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Origins of One Dimensional Instability in Stationary Shock and Slowly Moving Shock

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Shock instabilities in the numerical sense include the carbuncle phenomenon and the slowly moving shocks. The carbuncle phenomenon is a term referred to the protruding formation at the stagnation region in addition to the continuous bow shock when simulating a high-speed flow over a blunt body. Most schemes formulated to cure this problem only focus on the dissipation methods without properly indulged into the real cause, which could also be the root problem for the slowly moving shock. Therefore, this paper attempted to find the source of the problem by firstly analyzing the governing equations starting from 1D case. After using perturbation mechanism on the conservative variables, several factors were found and one of them is caused by perturbation in density. Then, a dissipation was added to the RHS (right-hand side) of the continuity equation to remove the perturbation. This artificial dissipation has shown stable solutions for both stationay and slowly moving shock problems.
Keywords: artificial dissipation, carbuncle phenomenon, shock anomaly, slowly moving shock.

I. Introduction

Three inevitable shock anomalies throughout the last three decades in studying the compressible flow are hypersonic wall heating by Noh (1987), slowly moving shocks by Woodward and Colella (1984) and carbuncle phenomenon by Peery and Imlay (1988). Even though the system of governing equations for each problem is the same, many approaches of fixing them are uniquely schemed to solve a particular problem. For example, in carbuncle phenomenon, Liou and J.R (1993) used a definition of separating pressure term from the mass flux but when Kitamura and Shima (2012) used the AUSM+-up scheme on the slowly moving shock, the instability is still contrived. Many previous works concluded that there are many underlaying factors and sources that contributed to the anomalous behavior and each of them are intricately emerged depending on the type of the problems. Furthermore, a stability analysis by Agrawal and Srinivasan (2017) also focused on the numerical schemes that are carbuncle prone, therefore, the root can only be found from that particular scheme instead of looking at the bigger picture.

The carbuncle is apparent in a fully 2D case where it manifested visibly by the naked eyes from its protruding behavior. On the hands, several works by Ismail and Roe (2009), Kitamura et al. (2007), Zaide and Roe (2011) and Wahi and Ismail (2012) have shown that the underlying problem is substantial in 1D case. Their evisceration in 1D using intermediate perturbation from Ismail and Roe (2009) in many carbuncle prone schemes found that the problem in 1D is caused by the inaccountability in internal shock structure that is inherent in Godunov-type schemes compared to the other schemes such as HLLE built by Wendroff (1999). This internal structure perhaps seems to not adhere to the second law of thermodynamics. Even though more precise discrete entropy control were introduced, some schemes still show signs of instability as stated by Agrawal and Srinivasan (2017). Consequently, a somewhat 2D case was introduced by Kitamura et al. (2007) and it is called 1.5D to further study the behavior if any additional factors play into the role. Any schemes that fails to stabilize in 1D also fails to stabilize in 1.5D. Moreover, some schemes that is strongly stable in 1D have started to show signs of instability under extreme perturbation in 1.5D such as the AUSM's type flux schemes by Liou and J.R (1993). For 1.5D case, an extreme perturbation would be the intermediate point perturbation taken from Ismail and Roe (2009) and Wahi and Ismail (2012) of 1D case. On the other hand, most schemes in CFD would produce spurious post-shock oscillations when simulating a slowly moving shock, first reported by Woodward and Colella (1984). These notorious oscillations would grow over time and may lead to instability as reported by Roberts (1990) and Stiriba and Donat (2003). In fact, it worsen with minimally dissipative schemes such as the Roe-flux as reported by Arora and Roe (1997). Johnsen and Lele (2008) reported that most fixes include adding imprecise numerical dissipations to remove the oscillations usually do not work when dealing with the carbuncle phenomenon.

Since all the numerical calculations started from the exact same governing equations, we believe that if we can find one source of the problem from its origin and using a scheme that can handle the source, at least a reasonable explanation of the erratic behavior can be made thus possibly a robust scheme can be formulated. The analysis from this paper took the liberty from the works by D'iakov (1958), Kontorovich (1957) and Swan and Fowles (1975) that gives an idea that the small perturbation in the variables from the Euler equations may be one of the culprits for carbuncle. Furthermore, Dumbser et al. (2004) also used this type perturbation to use it in their matrix stability analysis.

This paper is organized such that in the first section, a review on the previous study about the shock anomalies in the shock capturing schemes. These reviews included possible causes of the problem as well as critics regarding the causes. Then in the next section, a perturbation analysis is conducted on 1D and 2D Euler equations. The purpose of this part is to study any possible sources of the problem. The third section is the numerical experiment to mimick the perturbation done in the previous section to potray the issue. Then, the fourth section is the numerical experiment with the addition of artificial dissipation to remove the main cause found in perturbation analysis. Solutions before and after the fix will be compared. The last section is the conclusion followed by acknowledgment.

II. Perturbation Analysis on the System of Euler Equations

The von Neumann's stability analysis is essential yet insufficient on non-linear discretized schemes as mentioned in Hirsch (1988). For example a stable scheme from the linear equation may produced an unstable solution from its non-linear version. This behavior is apparent in neutrally stable scheme and additional dissipation is necessary to stabilize it. This is also applied to our case. Hence, we linearized the Euler equations by using the definition of 'small disturbance' as done by D'iakov (1958) such that

$$\phi = \overline{\phi} + \phi' \quad , \tag{1}$$

where ϕ is the any quantity to be linearized, $\overline{\phi}$ is the mean value and ϕ' is the small disturbance.

A. The Work of Dyakov and Previous Researchers

The work by D'iakov (1958) was based on very small arbitrary perturbation on the system of steady Euler equations using the method of characteristics based on linearizing the primary variables. In short, D'iakov discovered the conditions of instability of shock waves:

$$m^2(\frac{\partial V_2}{\partial P_2})_H < -1, \text{ and } m^2(\frac{\partial V_2}{\partial P_2})_H > 1+2M_2 \quad , \tag{2}$$

where m is the mass flux, and M_2, V_2, P_2 are the Mach number, specific volume and pressure of the fluid behind a shock. The derivatives are calculated along the Hugoniot curve in the pressure-volume plane.

Kontorovich (1957) then provided a more accurate stability requirement,

$$\frac{1-M_2^2-XM_2^2}{1-M_2^2+XM_2^2} < m^2(\frac{\partial V_2}{\partial P_2})_H > 1+2M_2 \quad , \tag{3}$$

where $X = \frac{V_1}{V_2}$ is the gas compression at a shock. In short, shock instability depends on the jump conditions imposed by the shock, which in turn depend on the equation of state. This implies that all shocks are stable in ideal gas conditions.

However, Robinet et al. (2000) discovered that an unstable mode does exist even for the ideal gas for a particular value of the upstream Mach number M_1. We intend to explore more on the discovery of Robinet. They are other extensive research work done in this shock instability analysis which we shall not discuss herein but the details can be obtained in Lubchich (2004).

B. Stability Analysis Using Conservative Variables

The first approach of our analysis is to perturb 1D Euler equations defined as

$$\begin{pmatrix} \rho \\ \rho u \\ \rho E \end{pmatrix}_t + \begin{pmatrix} \rho u \\ \rho u^2 + p \\ \rho u H \end{pmatrix}_x = 0. \tag{4}$$

where $E = e + \frac{u^2}{2}$ is the total energy per unit volume and $p = \rho e(\gamma - 1)$ is the pressure from equation of state in ideal gas. $H = E + \frac{p}{\rho}$ is the total enthalphy. The arbitrary disturbance in Equation (1) may be resolved into independent modes with corresponding wave numbers, frequencies and eigenvectors which express into

$$\phi' = \exp[(ikx - \omega t)]\mathbf{R} \tag{5}$$

Noted that the above equation is slightly differ with D'iakov (1958) and Swan and Fowles (1975) because we followed the assumption made by Ismail and Roe (2005) such that solution oscillates only in space. Nonetheless, the Equation (5) is plugged into Equation (4) to find the possible unstable mode. This unstable modes are calculated by the singularity of determinant of the eigenvector matrix. In our analysis, we shall assume that the Jacobian of Equation (4) would include perturbed values about a linearized state, similar to the approach of Rayleigh (1894, 1964) and McCartin (2009) in understanding the vibration of a Hamiltonian system and Schrodinger's matrix perturbation theory in Schrodinger (1982) and the work inspired by them after that such as Stewart and Sun (1990) and Cui and Liang (1993). This is where our work differs from previous work in studying shock-instability of the system of Euler equations in which the entries within the Jacobian are averaged states without any perturbation.

C. Analysis on 1D Euler Equations

The perturbed Euler equations in Equation (4) are written into quasi-linear form given by

$$\mathbf{U}_t + \tilde{A}\mathbf{U}_x = 0 \quad , \tag{6}$$

where $\mathbf{U} = \overline{\mathbf{U}} + \mathbf{U}' = [\overline{\rho} + \rho', \overline{\rho u} + (\rho u)', \overline{\rho E} + (\rho E)']$ and $\tilde{A}$ is the Jacobian matrix evaluated from the linearized (with perturbation) conservative variables. We then define the ratio of each conservative variable linearization to the density linearization. The first is the momentum ratio as j_u, secondly the energy ratio as j_e, then the enthalphy ratio as j_h and finally the the inverse of the density linearization as

j_r. The expression for these ratios are given in Equation (7).

$$
\begin{aligned}
j_u &= \frac{\overline{\rho u} + (\rho u)'}{\overline{\rho} + \rho'} \quad , \\
j_r &= \frac{1}{\overline{\rho} + \rho'} \quad , \\
j_e &= \frac{\overline{\rho E} + (\rho E)'}{\overline{\rho} + \rho'} \\
&= \frac{p}{(\gamma - 1)(\overline{\rho} + \rho')} + \frac{1}{2}\frac{(\overline{\rho u} + (\rho u)')^2}{(\overline{\rho} + \rho')^2} \\
&= \frac{p j_r}{\gamma - 1} + \frac{1}{2} j_u^2 \quad , \\
j_h &= \frac{\overline{H} + (H)'}{\overline{\rho} + \rho'} = j_e + p j_r .
\end{aligned}
\tag{7}
$$

Thus, $\tilde{A}$ can be written as

$$
\begin{pmatrix}
0 & 1 & 0 \\
\frac{\gamma - 3}{2} j_u^2 & (3 - \gamma) j_u & \gamma - 1 \\
j_u \left(\frac{\gamma - 1}{2} j_u^2 - j_h \right) & j_h - (\gamma - 1) j_u^2 & j_u \gamma
\end{pmatrix} .
\tag{8}
$$

The Equation (5) is plugged into Equation (6) giving an expression of

$$[ik\tilde{A} - \omega I]\mathbf{U}' = 0. \tag{9}$$

For nontrivial solutions, we set the determinant of the coefficient matrix in Equation (9) above to be zero with the resulting eigenvalues are listed as follows,

$$
\begin{aligned}
&\det(ik\tilde{A} - \omega I) \\
&= [(ikj_u - \omega)^2 - k^2 a^2](ikj_u - \omega) \\
&= 0.
\end{aligned}
\tag{10}
$$

The above expression presents that we have three expression of roots that are $\omega = ikj_u$ and $\omega = ik(j_u \pm a)$. For 1D case, we can see that the shear wave factor is absent but there is still the entropy wave and the other roots correspond to the acoustic wave. The value when $\omega = ikj_u$ will result into three repeated roots. That is to say the entropy and the acoustic waves are similar. After that, we check the determinant of right eigenvectors and putting them into a square matrix given by

$$
\mathbf{R} = \begin{bmatrix}
1 & 1 & 1 \\
j_u - a & j_u & j_u + a \\
j_h - a j_u & \frac{1}{2} j_u^2 & j_h + a j_u
\end{bmatrix} .
\tag{11}
$$

Problems would arise when the eigenvectors above are linearly dependent, particularly when there is a dependent mode in the form of resonance which may give birth to growth and instability.

$$
\begin{aligned}
\det(\mathbf{R}) &= 2a\gamma \left(\frac{j_u^2}{2} - j_e \right) \\
&= 2a\gamma \left(\frac{p j_r}{\gamma - 1} \right) = 0,
\end{aligned}
\tag{12}
$$

or when $det(\mathbf{R}) \to \infty$ which is true when

1. The acoustic waves coincide with the entropy wave.

2. The pressure or speed of sound $\to 0$, i.e approaching vacuum state

3. The fluids specific heat ratio $\gamma \to 1$

4. The inverse of density linearization $j_r = \dfrac{1}{\overline{\rho} + \rho'} \to \epsilon$, where ϵ is a small number.

The first three conditions have been reported before in Ismail and Roe (2005), but to the authors' best of knowledge, the last condition has never been found until now. In other words, mathematically speaking, the continuous growth of the density fluctuation within the system may drive the inverse of mass linearization to a very large number in which will eventually lead to instability. Even though one may argue that in numerical calculation, the accretion of disturbance of the density is finite hence its inverse is impossible to become zero, the conclusion made here is based on the expression given by the determinant as a guideline in understanding any possible explanations for the anomalous behavior in the shock problem. Therefore, from this analysis alone, we can try to initiate a method to reduce the growth formation from the density.

III. Numerical Experiments

This section is divided into two parts which are the 1D stationary shock followed by 1D slowly moving shocks. The numerical schemes used are four: Roe's flux from Roe (1981), AUSM+ from Liou (1996), AUSM+-up from Liou (2006) and Entropy Consistent(EC) flux from Ismail and Roe (2009).

A. Initial Conditions

In the first case, the stationary shock took the Rankine-Hugoniot jump condition for $\Delta F = 0$. Then, after derivation using the condition, the pre-shock and post-shock profiles are expressed as follow:

$$\begin{aligned} U_0 &= \begin{bmatrix} 1.0 & 1.0 & 1/2+\eta \end{bmatrix} \\ U_1 &= \begin{bmatrix} f(M_{pre}) & 1.0 & g(M_0)\eta + \dfrac{1}{2f(M_0)} \end{bmatrix}, \end{aligned} \tag{13}$$

where

$$\begin{aligned} f(M_0) &= \left(\frac{1}{\dfrac{2}{(\gamma+1)M_0^2} + \dfrac{\gamma-1}{\gamma+1}} \right), \\ g(M_0) &= \frac{2\gamma M_0^2}{\gamma+1} - \frac{\gamma-1}{\gamma+1}, \\ \eta &= \frac{1}{\gamma(\gamma-1)M_0^2}. \end{aligned} \tag{14}$$

The incoming Mach number was taken to be $M_0 = 5.0$, with CFL condition of $\nu = 0.2$ and 25 computational square grid cells. The 1D slowly moving shock numerical setup is from Jin and Liu (1996) such that the left and the right conservative variables' profiles are given as follow:

$$\begin{aligned} U_0 &= \begin{bmatrix} 3.86 & -3.1266 & 27.093 \end{bmatrix}, \\ U_1 &= \begin{bmatrix} 1.0 & -3.44 & 8.4168 \end{bmatrix}. \end{aligned} \tag{15}$$

The subscript 0 and 1 refer to the pre-shock and post-shock profile respectively. 200 grid cells were used with CFL number of 0.1. In addition, all types of limiters were excluded in all cases in order to follow as close as possible to the Euler equations.

B. Boundary Conditions

In stationary carbuncle, the boundary conditions for inlet and outlet for all variables were being kept constant at the exact Rankine-Hugoniot relation using the ghost cells on the left and on the right. This conditions followed the configuration of stability analysis numerical setup by Dumbser et al. (2004). Whereas, for the second case, the left and right boundaries were set to zero gradient.

C. Perturbation Procedures to Induce Shock Instability in 1D stationary shock

The random perturbation as done by Dumbser et al. (2004) as expressed below

$$\mathbf{U}_s = \overline{\mathbf{U}} + \epsilon \mathbf{U}. \tag{16}$$

This method mimics the fluctuations from its mean value which is in good agreement with the linearization process. The ϵ has an interval of $[10^{-3}, 10^{-6}]$ as practised by Kitamura and Shima (2012). In addition, a second test is also being administered by introducing an intermediate point at the shock location to mimic the fully $2D$ grid orientation as done by Ismail et al.

(2006) unbiasedly on all conservative variables such that

$$\mathbf{U}_s = \delta \mathbf{U}_0 + (1 - \delta)\mathbf{U}_1. \qquad (17)$$

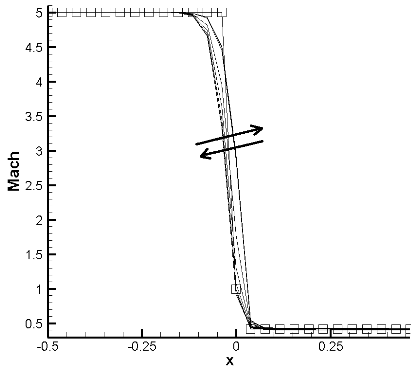
where the subscript s, 0 and 1 refer to the shock location, pre-shock and post-shock profile respectively. The range of δ is from 0.0 to 1.0. The perturbations will be done with various combinations. We start with perturbations to only the density, followed by momentum, energy, combinations of any two conserved variables and finally all conserved variables.

D. Observations on Shock Instability

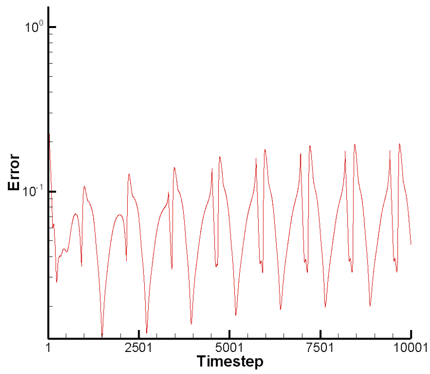
The results of the anomalous behavior are already established in their respective literatures. However, they are still to be included in Section(IV.) for comparison. The tests begin with 1D stationary carbuncle then 1D slowly moving shocks. Furthermore, the perturbation using intermediate point has shown much severe instability compared to the random perturbation for the stationary shock.

1. 1D Stationary Shock

We present two sample figures as a sample of results for Mach 5 shock profile that leads to shock instability from the initial conditions as expressed in Equation (13,14) with boundary conditions as explained in Section (B.). It is worth to highlight that any perturbation which involves the density variable (either alone, or combined with other conserved variables) will lead to shock instability. From our experience, perturbing just the momentum variables alone will not yield instability. Perturbations to the energy variable will yield instability after a very long time. Once instability kicks in, there is a distinct evolutionary pattern for almost all conditions.



(a) Mach profile



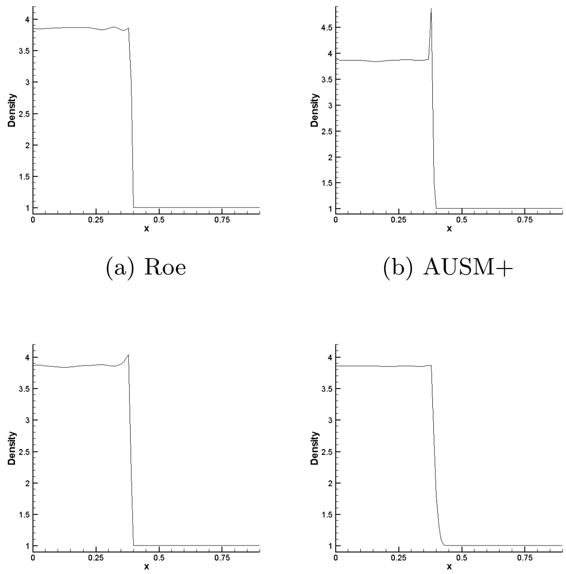
(b) Residual error

Figure 1: Solution for Mach number using $\delta = 0.7$

Figure(1) present the results of perturbing the shock using intermediate perturbation of value $\delta = 0.7$. All schemes were showing similar behavior where the intermediate shock profile is moving between the left and right as indicated by the arrows in Figure(1a). This motion is evidently represented by the uniform pulsation of the residual error in Figure(1b). Even after 10,000 timesteps, the solution never shown any sign to attenuate.

2. 1D Moving Shocks

The results for Roe's flux, two AUSM's family flux and the EC's flux are displayed in Figure (2).



(a) Roe (b) AUSM+

(c) AUSM+-up (d) EC

Figure 2: The results of slowly moving shock in 1D taken at 1000 timesteps.

All the solutions in Figure(2) presented the instability in the slowly moving shock cases where oscillation near the jump is apparent. The EC's flux scheme has shown the best result among others where the amplitude of the oscillation is much less and occasionally showing a small overshoot.

IV. Adding Dissipation

Adding an artificial dissipation is widely practiced such as in von Neumann and Richtmyer (1950) as well as in Noh (1987). This practiced is exploited by Rodionov (2017) who used a modified von Neumann and Richtmeyer artificial viscosity on the right hand equations to suit any given schemes. The fix in this paper however, is based on the founded cause on the governing equations and to focus on eradicating the source perhaps for both the stationary and slowly moving shock problems.

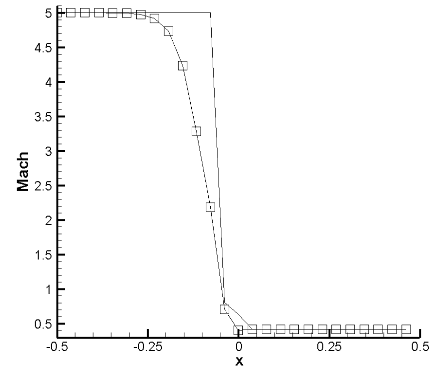
We attempted to remove the instability by adding a diffusive factor on the right-hand-side of Equation (6) only to the mass equation such that

$$\varsigma(\rho_{xx}). \tag{18}$$

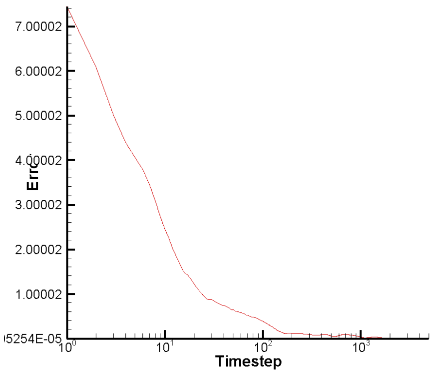
Then, a pure central-differencing was used to evaluate ρ_{xx}. The purpose of adding this diffusion to the density equation is to show that we can focus on a problematic factor in one variable instead of the whole system; thus, keeping the dissipation minimal. This dissipation addendum is merely to prove our point that density is one of the potential reasons of shock instability and not necessarily the best method for the carbuncle problem. Moreover, adding diffusion on all conservative equations would be overkill.

A. 1D Stationary Shock

We found that the range value of $\varsigma = 0.06$ is ample to resist the recurring instability for all schemes in this stationary case. The computed solutions in the following figures are being compared to the state before and after the inclusion of density dissipation.

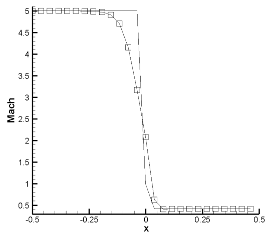


(a) Mach profile

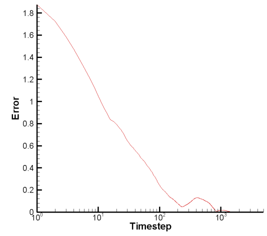


(b) Residual error

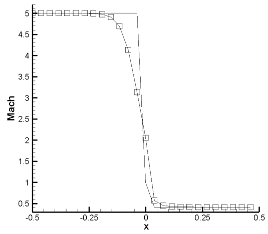
Figure 3: The results of stationary shock after adding artificial dissipation on density using Roe's flux scheme.



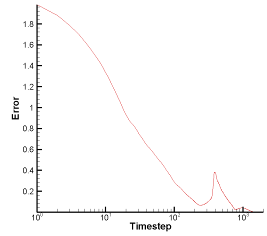
(a) AUSM$^+$



(b) Residual error



(c) AUSM^{+-}up

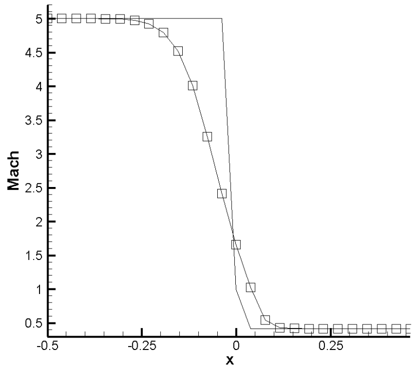


(d) Residual error

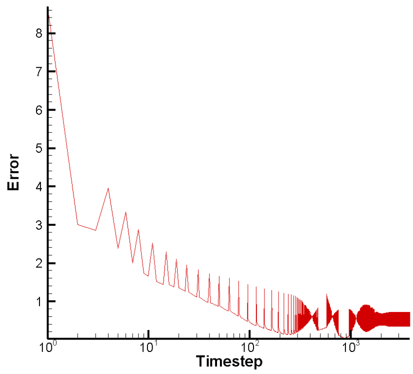
Figure 4: The Mach's profiles and their respective residual error after adding artificial dissipation on density using AUSM$^+$ and AUSM^{+-}up.

Figure (3) shows the Mach's profile using $\varsigma = 0.06$. The dissipation addition is quite evident especially at the pre-shock location. However, the pulsation is quickly ceased after 1000 timesteps as demonstrated in Figure (3b). Furthermoret, we have explored adding the same dissipation technique to only momentum (or energy or both) equation(s) but the instability still persists for both shocks. This perhaps explain why the carbuncle is not removed for Navier-Stokes calculations as reported in Pandolfi and D'Ambrosio (2001). Nonetheless, the results for other schemes are presented by following figures.

Figure(4) above display the Mach's profile and residual error for AUSM$^+$ and AUSM^{+-}up. Both solutions stabilize at similar timesteps as Roe's scheme. The last scheme is the Entropy-Consistent and its solutions are displayed in the following figures.



(a) Mach pofile



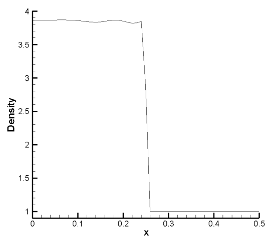
(b) Residual error

Figure 5: The profile of Mach number and its corresponding residual error using E-C scheme.

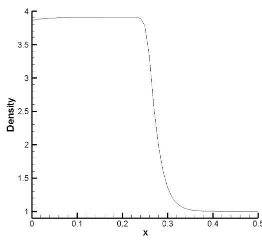
Figure(5a) shows similar Mach's profile compared to the previous three schemes. However, the residual error from Figure(5b) differs from the others. To confirm that the solution is stable, we perused its animated solution which did not show any sign of instability. Therefore, we concluded that the solution is indeed stable. At the moment, the only explanation of its residual behavior is due to its consistent entropy production which is absent from other schemes.

B. 1D Slowly Moving Shocks

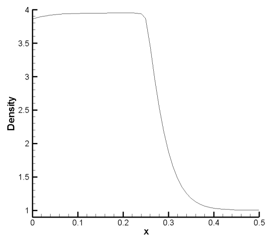
The dissipation ς is ranging from 0.2 to 0.4. These values are quite higher than the stationary tests, but they gave the best solutions among others. Moreover, the unstable solutions for all schemes are similarly formed, therefore only the unstable from Roe's flux scheme will be displayed for reference. The solutions are being displayed as follow.



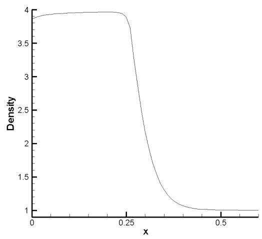
(a) Solution without ς



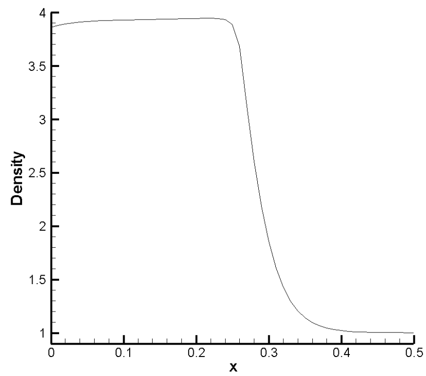
(b) Roe's Flux, $\varsigma = 0.2$



(c) AUSM+,$\varsigma = 0.35$



(d) AUSM+-up, $\varsigma = 0.35$



(e) EC's flux,$\varsigma = 0.3$

Figure 6: Comparison of the three different schemes after being introduced the density dissipation factor of various values.

Figure (6a) shows the oscillation near the jump for slowly moving shock using Roe's flux (other schemes also experienced a similar behavior) when dissipative factor was inserted. Though imperfect, this oscillatory behavior is removed when density dissipation was inserted as can be seen from the other figures.

Conclusion

For the systems of Euler equations, we have found that the growth of density fluctuations is one possible root of shock instability. This crucial and necessary discovery in analyzing the anomalies in shock capturing schemes which has not been found in the previous studies. In one dimension, other than approaching vacuum state, instability is solely due to the growth in density fluctuations. This can be depicted numerically in the form of slowly moving shocks. Applying the artificial dissipation to the density equation seems to remove the post shock oscillations and hence the instability. This is significant contribution to initiate a proper cure to the carbuncle phenomena and the slowly moving shocks especially in higher dimension. All in all, we would like to highlight the fact that by using the linearized analysis on the system of Euler equations, we have found a common factor contributing to the shock instability and the problem is solved by treating the proper source.

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