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Elicitation of Bayesian Belief Network (EBBN) using Z-Number Approach

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Bayesian Network (BN) is established in a wide variety of applications to provide cause-effect relationships of variables in a compact manner. It makes use of expert domain knowledge when actual data is not available. One of the available methods in reducing the expert burden in elicitation task of BN is an Elicitation of Bayesian Belief Network (EBBN). It requires only a limited amount of elicited probabilities from the expert to derive the conditional probability values of the variables. The current elicited-probability methods are stated in a crisp way, but expert opinion is usually expressed in linguistic terms to illustrate the judgment. This study proposes a new elicitation procedure by incorporating EBBN with a Z-Number approach, a 2-tuple fuzzy numbers to represent the linguistic terms. Besides the human subjective judgment, the Z-Number has advantages of including the confidence of the evaluation, thus providing a more reliable final outcome. A case study example of elicitation on a well-known medical diagnostic network is presented to illustrate how the method works in practice.
Keywords: Bayesian Network, EBBN, Expert elicitation, Z-Number, Linguistic terms.

I. Introduction

Bayesian Network (BN) is a causal network or graph of dependency based on uncertainty that can represent discrete and continuous variables. It is able to capture the probabilistic relationship between variables, combine different sources of data and does not require a specific distribution type to the data like other statistical techniques (Neapolitan, 2004). A BN gives a graphical representation of events which contains the parent's node (variables that are causes of a particular node) and child's node (the consequence of that node) and causal relationship between the two nodes is quantified by the conditional probabilities which are represented in a conditional probability table (CPT). Each probability in a CPT represents the probability of a certain state in a child node given a set of parent states. The causal relationship and values of the CPT are usually estimated from available data, or can be determined on the basis of expert knowledge.

The procedure to obtain information in BN from expert is called expert elicitation (Fenton and Neil, 2011). However, the involvement of expert elicitation is a big challenge such that the elicitation task requires a tiresome or repetitive work to perform transmission of knowledge even for a simple BN (Jensen, 1996). One of the ways is reducing the burden of the experts by decreasing the number of conditional probabilities to be elicited (Mkrtchyan et al., 2015). In the discrete case of BN, among methods available to generate a full CPT from a few information such as likelihood method

(Kemp-Benedict, 2008), weighted-sum algorithm (Das, 2004) and Elicitation of Bayesian Belief Network (EBBN) (Wisse et al., 2008). The likelihood method requires experts to provide weighting factor for each state in child and parents nodes, whereas the weighted sum algorithm is based on the concept of compatible parental configuration. Both of these methods can be hard for an expert to determine the required information. The EBBN has advantages as it requires only small number of probability values to be elicited in order to generate full CPT.

In the traditional EBBN procedure, expert has to give a precise numerical probability values to infer their belief. Ramli, Ghani, Hashim, and Zulkarnain (2015) used a well-known numerical probability scale in EBBN procedure as an aided tool to simplify the task of the probability assessment. Probability scale is a horizontal or vertical line divided into several intervals showing the linguistic terms (such as improbable, uncertain, expected, probable) assigned to corresponding probability values to help expert put his belief (Renooij and Witteman, 1999). They found that the use of a probability scale tends to have a scale bias, i.e. expert tend to use the point-based value assigned without considering the spread of other values over the scale. Moreover, non-statistician expert struggled to provide the subjective probabilities especially when involving dependability of many variables.

Due to inefficient utilisation of precise probability value for an expert elicitation process in a BN, fuzzy sets has been employed to provide subjective judgments to infer the belief (Kabir et al., 2015, Yang et al., 2008, Yazdi and Kabir, 2017, Zoullouti et al., 2015). A fuzzy probability is a representation of imprecise linguistic probabilities with a fuzzy subset. However, fuzzy sets face the limitation of not taking into account the degree of reliability or confidence of the judgment. Zadeh (2011) introduced Z-number, a new concept of fuzzy numbers that able to manage the uncertainty of information by adding a reliability level to fuzzy linguistic values. A Z-number is a 2-tuple fuzzy numbers, where the first component is the restriction of the information and the second part is the reliability, confidence or strength of truth of the first component. There are few studies applying Z-numbers in different domains such as decision-making problems (Aboutorab et al., 2018, Azadeh et al., 2013, Kang et al., 2012, Mohamad et al., 2014, Yaakob and Gegov, 2015), earned value management (Salari et al., 2014) and psychological research (Aliev and Memmedova, 2015). Research shows that Z-number has the ability to describe the human judgment since it includes the reliability of information.

In this study, we propose a new elicitation method by integrating the Z-number concept in EBBN elicitation procedure, and a procedure framework is proposed. This procedure consists of four major phases, i) identification of uncertainty problems in a BN model, ii) expert assessment based on the Z-numbers concept, iii) conversion of a Z-number to a crisp value, and iv) the derivation of CPT through EBBN method. This procedure will effectively overcome the address constraint in traditional EBBN elicitation method.

II. The EBBN Method

The EBBN method is developed by Wisse et al. (2008) and has been patented by United States Patent in 2013 (Wisse, 2013). Since then, the method has been recognized as one of the methods that able to reduce the expert assessment burden in eliciting a BN model (Hansson and Sjökvist, 2013, Knochenhauer et al., 2013, Werner et al., 2017, Zhang and Thai, 2016). It requires only a limited amount of elicited probabilities from expert and uses piece-wise linear interpolation function to derive the full CPT. Some definitions of the terms in the EBBN are defined as follows:

X_c — Discrete random variable of a child node, with $x_{c,min}$ and $x_{c,max}$ are the minimum and maximum value of X_c, respectively

$pa(X_c)$	The parent nodes of the X_c
$P(X_c\|a_{x_c})$	The probabilities of child node in the states of parent nodes, $pa(X_c)$
a_{x_c}	The assignment of the states of the parent nodes
a_{neg}	The assignment of the best combination of the states of parent nodes for low state ordered of X_c
a_{pos}	The assignment of the best combination of the states of parent nodes for high state ordered of X_c
$a_{neg,k+}$	The assignment of $pa(X_c)$ in which X_k in its most favorable state and all $X_m \in pa(X_c) \sim X_k$ are in their least favorable state for low values of X_c
$S^+(X_k, X_c)$	The parent nodes, $X_k \in pa(X_c)$ has a positive influence on X_c, i.e. observing a higher value for X_k does not reduce the higher values of X_c, regardless the values of other variables $pa(X_c) \sim X_k$
$S^-(X_k, X_c)$	The parent nodes, $X_k \in pa(X_c)$ has a negative influence on X_c, i.e. observing a higher value for X_k does reduce the higher values of X_c, regardless the values of other variables $pa(X_c) \sim X_k$

In a nutshell, the EBBN method starts with minimum assessment from the expert before the full CPT for a network can be determined. It starts with ordering of the states of the child node, X_c and the states of the parent node, X_k and determine that X_k has either positive, $S^+(X_k, X_c)$ or negative influence, $S^-(X_k, X_c)$ on X_c. Next, the probabilities of child node in the best states of parent nodes, $P(X_c|a_{x_c})$ and probabilities to determine the parent's weight, $P(X_c = x_{c,max}|a_{neg,k+})$ and $P(X_c = x_{c,min}|a_{neg,k+})$ are assessed. The weight for each parent node $X_k \in pa(X_c)$ is calculated as:

$$w_k = \frac{1}{2}\frac{\delta_k^+}{\sum_{l:X_l \in pa(X_c \delta_l^+}} + \frac{1}{2}\frac{\delta_k^-}{\sum l{:}X_l \in pa(X_c)\delta_l^-} \quad (1)$$

with

$$\delta_k^+ = P(X_c = x_{c,max}|a_{neg,k+}) - P(X_c = x_{c,max}|a_{neg})$$

$$\delta_k^- = P(X_c = x_{c,min}|a_{neg}) - P(X_c = x_{c,min}|a_{neg,k+})$$

A CPT can be determined in two steps procedure. In the first step, $P(X_c = x_c)$ is estimated as a function of joint influence factor I_{joint} , $f_{x_c}(I_{joint})$ for each sate x_c of X_c. I_{joint} is a function to express the general tendency of all parents' influences together, $pa(X_c)$ on X_c. It can be derived as:

$$I_{joint}(a) = \frac{\sum_{k:X_k \in pa(X_C)} I_{ind}(x_k).(rank(x_k)-1)}{\sum_{k:X_k \in pa(X_C)}(rank(x_{k,max})-1)} \quad (2)$$

The function can takes on values in the range of 0 to 1, i.e. $I_{joint}(a) \in (0,1)$, indeed $I_{joint}(a_{neg}) = 0$ and $I_{joint}(a_{pos}) = 1$. In addition, individual influence factor, I_{ind} that contains information about the influence of each parents individually are also calculated, as follows:

$$I_{ind}(x_k) = \begin{cases} \frac{rank(x_k)-1)}{(rank(x_{k,max})-1)} & \text{if } S^+(X_k, X_c) \\ \frac{rank(x_{k,max})-rank(x_k)}{rank(x_{k,max})-1} & \text{if } S^-(X_k, X_c) \end{cases} \quad (3)$$

where x_k, max is the highest value of X_k. The function can takes on values in the range of 0 to 1, i.e. $I_{ind}(x_k) \in (0,1)$. To account for both the general tendency and the individual influences, the intervals of each parent nodes are determined as $[min(I_{ind}(x_k), I_{joint}(a)), max(I_{ind}(x_k), I_{joint}(a))]$. The piece-wise linear function $f_{x_c} : [0,1] \to [0,1]$ is then constructed through this intervals to obtain the full CPT as the weighted average over the distributions $P_k(X_c|a)$. The use of linear interpolations ensures that $\sum^{x_c} f_{x_c}(i) = 1$ coherent with total of probabilities of different values of X_c , $\sum^{x_c} P(X_c) = 1$. Details of EBBN procedure are discussed in the following.

III. The Z-numbers concept

Zadeh Zadeh (1965) introduced a concept of fuzzy set to deal with uncertain conditions of real life situations. In addition, his his new novel notion of Z-numbers has a greater capability to express the impreciseness of real life problems since it includes the reliability of information (Zadeh, 2011). This section discussed some definitions and basic concepts of fuzzy set, linguistic terms, fuzzy number and Z-numbers as follows, reviewed from García-Cascales and Lamata (2007), Wang (1997), Zadeh (1965, 2011):

Definition 1. *A fuzzy set of A in universe of discourse X is illustrated as:*

$$A = \langle x, \mu_A(x) \rangle \,|x \in X$$

where $\mu_A(x)$ is the membership function of A which maps each element x in X to a real interval [0,1].

Definition 2. *A fuzzy number A is a fuzzy subset of X can be described using triangular fuzzy number, $\widetilde{A} = (a_1, a_2, a_3)$, where the membership function $\mu_{\widetilde{A}}(x)$ as:*

$$\mu_{\widetilde{A}}(x) = \begin{cases} 0 & x < a_1 \\ \frac{x-a_1}{a_2-a_1} & a_1 < x \leq a_2 \\ \frac{a_3-x}{a_3-a_2} & a_2 < x \leq a_3 \\ 0 & x > a_3 \end{cases}$$

Definition 3. *A linguistic variable is variable with value is expressed in linguistic terms such as low, medium, high, etc. Generally, linguistic term is represented by fuzzy numbers.*

Definition 4. *A Z-number denoted by $Z = (\widetilde{A}, \widetilde{R})$ is an ordered pair of fuzzy numbers that contains two main components. The first component is the fuzzy restriction on the values of X can take, $\widetilde{A}$; while the second component is the reliability or the strength of confidence of the first component, $\widetilde{R}$. Generally, both component of $\widetilde{A}$ and $\widetilde{R}$ are described in linguistic terms and presented in a fuzzy number form.*

IV. Proposed Procedure

The proposed elicitation in EBBN procedure using Z-numbers is defined as following four major steps:

STEP 1: Identify the uncertainty problems in a BN model

The identification of the uncertainty problems in a BN model determines whether the expert elicitation method is relevant approach to deal with it. For applying EBBN method to solve the uncertainty, the BN conceptual model that contained must that satisfy the following requirements: 1) the CPT of the child node can be subjectively assessed, 2) the CPT of the child node is needed to have two or more parents, and 3) the states of both parent and child node can be ordered.

STEP 2: Expert assessment of probabilities using Z-numbers

The EBBN method requires the expert to assess both qualitative information (such as ordering of the states of parent and child node, type of influence and dominance of parent node) and quantitative information (typical probabilities). Without loss of generality, the Z-numbers comes in when it only involves probability assessment in the beginning part of the procedure. Instead of providing numerical data, expert use Z-number concept which is expressed in linguistic terms to represent their belief. The following assessment is needed from the expert for the derivation of the CPT of child node:

i. *Ordering.* First, the method requires that the states of the child node, X_c and the states of the parent nodes, $X_k \in pa(X_c)$ be ordered from the lowest to the highest value. Note that X_k can have influence on X_c, either a positive, $S^+(X_k, X_c)$ or a negative, $S^-(X_k, X_c)$

ii. *Typical probabilities.* The assignment of best combination of the states of parent nodes for each state in child nodes, $pa(X_c) = a_{x_c}$ is determined, next the probability of $P(X_c|a_{x_c})$ is assessed

iii. *Weights.* The determination of each parent's weight w_k is by using Eq. (1). In this case, the values of $P(X_c = x_{c,max}|a_{neg,k+})$ and $P(X_c = x_{c,min}|a_{neg,k+})$ are assessed.

iv. *Dominance*. The EBBN method includes the expression of dominance of parent node, whether X_k has either a positive or negative dominance over X_c

The assessment of probabilities in Step (ii) and (iii) are replaced with assessment using Z-numbers form, $Z = (\widetilde{A}, \widetilde{R})$. $\widetilde{A}$ represents the restriction values of the likelihood of occurrence in the child node given the parent node; while $\widetilde{R}$ represents the reliability or the expert confidence of the judgment.

STEP 3: Converting Z-number to crisp value

In order to be able to generate the full CPT in EBBN method, the assessment using Z-number in STEP 2 must be converted into a crisp (probability) value. This study utilized transformation method proposed by Kang et al. (Kang et al., 2012) that make use canonical representation of multiplication operation on triangular fuzzy number from Chou (Chou, 2003) as follows:

$$
\begin{aligned}
w(Z_{ij}) &= w(\widetilde{A}, \widetilde{R}) \\
&= \widetilde{A} \bigotimes \widetilde{R} \\
&= (a^l_{ij}, a^m_{ij}, a^u_{ij}) \bigotimes (r^l_{ij}, r^m_{ij}, r^u_{ij}) \\
&= ((a^l_{ij} + 4 \times a^m_{ij} + a^u_{ij})/6) \times ((r^l_{ij} + 4 \times r^m_{ij} + r^u_{ij})/6)
\end{aligned} \tag{4}
$$

STEP 4: Derivation of CPT

The CPT can be determined in two steps procedure. In the first step, $P(X_c = x_c)$ is estimated as a function of joint influence factor, I_{joint} , $f_{x_c}(I_{joint})$ that represent general preferences of all parents' nodes. Next, individual influence factor, I_{ind} that contains information about the influence of each parents individually are also calculated. The piece-wise linear function $f_{x_c} : [0, 1] \rightarrow [0, 1]$ is constructed through $[min(I_{ind}(x_k), I_{joint}(a)), max(I_{ind}(x_k), I_{joint}(a))]$ to derive the desired CPT as the average probabilities, denoted with $\overline{P_k(X_c = x_c|a_{x_c})}$ as follows:

$$
\begin{aligned}
&P(X_c = x_c|pa(X_c) = a) \\
&= \sum_{i:X_{p_i}|pa(X_c)} w_i \frac{\int_{I_{min,k}}^{I_{max,k}} f_{x_c}(I_{joint}(a))dI_{joint}(a)}{I_{max,k} - I_{min,k}}
\end{aligned} \tag{5}
$$

where

$$
\begin{aligned}
I_{max,k} &= \max(I_{ind}(x^j_{p_i}), I_{joint}(a)) \\
I_{min,k} &= \min(I_{ind}(x^j_{p_i}), I_{joint}(a))
\end{aligned}
$$

V. Illustrative Example

This section presents a case study to demonstrate the application of the proposed method. We use the Cardiac output node from a well-known ALARM (A Logic Alarm Reduction Mechanism) network available in Netica (Norsys Software Corp., 2018). The ALARM network is a complex belief network of medical diagnostic system for patient monitoring (Beinlich et al., 1989). As in Figure **??**, the node Cardiac output is depends on Heart rate (HR) node and Stroke volume (SV) node. Each of them represented with three states level [low, normal, high]. In this example, we will show how to determine the full CPT for Cardiac output in normal condition when HR is low and SV is normal, $P(X_c = normal|a_{lownormal})$.

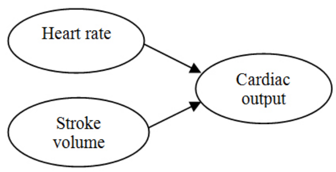

Figure 1: The node Cardiac output from the network ALARM

A. States ordering

Firstly, the method requires that the states of both parent and child nodes is ordered form low to high the values and determine its influences.

In this study, the states of the child node is ordered as $X_c = [low, normal, high]$ and the assignment states of parents a_{x_c} are $a_{low} = [HR = low, SV = low]$, $a_{normal} = [HR = normal, SV = normal]$ and $a_{high} = [HR = high, SV = high]$. Both parent nodes have positive influences, $S^{+}(X_k, X_c)$ to the Cardiac node.

B. Expert assessment

For the $P(X_{cardiac}|a_{x_c})$ assessment, five linguistic terms from Chang and Hwang (Chen and Hwang, 1992)) is being adopted to represent both of the restriction, $\widetilde{A}$ and reliability part, $\widetilde{R}$. Each linguistic term is represented by the triangular fuzzy numbers in Table 1 and graphically as in Figure 2. Table 2 shows the assessment made by the expert using Z-number concept.

Table 1: Linguistic terms for restrictions and reliability part

Linguistic terms	Likelihood of the child states, $\widetilde{A}$	Reliability, $\widetilde{R}$
Very Low (VL)	(0, 0.1, 0.2)	(0, 0.1, 0.2)
Low (L)	(0.1, 0.25, 0.4)	(0.1, 0.25, 0.4)
Medium (M)	(0.3, 0.5, 0.7)	(0.3, 0.5, 0.7)
High (H)	(0.6, 0.75, 0.9)	(0.6, 0.75, 0.9)
Very High (VH)	(0.8, 0.9, 1.0)	(0.8, 0.9, 1.0)

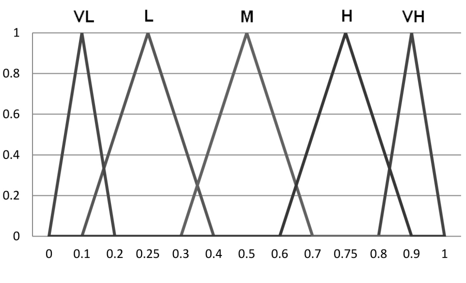

Figure 2: Linguistic terms representation

C. Converting Z-Number to crisp value

The Z-number forms are transformed into crisp (probability) values using Eq.

Table 2: Z-number assessment for node "Cardiac" and converted probability values

$X_{cardiac}$	$P(X_{cardiac}|a_{x_c})$	Z-Number assessment	Normalized probability value
low	$P(X_c = low|a_{low})$	(VH,VH)	0.82
	$P(X_c = normal|a_{low})$	(VL,VH)	0.09
	$P(X_c = high|a_{low})$	(VL,VH)	0.09
normal	$P(X_c = low|a_{normal})$	(VL,VH)	0.11
	$P(X_c = normal|a_{normal})$	(VH,H)	0.79
	$P(X_c = high|a_{normal})$	(VL,VH)	0.11
high	$P(X_c = low|a_{high})$	(VL,VH)	0.09
	$P(X_c = normal|a_{high})$	(VL,VH)	0.09
	$P(X_c = high|a_{high})$	(VH,VH)	0.82

(4). For example, the assessment of $P(X_c = normal|a_{normal} = (VH, H) = ((0.8, 0.9, 1.0), (0.6, 0.75, 0.9))$ is converted as follows:

$$P(X_c = normal|a_{normal})$$
$$= ((0.8 + 4 \times 0.9 + 1.0)/6) \times ((0.6 + 4 \times 0.75 + 0.9)/6)$$
$$= 0.675$$

While the Z-number assessment for $P(X_c = low|a_{normal}) = (VL, VH)$ and $P(X_c = high|a_{normal}) = (VL, VH)$ are converted to 0.09. We have employed a simple normalized procedure to ensure that the total probability of the transformed values in any row of the CPT is 1.0. The normalized value is obtained as follows:

$$P(X_c = low|a_{normal}) = \frac{P(X_c = low|a_{normal})}{\sum P(a_{normal})}$$
$$= 0.675/((0.675 + 0.09 + 0.09))$$
$$= 0.79$$

Similarly, $P(X_c = normal|a_{normal})$ and $P(X_c = high|a_{normal})$ are obtained as 0.11. Thus, $\sum P(a_{normal}) = 1.0$. The normalized converted values of other assessments can be found in the last column in Table 2.

D. Deriving the CPT

Once the crisp probability values are obtained, following related functions can be calculated:

i. w_k using Eq. (1), we obtain the following weights of $HR = 0.7708$ and $SV = 0.2292$.

ii. $I_{joint}(a)$ using Eq. (2), we have the following $I_{joint}a_{low} = 0$, $I_{joint}a_{normal} = 0.25$ and $I_{joint}a_{high} = 1$

iii. $I_{ind}(x_k)$ for each state of parent nodes are obtained using Eq. (3): $I_{ind}HR$ and $I_{ind}SV$=[0 0.5 1]

The illustration of piece-wise linear function $f_{normal} : [0, 1]$ through the HR and SV intervals in obtaining $P(X_c = normal|a_{lownormal})$ are shown in Figure 3. For the respective CPT of $P(X_c = normal|a_{lownormal})$, the intervals for parent nodes are $HR = [0, 0.125]$ and $SV = [0.125, 0.25, 0.5]$. Using Eq. (5), the conditional probability is obtained as:

$$P(X_c = normal|a_{lownormal}) = \sum_{k:HR,SV} \overline{w_k.P(X_c = normal|a_{lownormal})}$$
$$= (0.7708 \times 0.265) + (0.2292 \times 0.654)$$
$$= 0.354.$$

Table 4 in Appendix shows the full CPT values for the node Cardiac output using the proposed method and obtained from original database in the Netical tool. When we look up the probabilities in the original CPT, we find that the generated probabilities from the proposed method are 'not far off'. Next we will investigate how well our proposed method approximates the probabilities.

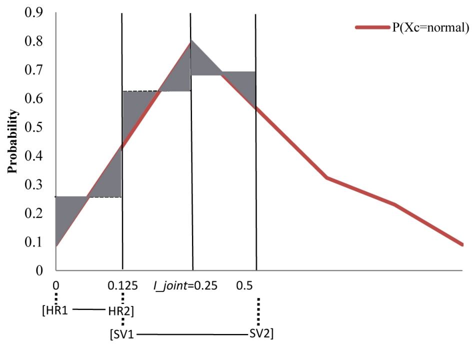

Figure 3: Linear function of f_{normal} through the $HR = low$ and $SV = normal$ intervals

E. Performance of proposed method

To further validate and assess the performance the proposed method, we conducted two types of analyses: performance measures and sensitivity analysis. For comparison, full CPT of the Cardiac output from the ALARM network is obtained based on two different elicitation methods: the EBBN using crisp probability values and EBBN using Z-number (see Table 4 in Appendix).

1. Performance measure

Assuming that we have the knowledge of the true probability value for a certain CPT, we can assess the quality of approximation to that CPT using specific statistical measures. The most common performance measures are error-based and distance/dissimilarity metrics. Hansson and Sjökvist (2013) used mean absolute difference between the original CPT and the CPT generated by elicitation methods. Zagorecki and Druzdzel (2013) used Euclidean Distance and Kullback-Leibler divergence to measure the dissimilarity between two CPTs. In this study, we have used the following three measures to assess the performance of the proposed method:

(a) Mean Absolute Error (MAE): is an average of the absolute difference between the predicted and the true value

(b) Mean Absolute Percentage Error (MAPE): is an average absolute percent error for each pair of data point

(c) Root Mean Square Error (RMSE): or Root Mean Square Deviation (RMSD) is a measure of the variance of the residuals (prediction errors). As the square root of a variance, RMSE can be interpreted as the standard deviation of the unexplained variance, and indicates how close data points are to the predicted values

(d) Normalized Root Mean Squared Error (NRMSE): computes the normalized root mean squared error where lower values indicate less residual variance

(e) Euclidean distance: or Euclidean metric examines the root of square differences between two points

(f) Kullback-Leibler (KL) divergence: also called relative entropy is a measure of differences between two probability distribution. It origins from information theory that can helps us to measure just how much information we lose when we choose an approximation

(g) Jensen-Shannon (JS) divergence: a measure to assess the similarity between two conditional probability distributions based on Kullback-Leibler divergence and Shannon's entropy. We use the square root of JSD that can be used as a pair-wise distance metric and satisfies properties of a distance measure.

For the above three measures, the smaller the measure, the better performance of the elicitation method. Results of the comparison of the performance of the two elicitation methods are presented in Table 3.

Table 3: Performance of the elicitation methods on the measures

Measure	Crisp EBBN	Z-number EBBN
MAE	0.1896	0.2293
MAPE	4.9983	7.1719
RMSE	0.2893	0.2886
NRMSE	0.2982	0.2975
Euclidean	1.5032	1.4996
KL	0.4297	0.4276
JS	0.3213	0.3393

Note: Underlined values indicate the best score for the two methods.

The underlined number in Table 3 indicates that the corresponding elicitation method has the best performance for that measure. Crisp EBBN have a better performance based on MAE, MAPE and JS divergence measures, while Z-number EBBN is better according to RMSE, NRMSE, Euclidean and KL divergence measure. For four of the measures, the Z-number EBBN are distinctly better however the result of crisp EBBN are relatively close by and consistent.

2. Sensitivity analysis

Sensitivity analysis refers to procedure in investigating the effects of inaccuracies in the BN parameters on the model's output. It determines how posterior probability change when prior probability of parent nodes take different value (Ren et al., 2008). In this case study, we choose one situation where "High" Cardiac output, a_{high} has been observed and we want to calculate the posterior probability of $P(HR_{high}|a_{high})$ and $P(SV_{high}|a_{high})$. The assumption that the prior probability using crisp EBBN is subject to uncertainty of ± 0.5 and using Z-number EBBN is subject to five different linguistic assessment has been made (see Table 5 in Appendix). As can be seen in the last column in Table 5, the change between prior and posterior probabilities of parent nodes HR_{high} and SV_{high} for both elicitation methods clearly indicate that the posterior probability steadily increases with respect to prior probability. This trend shows that the proposed assigned probabilities are rational and stable to input variability (Ren et al., 2008).

Derived from above performance measure and sensitivity analysis, we conclude that the Z-number EBBN has a comparable performance to traditional crisp EBBN elicitation method, thus can be an acceptable elicitation method.

VI. Conclusion

A new expert elicitation method by integrating the EBBN and Z-number concept is presented in this paper. The Z-number uses linguistic terms to represent the expert's evaluation of the information and also the expert confidence in providing the evaluation. The proposed procedure consists of four important steps. In the first step, the uncertainty problem in a particular BN model that need the expert elicitation is identified. The model structure also must have fulfilled the EBBN requirements. Then, expert making assessment of required CPT value using the Z-number concept. Next, in order to generate precise probability value in CPT, the Z-number assessment is converted to a crisp value. Finally in the third stage, the full CPT is derived as the weighted average over the conditional probability distributions. A case study example of well-known belief network, an ALARM network shows that the procedure works in practice. The performance of the method is comparable with the traditional EBBN method. The elicitation approach in this study can fill the methodological gaps in expert elicitation approach of a BN model. By integrating Z-numbers, we believe it have added two features that benefits the classical EBBN method. Firstly, it is possible to address human uncertainty in judgment by means of linguistic variable. Secondly, the component of reliability of Z-numbers able to include expert's confidence of their estimates. In the future, we would like to test the method on more examples and improve the performance of the method.

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Appendix

Table 4: The CPT for the node Cardiac output from the network ALARM database, generated from the traditional EBBN and proposed method

Heart Rate	Stroke Volume	Cardiac Output								
		Original database			EBBN			Z-number EBBN		
		Low	**Normal**	**High**	**Low**	**Normal**	**High**	**Low**	**Normal**	**High**
Low	Low	**0.98**	**0.01**	**0.01**	**0.95**	**0.03**	**0.02**	**0.82**	**0.09**	**0.09**
Low	Normal	0.95	0.04	0.01	0.63	0.32	0.04	0.53	0.35	0.12
Low	High	0.30	0.69	0.01	0.24	0.56	0.20	0.24	0.50	0.26
Normal	Low	0.95	0.04	0.01	0.22	0.66	0.12	0.28	0.56	0.17
Normal	Normal	**0.04**	**0.95**	**0.01**	**0.06**	**0.90**	**0.04**	**0.11**	**0.79**	**0.11**
Normal	High	0.01	0.30	0.69	0.04	0.49	0.47	0.10	0.44	0.46
High	Low	0.8	0.19	0.01	0.07	0.37	0.56	0.14	0.38	0.48
High	Normal	0.01	0.04	0.95	0.03	0.29	0.68	0.10	0.32	0.58
High	High	**0.01**	**0.01**	**0.98**	**0.02**	**0.03**	**0.95**	**0.09**	**0.09**	**0.81**

Note: Bold numbers indicate assigned probabilities replaced the generated probabilities.

Table 5: Sensitivity analysis result between prior and posterior probabilities

Variable state	Crisp EBBN			Z-number EBBN			
	Prior probabilities	Posterior probabilities	Change of prior and posterior probabilities (%)	Z-number assessment	Converted prior probabilities	Posterior probabilities	Change of prior and posterior probabilities (%)
HR_{high}	0.20	0.5529	1.7644	(L,M)	0.13	0.4331	2.4645
	0.25	0.6162	1.4649	(L,VH)	0.23	0.5926	1.6338
	0.30	0.6672	1.2239	(M,H)	0.38	0.7308	0.9487
	0.35	0.7090	1.0258	(M,VH)	0.45	0.7738	0.7196
	0.40	0.7441	0.8602	(H,H)	0.56	0.8418	0.4965
SV_{high}	0.20	0.3751	0.8754	(L,M)	0.13	0.2641	1.1129
	0.25	0.4444	0.7778	(L,VH)	0.23	0.4176	0.8559
	0.30	0.5069	0.6898	(M,H)	0.38	0.5950	0.5867
	0.35	0.5635	0.6101	(M,VH)	0.45	0.6621	0.4714
	0.40	0.6150	0.5376	(H,H)	0.56	0.7530	0.3386