity
κ_{T_z}	vertical thermal diffusivity
μ	dynamic viscosity
μ_m	magnetic permeability
ν	kinematic viscosity, $\frac{\mu}{p_0}$

ξ	mechanical anisotropy parameter, $\frac{K_x}{K_z}$
ρ_0	reference density
ϕ	porosity
ΔT	temperature difference between the wall
ΔS	concentration difference between the wall
$_b$	basic state
$_c$	critical value
$'$	perturbation quantities