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Robustness Analysis of Model Parameters for Sediment Transport Equation Development

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Robustness analysis of model parameters for sediment transport equation development is carried out using 256 hydraulics and sediment data from twelve Malaysian rivers. The model parameters used in the analyses include parameters in equations by Ackers-White, Brownlie, Engelund-Hansen, Graf, Molinas-Wu, Karim-Kennedy, Yang, Ariffin and Sinnakaudan. Seven parameters in five parameter classes were initially tested. Robustness of the model parameters was measured on the statistical relations through Evolutionary Polynomial Regression (EPR) technique and further examined using the discrepancy ratio of the predicted versus the measured values. Results from analyses suggest $\frac{U^*}{V}$ (ratio of shear velocity to flow velocity) and $\frac{R}{d_{50}}$ (ratio of hydraulic radius to mean sediment diameter) to be the most significant and influential parameters for the development of sediment transport equation.

Keywords: reliability assessment; sediment transport

I. INTRODUCTION

Sediment transport is important in the fields of sedimentary geology, geomorphology, civil and environmental engineering. Knowledge of sediment transport is essential to help solve problems of deposition in navigation canals obstructing water traffic. Deposition problems in lakes are causing overflow with even brief storm events which affect the property at the perimeter of the lake. Local scouring around hydraulic structures and bridge piers, as well as bed and bank instability, is resulting from head-cutting due to sand and gravel mining activities. In sediment transport analysis, two types of loads are considered in the calculation, and they are the suspended load and bed load. Suspended load are loads that move in suspension with a diameter size of 0.0625mm and larger. Usually, sand size of 2mm diameter and smaller would remain buoyant and easily lifted depending on the hydraulics force of water. While bed loads are the bigger fractions that move by the traction force of the flow. Having to choose the most reliable model require a model assessment to be carried out. Assessment can be carried out using statistical relations and EPR technique of all model parameters of an equation.

EPR is a data-driven hybrid regression technique developed by Giustolisi and Savic (2006). EPR has been used successfully in solving several problems in civil engineering, e.g. (Ghorbani & Hasanzadehshooiili 2018; Doglioni & Simeone 2017; Yin *et al.*, 2016; Giustolisi *et al.*, 2008; Savic *et al.*, 2006). It constructs symbolic models by integrating the best features of numerical regression (Draper & Smith, 1998), with genetic programming and symbolic regression (Koza, 1992).

This paper aims to establish the most significant and influential parameters for use in the development of

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sediment transport equation. Parameter test analyses are carried out using statistical analysis and EPR technique.

II. SEDIMENT TRANSPORT EQUATIONS – EVALUATIONS AND PERFORMANCES

Sediment transport equations were developed mainly from flume experiments of shallow flows with depths not exceeding 0.5m (Ackers & White 1973; Yang 1973; Engelund & Hansen 1967). The derived equations are only suitable for use in channels of uniform flow and cross-section. Some adjustment on the predicted values may be required if used on natural rivers.

Evaluations of established sediment transport equations for use in Malaysian rivers have been carried out in the past (Department of Drainage and Irrigation 2009; Chang *et al.*, 2005). The studies have identified two equations, Yang and Engelund-Hansen of acceptable performance. Yang derived his equation using data from the Yellow river consisting primarily of fine silts and clays. The suitability of Yang equation to predict sediment load in Malaysian rivers can be attributed to the similarity in sediment characteristics to China where most upland erosions originated from the loess region. The local researchers, Saleh *et al.* (2017), Sinnakaudan *et al.* (2006) and Ariffin (2004) have made efforts to develop sediment transport equations that are exclusive for Malaysian rivers.

Table 1 illustrates the performance of nine sediment transport equations on Malaysian rivers, namely of Sungai Perak, Sungai Kemaman, Sungai Pergau and Sungai Kurau and the corresponding model parameters modified after Saleh *et al.* (2017). The nine equations used in analyses are Molinas and Wu (2001), Karim (1998), Yang (1973), Graf (1971) and Engelund and Hansen (1967). Table 2 shows the data range for d_{50} used in the analyses (Saleh 2016).

Table 1. Performance of nine sediment transport equations on Malaysian rivers and the corresponding model parameters (modified after Saleh *et al.*, 2017)

Equation	Model parameters	Data within (0.5-2.0)DR	Percentage
Ackers-White (1973)	$\frac{V}{u_*}, \frac{F_{gr}}{A}, d_{35}, d_{50}, \frac{\Delta g}{\upsilon^2}$	14	40
Ariffin (2004)	$R/d_{50}, U^*/W_s, U^*/V, V^2/gy_o$	29	83
Engelund and Hansen (1967)	$\gamma_s, V^2, \sqrt{d_{50}/g(\gamma_s/\gamma_w)}, \sqrt[1.5]{\tau/(\gamma_s-\gamma_w)d_{50}}$	6	17
Graf (1971)	$(S_s-1)d_{50}/RS_o, C_vVR/\sqrt{g(S_s-1)d_{50}^{\ 3}}$	4	11
Molinas and Wu (2001)	V, g, w_s, d_{50}, h	22	63
Karim (1998)	$\frac{u_*}{w_s}, V, g, d_{50}$	20	57
Sinnakaudan*et al.* (2006)	$VS_o/\omega_s, R/d_{50}, \sqrt{g(S_s-1)d_{50}^{\ 3}}/VR$	24	69
Yang (1973)	$, \frac{u_*}{W_S}, \frac{VS}{W_s}, R_e, \frac{w_s d_{50}}{\upsilon}$	9	26

Table 2. Total bed material load and data range for d_{50} (Saleh 2016)

Equation	Data Range (mm)
Graf	$0.09 < d_{50} < 2.78$
Engelund and Hansen (E-H)	$0.19 < d_{50} < 0.93$
Yang	$0.137 < d_{50} < 1.71$
Ackers and White	$0.04 < d_{50} < 4.94$
Ariffin	$0.37 < d_{50} < 4.00$
Sinnakaudan*etal.*	$0.3711 < d_{50} < 4.00$

III. DATA SELECTION

Data used in this study comprised of data from the works of Saleh *et al.* (2017), Department of Irrigation and Drainage (2013), Ibrahim (2012) and Ariffin (2004). There are 256 hydraulics, and sediment data measured from twelve rivers in Malaysia and the data range is given in Table 3.

Table 3. Data range used in analyses

Parameter	Range
Total sediment load, T_j(kg/s)	0.0333-119.601
Flow, Q (m^3/s)	0.737-87.792
Velocity, V (m/s)	0.194-1.422
Depth of water, y_o (m)	0.22-3.23
Particle mean size, d_{50} (mm)	0.37-4.9109
Water surface slope, S_o	0.0003-0.0167
Fall velocity, W_s(m/s)	0.043-21.157
Hydraulic radius, R (m)	0.21-2.66

A. Performance of Sediment Transport Equations on Twelve Rivers by Various Investigators

Table 4 shows the range of hydraulics and sediment data used in the analyses that include width, velocity and median size of sediment load. Table 5 illustrates the performance of selected equations on Malaysian river data.

Table 4. Range of hydraulics and sediment data used in analyses

No	River	Total no of data	Width (m)	Velocity (m/s)	Median sediment size,d_{50} (mm)
1	Raia	41	17.3 25.6	0.478 – 0.76	0.5 – 1.6
2	Kulim	16	14- 18	0.303 – 0.872	3.00- 4.00
3	Kinta	20	24.6 - 28	0.42 – 0.651	0.4 – 1.
4	Kerayong	24	18	0.218 – 0.586	2.8 -3.0
5	Bernam	36	12 - 20	0.266 – 0.868	0.526 – 2.471
6	Semenyih	50	13.5 - 15	0.447 – 0.852	0.879 – 2 .288
7	Kampar	21	20.2 - 21	0.592 – 0.71	0.85 – 1.1
8	Langat	23	17 - 30	0.536 – 1.422	0.37- 2.13
9	Lui	92	15 -17	0.194 – 1.029	0.502 – 1.758
10	Jeli	3	31 - 46	0.43 -0.69	1.01- 2.62
11	Kurau	8	9 - 11	0.45 – 0.636	0.715 – 1.27
12	Pari	56	18 – 20.3	0.461 – 1.26	0.85 -3.1

B. Robustness Measurement

The robustness measurement of all model parameters based on five parameter classes was derived from studies carried out by Ariffin (2017; 2004), Azamathulla *et al.* (2010), Sulaiman (2009), Sinnakaudan *et al.* (2006), and Chang *et al.* (2005). The variables are, relative roughness on the bed (R/d_{50}) in flow resistance parameter class, stream-width ratio (B/y_o) in conveyance and shape class which are shear velocity ratio to fall velocity (U^*/ω_s) and fall velocity to shear velocity (ω_s/U^*), in sediment properties class is ratio of shear stress to average velocity (U^*/V) and dimensionless unit stream power (VS_o/ω_s) in mobility class and the last variable is velocity head ($v^2/2g$). The output variable selected for this model is concentration by volume or volumetric concentration (ratio of total sediment transport rate to flow rate) (Qt/Q).

Data were randomly divided into two sets: a training set for model calibration and an independent validation set for model verification. In dividing the data into their sets, the training and testing sets were selected to be statistically consistent; thus, represent the same statistical population, as recommended by Shahin *et al.* (2004). In total, 174 data cases (68%) of the 256 data cases are used for training and balance 82 data cases (32%) for use for validation. The statistical analysis showing measures of central tendency (mean values) and variability (standard deviation, minimum and maximum values) is given in Table 6. Table 7 lists the model parameters used in the robustness analysis.

Table 5. Performance of selected equations on Malaysian river data

No.	River	Total No of data	Percentage of data within DR of 0.5 – 2.0				
			Engelund and Hansen (1967)	Graf (1971)	Ariffin (2004)	Chang *et al.* (2005)	Sinnakaudan *et al.* (2006)
1	Raia	41	0	0	61	63	66
2	Kulim	16	0	56	88	0	75
3	Kinta	20	20	30	45	0	30
4	Kerayong	24	21	50	83	0	58
5	Bernam	36	39	17	25	11	28
6	Semenyih	50	30	8	56	30	68
7	Kampar	21	38	0	0	28	48
8	Langat	23	17	0	43	13	57
9	Lui	92	14	2	46	22	63
10	Jeli	3	33	67	0	0	33
11	Kurau	8	0	0	0	38	88
12	Pari	56	43	25	34	2	73

Table 6. Measures of central tendency and variability

Model parameters	Measures of central tendency and variability				
	Mean	Standard Deviation	Minimum	Maximum	Range
Total load, *(Qt/Q)*					
Training set	0.0003569	0.0001926	0.00010	0.00093	0.00
Testing set	0.0003974	0.0002484	0.00009	0.00215	0.00
Ratio of stream radius to median diameter of bed material (R/d_{50})					
Training set	428.19	216.87	106.90	1229.73	1122.84
Testing set	642.33	445.03	213.35	2800.00	2586.65
Stream width ratio to water depth (B/y_o)					
Training set	32.29	13.72	10.86	87.76	76.91
Testing set	33.08	14.89	9.29	74.70	65.41
Ratio of shear velocity to fall velocity (U^*/ω_s)					
Training set	0.56	0.26	0.01	0.99	0.98
Testing set	1.40	0.61	0.00	5.44	5.43
Ratio of fall velocity to shear velocity (ω_s/U^*)					
Training set	4.95	15.85	1.01	153.47	152.45
Testing set	3.41	24.33	0.18	225.08	224.89
Ratio of shear velocity to average velocity (U^*/V)					
Training set	0.18	0.08	0.07	0.47	0.39
Testing set	0.26	0.11	0.10	0.73	0.63
Dimensionless unit stream power $(VS_o/\ \omega_s)$					
Training set	0.01	0.01	0.00	0.02	0.02
Testing set	0.03	0.02	0.00	0.12	0.12
Velocity head$(V^2/\ 2g)$					
Training set	0.07	0.03	0.01	0.16	0.14
Testing set	0.08	0.03	0.01	0.14	0.13

Table 7. Parameters used in robustness analysis

Name	(R/d_{50})	(B/y_0)	(U^*/ω_s)	(ω_s/U^*)	(U^*/V)	(VS_0/ω_s)	$(V^2/2g)$
Model1	/	/	/	/	/	/	/
Model2	/	/	/		/		
Model3	/	/	/	/	/		
Model4	/	/	/	/	/	/	
Model5	/	/	/	/			
Model6	/	/	/	/		/	
Model7	/	/	/	/		/	/
Model8	/	/	/				/
Model9	/	/	/	/			/
Model10	/	/	/	/	/		/
Model11	/	/	/		/	/	
Model12	/	/	/			/	
Model13	/	/	/		/	/	/

Selection of the model parameters was carried out by trial-and-error approach in which a series of EPR models were trained using functions given in Table 8. A more detailed description of functions used for parameter selection can be found in the EPR Toolbox manual (Laucelli *et al.*, 2009).

Table 8. Functions used for parameter selection

Function	Expression Structure
f_0	$Y = sum(a_i*X_1*X_2*f(X_1)*f(X_2))+a_0$
f_1	$Y = sum(a_i*f(X_1*X_2))+a_0$
f_2	$Y = sum(a_i*X_1*X_2*f(X_1*X_2))+a_0$
LS	Least square
LSN	Non-negative least square

C. Model Approximations

Models are approximated using Least Square (LS) and Non-Negative Least Square (LSN) methods. Accuracy of the models was measured using the discrepancy ratio of 0.5 – 2.0. Discrepancy ratio is the ratio of predicted to measured values. Performance of all models is shown in Table 9. Table 10 illustrates the best performing models of the model groups with the corresponding model exponential relations. The model parameters established from the robustness test are used as inputs in the EPR model. A total of 666 new models from 13 groups have been generated using 174 data in training set using the functions given in Table 8.

Table 9. Performance results of the EPR models during training

Model group	Accuracy DR (0.5-2.0) (%)											
Model1	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
f0LS	70.11	70.11	70.11	70.11	74.71	77.59	75.86					
f1LS	69.94	69.94	69.94	69.94	74.56	75.72						
f2LS	71.10	71.10	58.38	71.10	71.10	65.32	63.01	63.01	75.72			
f0LSN	69.54	66.09	66.09	64.37	67.24	68.39	72.99	72.41	71.84	74.14	75.29	
f1LSN	69.94	66.47	66.47	64.74	67.63	69.94	73.99	73.41	72.83	75.14	76.30	
f2LSN	71.10	67.03	67.63	65.90	68.79	69.94	73.99	73.41	72.83	76.88	72.25	76.30
Model2	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
f0LS	69.54	69.54	69.54	69.54	69.54	69.54	69.54	69.54				

f1LS	69.94	69.94	69.94	69.94	69.94	69.94	69.94	69.94				
f2LS	71.10	87.86	87.86	78.61	78.61	**100.00**	**100.00**					
f0LSN	69.54	66.09	65.52	64.94	65.52	67.24	64.94	64.37	69.54			
f1LSN	69.94	66.47	65.90	65.32	65.90	67.63	65.32	64.74	69.94			
f2LSN	71.1	67.73	67.05	66.47	67.05	69.36	67.05					
Model3	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
f0LS	70.11	86.78	86.78	77.01	77.01	98.28	90.23					
f1LS	69.94	86.71	86.71	76.88	76.88	98.27	90.17					
f2LS	71.10	71.10	71.10	71.10	71.10	71.10	71.10	71.10	71.10			
f0LSN	69.54	66.09	65.52	64.94	65.52	67.24	65.52	64.94	70.11			
f1LSN	69.94	66.47	65.90	65.32	65.90	69.36	66.47	65.90	71.86			
f2LSN	71.10	67.63	67.05	66.47	67.05	69.36	67.05					
Model4	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
f0LS	70.11	70.11	70.11	70.11	70.11	70.11						
f1LS	69.94	69.94	69.94	69.94	69.94	69.94	69.94	65.90				
f2LS	71.10	71.10	76.88	76.88	76.30	76.30	75.14	72.83				
f0LSN	69.54	66.09	64.94	64.37	66.09	63.79	72.99	73.56	74.71	71.26		
f1LSN	69.94	69.47	65.32	64.74	66.47	65.32	74.57	74.57	76.30	72.83		
f2LSN	71.10	67.63	66.47	65.90	67.63	65.32	74.57	75.57	76.30	72.83		
Model5	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
f0LS	61.49	86.78	77.01	77.01								
f1LS												
f2LS												
f0LSN	60.92	66.09	66.67	67.82	67.24	67.82	68.39	68.39				
f1LSN	61.27	66.47	67.05	68.21	67.63	69.36	69.36					
f2LSN	67.63	68.21	68.21	68.79	69.36	69.36						
Model6	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
f0LS	61.49	86.78	67.82	77.01	77.01	67.82						
f1LS	61.27	86.71	67.63	76.88	76.88	67.63						
f2LS	67.05	87.28	77.46	78.03	78.03							
f0LSN	60.92	66.09	66.09	65.52	70.11	67.82	68.39	68.39	68.39	68.39	68.39	
f1LSN	61.27	66.47	66.47	65.90	70.52	69.36	69.36	69.36	69.36	69.36		
f2LSN	61.63	67.63	67.63	67.05	71.68	69.36	69.36	69.36				
Model7	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
f0LS	67.24	0	68.39	64.37	70.11	70.11	70.11	70.11				
f1LS	66.47	0	67.03	63.58	69.36	69.36	69.36	69.36				
f2LS	67.63	67.63	69.36	65.32	67.63	67.63						
f0LSN	66.67	66.09	66.67	72.99	72.99	69.54	66.67	71.26	70.69	68.39		
f1LSN	67.05	66.47	67.05	73.41	71.10	71.10	67.63	72.25	71.68	69.36		
f2LSN	67.63	67.63	68.21	67.05	69.05	69.36	69.36	76.30	73.41	73.99	74.57	
Model8	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**

f0LS	67.24	0	68.39	64.37	70.11	70.111	70.11					
f1LS	67.05	0	68.21	64.16	69.94	69.94	69.94					
f2LS	67.36	67.63	69.36	65.32	71.10	71.10	67.63					
f0LSN	66.67	66.09	67.24	70.69	70.69	70.69						
f1LSN	67.05	66.47	67.63	71.10	71.10	71.10						
f2LSN	67.05	67.05	68.21	71.68	71.68	71.68						
Model9	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
f0LS	67.24	86.78	0	66.09	66.09	66.67	66.67	66.67	66.67			
f1LS	67.05	86.71	0	65.90	65.90	66.47	66.47	66.47	66.47			
f2LS	67.63	67.63	67.63	67.63	67.63	67.63	67.63					
f0LSN	66.67	66.09	67.24	70.69	70.69	68.97	68.97					
f1LSN	67.05	66.47	67.63	71.10	71.10	70.52	69.94					
f2LSN	67.63	67.63	68.79	72.25	72.25	70.52	68.79	69.94				
Model10	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
f0LS	70.11	70.11	57.47	77.01	77.01	76.44	72.99					
f1LS	69.94	69.94	57.23	76.88	76.88	76.30	72.83					
f2LS	71.10	71.10	67.63	67.63	71.10	75.72	78.61	76.88				
f0LSN	68.97	65.52	66.09	66.09	67.24	64.94	71.84	72.99	69.54	68.39	72.41	
f1LSN	69.94	66.47	66.47	66.47	67.63	66.47	72.83	73.99	70.52	69.36	73.99	
f2LSN	71.10	67.63	67.63	65.90	68.79	66.47	73.99	73.41	72.83	72.25	70.52	
Model11	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
f0LS	70.11	70.11	70.11	70.11	70.11	70.11	66.09	64.94				
f1LS	69.94	69.94	69.94	69.94	69.94	69.94	65.90	67.40				
f2LS	71.10	87.86	87.86	**100.00**	60.12	73.99						
f0LSN	69.54	66.09	64.94	64.37	66.09	63.79	72.99	73.56	74.71			
f1LSN	69.54	66.47	65.32	64.74	66.47	65.32	74.57	74.57	76.30			
f2LSN	70.52	67.05	65.90	65.32	67.05	74.57	73.99	73.99	75.72			
Model12	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
f0LS	62.43	87.86	68.79	78.61	78.61	72.25	69.36	71.86	68.21	67.63		
f1LS	62.43	87.86	68.79	78.61	78.61	72.25	69.36	71.86	68.21			
f2LS	67.63	87.86	78.03	78.61	78.61	76.72	75.14					
f0LSN	60.92	65.52	65.52	65.52	70.11	67.82	68.39					
f1LSN	61.27	65.90	65.90	65.90	70.52	69.36	69.36					
f2LSN	67.63	67.63	67.63	68.21	67.05	67.63	67.63	67.05				
Model13	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
f0LS	71.10	87.86	0	0	78.61	77.46	73.99	80.35	80.35	80.35	71.10	
f1LS	71.10	87.86	0	0	78.61	77.46	73.99	80.35	80.35	80.35	71.10	
f2LS	71.10	71.10	67.63	67.63	71.10	65.32	64.74	63.58	75.72	67.63	67.63	60.21
f0LSN	69.54	66.09	64.94	68.97	68.97	67.21	71.84	71.26	68.97	68.39	71.84	
f1LSN	69.54	66.47	65.32	69.36	69.36	68.79	72.83	72.25	69.94	69.36	72.83	
f2LSN	71.10	67.63	66.47	70.52	70.52	73.99	73.99	73.99	72.25	70.52	72.83	

Table 10. Model Exponential Relations of the performing model

Group Name	Model No	Percent Accuracy	No of Input	Model Exponential Relations of the performing model
ModelfoLS	6	77.59	7	$0.005722\left(\frac{u^*}{v}\right)^{0.5}\left(\frac{v^2}{gy}\right)^{0.5} +- 1.288\text{e-}005\left(\frac{R}{d_{50}}\right)\left(\frac{v^2}{gy}\right) + 8.7177\text{e-}011\frac{\left(\frac{R}{d_{50}}\right)^{1.5}}{\left(\frac{u^*}{v}\right)^{2}\left(\frac{v^2}{gy}\right)^{0.5}}$
Modelf1LS	6	75.72	7	$0.005722\left(\frac{u^*}{v}\right)^{0.5}\left(\frac{v^2}{gy}\right)^{0.5} +- 1.288\text{e-}005\left(\frac{R}{d_{50}}\right)\left(\frac{v^2}{gy}\right) + 8.7177\text{e-}011\frac{\left(\frac{R}{d_{50}}\right)^{1.5}}{\left(\frac{u^*}{v}\right)^{2}\left(\frac{v^2}{gy}\right)^{0.5}}$
Modelf2LS	9	75.72	7	$0.0047067\left(\frac{u^*}{v}\right)^{0.5}\left(\frac{v^2}{gy}\right)^{0.5} +- 1.1407\text{e-}006\left(\frac{R}{d_{50}}\right)^{1.5}\left(\frac{v^2}{gy}\right)^{1.5} + 6.7653\text{e-}013\frac{\left(\frac{R}{d_{50}}\right)^{1.5}\left(\frac{u^*}{\omega_s}\right)^{1.5}\left(\frac{v^2}{gy}\right)^{0.5}}{\left(\frac{u^*}{v}\right)^{2}\left(\frac{vs}{\omega}\right)^{1.5}}$
ModelfoLSN	11	75.29	7	$0.13155\frac{\left(\frac{u^*}{\omega_s}\right)^{0.5}\left(\frac{v^2}{gy}\right)}{\left(\frac{R}{d_{50}}\right)^{1.5}\left(\frac{u^*}{v}\right)^{2}}+7.7383\frac{\left(\frac{u^*}{v}\right)^{2}\left(\frac{v^2}{gy}\right)}{\left(\frac{R}{d_{50}}\right)^{0.5}\left(\frac{B}{y}\right)^{0.5}} + 2.1708\text{e-}007\frac{\left(\frac{u^*}{\omega_s}\right)^{0.5}}{\left(\frac{u^*}{v}\right)^{1.5}\left(\frac{v^2}{gy}\right)^{1.5}}$
Modelf1LSN	11	76.30	7	$0.13155\frac{\left(\frac{u^*}{\omega_s}\right)^{0.5}\left(\frac{v^2}{gy}\right)}{\left(\frac{R}{d_{50}}\right)^{1.5}\left(\frac{u^*}{v}\right)^{2}}+7.7383\frac{\left(\frac{u^*}{v}\right)^{2}\left(\frac{v^2}{gy}\right)}{\left(\frac{R}{d_{50}}\right)^{0.5}\left(\frac{B}{y}\right)^{0.5}} + 2.1708\text{e-}007\frac{\left(\frac{u^*}{\omega_s}\right)^{0.5}}{\left(\frac{u^*}{v}\right)^{1.5}\left(\frac{v^2}{gy}\right)^{1.5}}$
Modelf2LSN	10	76.88	7	$8.9645\frac{\left(\frac{v^2}{gy}\right)^{1.5}}{\left(\frac{R}{d_{50}}\right)\left(\frac{B}{y}\right)^{0.5}}\left(\frac{\omega}{u^*}\right)^{1.5}+1.4889\frac{\left(\frac{u^*}{v}\right)^{2}\left(\frac{v^2}{gy}\right)^{0.5}}{\left(\frac{R}{d_{50}}\right)^{0.5}\left(\frac{B}{y}\right)^{0.5}} + 2.0793\text{e-}007\frac{\left(\frac{u^*}{\omega_s}\right)^{0.5}}{\left(\frac{u^*}{v}\right)^{1.5}\left(\frac{v^2}{gy}\right)^{1.5}}$
Model2foLS	8	69.54	4	$0.00096449\left(\frac{u^*}{v}\right)^{0.5} +0\frac{\left(\frac{R}{d_{50}}\right)^{1.5}}{\left(\frac{B}{y}\right)\left(\frac{u^*}{\omega_s}\right)^{0.5}} + 0\frac{\left(\frac{R}{d_{50}}\right)^{2}\left(\frac{u^*}{\omega_s}\right)^{1.5}\left(\frac{v^2}{gy}\right)^{0.5}}{\left(\frac{B}{y}\right)^{2}\left(\frac{u^*}{\omega_s}\right)^{0.5}\left(\frac{u^*}{v}\right)}$
Model2f1LS	8	69.94	4	$0.00096449\left(\frac{u^*}{v}\right)^{0.5} +0\frac{\left(\frac{R}{d_{50}}\right)^{1.5}}{\left(\frac{B}{y}\right)\left(\frac{u^*}{\omega_s}\right)^{0.5}} + 0\frac{\left(\frac{R}{d_{50}}\right)^{2}\left(\frac{u^*}{\omega_s}\right)^{1.5}\left(\frac{v^2}{gy}\right)^{0.5}}{\left(\frac{B}{y}\right)^{2}\left(\frac{u^*}{\omega_s}\right)^{0.5}\left(\frac{u^*}{v}\right)}$
Model2f2LS ***	**6**	**100**	**4**	$0\frac{1}{\left(\frac{R}{d_{50}}\right)^{2}}+0.015724\frac{\left(\frac{u^*}{v}\right)^{0.5}}{\left(\frac{R}{d_{50}}\right)^{0.5}} + 0\frac{\left(\frac{R}{d_{50}}\right)^{2}}{\left(\frac{B}{y}\right)^{2}\left(\frac{u^*}{v}\right)^{2}}$
Model2foLSN	1	69.54	4	$0.00096449\left(\frac{u^*}{v}\right)^{0.5}$
Model2f1LSN	1	69.94	4	$0.00096449\left(\frac{u^*}{v}\right)^{0.5}$
Model2f2LSN	1	71.10	4	$0.00096449\left(\frac{u^*}{v}\right)^{0.5}$
Model3foLS	6	98.28	5	$0\frac{1}{\left(\frac{R}{d_{50}}\right)^{2}}+0.015724\frac{\left(\frac{u^*}{v}\right)^{0.5}}{\left(\frac{R}{d_{50}}\right)^{0.5}} + 0\frac{\left(\frac{R}{d_{50}}\right)^{2}}{\left(\frac{B}{y}\right)^{2}\left(\frac{u^*}{v}\right)^{2}}$
Model3f1LS	6	98.27	5	$0\frac{1}{\left(\frac{R}{d_{50}}\right)^{2}}+0.015724\frac{\left(\frac{u^*}{v}\right)^{0.5}}{\left(\frac{R}{d_{50}}\right)^{0.5}} + 0\frac{\left(\frac{R}{d_{50}}\right)^{2}}{\left(\frac{B}{y}\right)^{2}\left(\frac{u^*}{v}\right)^{2}}$
Model3f2LS	1	71.10	5	$0.00096449\left(\frac{u^*}{v}\right)^{0.5}$
Model3foLSN	9	70.11	5	$31.2116\frac{\left(\frac{u^*}{\omega_s}\right)^{2}}{\left(\frac{R}{d_{50}}\right)^{2}\left(\frac{B}{y}\right)\left(\frac{u^*}{v}\right)^{2}}+0.00022426\left(\frac{B}{y}\right)^{0.5}\left(\frac{u^*}{v}\right) + 1.4844\text{e-}007\frac{\left(\frac{R}{d_{50}}\right)^{1.5}}{\left(\frac{B}{y}\right)^{2}\left(\frac{u^*}{v}\right)^{1.5}}$
Model3f1LSN	9	71.86	5	$31.2116\frac{\left(\frac{u^*}{\omega_s}\right)^{2}}{\left(\frac{R}{d_{50}}\right)^{2}\left(\frac{B}{y}\right)\left(\frac{u^*}{v}\right)^{2}}+0.00022426\left(\frac{B}{y}\right)^{0.5}\left(\frac{u^*}{v}\right) + 1.4844\text{e-}007\frac{\left(\frac{R}{d_{50}}\right)^{1.5}}{\left(\frac{B}{y}\right)^{2}\left(\frac{u^*}{v}\right)^{1.5}}$

Model3f2LSN	1	71.10	5	$0.00096449\left(\frac{u^*}{v}\right)^{0.5}$
Model4f0LS	1	70.11	6	$0.00096449\left(\frac{u^*}{v}\right)^{0.5}$
Model4f1LS	1	69.94	6	$0.00096449\left(\frac{u^*}{v}\right)^{0.5}$
Model4f2LS	3	76.88	6	$+-0.00011109\frac{1}{\left(\frac{\omega}{u^*}\right)^{1.5}\left(\frac{u^*}{v}\right)^{1.5}}+0.0010153\frac{1}{\left(\frac{\omega}{u^*}\right)^{0.5}}$
Model4f0LSN	9	74.71	6	$18.7575\frac{\left(\frac{vs}{\omega}\right)^{0.5}}{\left(\frac{R}{d_{50}}\right)\left(\frac{B}{y}\right)}+1.3436\text{e-}007\frac{1}{\left(\frac{B}{y}\right)\left(\frac{\omega}{u^*}\right)^{2}\left(\frac{u^*}{v}\right)^{1.5}\left(\frac{vs}{\omega}\right)^{1.5}}+0.00018294\left(\frac{B}{y}\right)^{0.5}\left(\frac{u^*}{v}\right)$
Model4f1LSN	9	76.30	6	$18.7575\frac{\left(\frac{vs}{\omega}\right)^{0.5}}{\left(\frac{R}{d_{50}}\right)\left(\frac{B}{y}\right)}+1.3436\text{e-}007\frac{1}{\left(\frac{B}{y}\right)\left(\frac{\omega}{u^*}\right)^{2}\left(\frac{u^*}{v}\right)^{1.5}\left(\frac{vs}{\omega}\right)^{1.5}}+0.00018294\left(\frac{B}{y}\right)^{0.5}\left(\frac{u^*}{v}\right)$
Model4f2LSN	9	76.30	6	$18.7575\frac{\left(\frac{vs}{\omega}\right)^{0.5}}{\left(\frac{R}{d_{50}}\right)\left(\frac{B}{y}\right)}+1.3436\text{e-}007\frac{\left(\frac{u^*}{\omega_s}\right)^{2}}{\left(\frac{B}{y}\right)\left(\frac{u^*}{v}\right)^{1.5}\left(\frac{vs}{\omega}\right)^{1.5}}+0.00018294\left(\frac{B}{y}\right)^{0.5}\left(\frac{u^*}{v}\right)$
Model5f0LS	2	86.78	4	$0.00041211\left(\frac{u^*}{\omega_s}\right)^{0.5}\left(\frac{\omega}{u^*}\right)^{0.5}$
Model5f1LS	-	-	-	-
Model5f2LS	-	-	-	-
Model5f0LSN	7	68.39	4	$3936.6496\frac{\left(\frac{u^*}{\omega_s}\right)^{2}}{\left(\frac{R}{d_{50}}\right)^{2}\left(\frac{B}{y}\right)^{1.5}}+0.0003356\left(\frac{u^*}{\omega_s}\right)^{0.5}\left(\frac{\omega}{u^*}\right)^{0.5}+1.5921\text{e-}005\frac{\left(\frac{R}{d_{50}}\right)}{\left(\frac{B}{y}\right)^{2}}$
Model5f1LSN	6	69.36	4	$3986.4618\frac{\left(\frac{u^*}{\omega_s}\right)^{2}}{\left(\frac{R}{d_{50}}\right)^{2}\left(\frac{B}{y}\right)^{1.5}}+0.0029567\frac{1}{\left(\frac{B}{y}\right)^{2}}+0.00034401\left(\frac{u^*}{\omega_s}\right)^{0.5}\left(\frac{\omega}{u^*}\right)^{0.5}$
Model5f2LSN	5	69.36	4	$3986.4618\frac{\left(\frac{u^*}{\omega_s}\right)^{2}}{\left(\frac{R}{d_{50}}\right)^{2}\left(\frac{B}{y}\right)^{1.5}}+0.0029567\frac{1}{\left(\frac{B}{y}\right)^{2}}+0.00034401\frac{1}{\left(\frac{B}{y}\right)}$
Model6f0LS	2	86.78	5	$0\frac{1}{\left(\frac{R}{d_{50}}\right)^{1.5}}+0.006834\frac{1}{\left(\frac{R}{d_{50}}\right)^{0.5}}$
Model6f1LS	2	86.71	5	$0\frac{1}{\left(\frac{R}{d_{50}}\right)^{1.5}}+0.006834\frac{1}{\left(\frac{R}{d_{50}}\right)^{0.5}}$
Model6f2LS	2	87.28	5	$0\frac{1}{\left(\frac{R}{d_{50}}\right)^{1.5}}+0.006834\frac{1}{\left(\frac{R}{d_{50}}\right)^{0.5}}$
Model6f0LSN	5	70.11	5	$0.31667\frac{\left(\frac{vs}{\omega}\right)}{\left(\frac{R}{d_{50}}\right)^{0.5}\left(\frac{u^*}{\omega_s}\right)}+2.461\text{e-}007\frac{1}{\left(\frac{vs}{\omega}\right)^{2}\left(\frac{vs}{\omega}\right)^{1.5}}$
Model6f1LSN	5	70.52	5	$0.31667\frac{\left(\frac{vs}{\omega}\right)}{\left(\frac{R}{d_{50}}\right)^{0.5}\left(\frac{u^*}{\omega_s}\right)}+2.461\text{e-}007\frac{1}{\left(\frac{vs}{\omega}\right)^{2}\left(\frac{vs}{\omega}\right)^{1.5}}$
Model6f2LSN	5	71.68	5	$0.31667\frac{\left(\frac{\omega}{u^*}\right)\left(\frac{vs}{\omega}\right)}{\left(\frac{R}{d_{50}}\right)^{0.5}}+2.461\text{e-}007\frac{\left(\frac{u^*}{\omega_s}\right)^{2}}{\left(\frac{vs}{\omega}\right)^{1.5}}$
Model7f0LS	5	70.11	6	$0\left(\frac{\omega}{u^*}\right)^{0.5}+0.0022795\left(\frac{R}{d_{50}}\right)^{0.5}\left(\frac{v^2}{gy}\right)^{0.5}+-2.622e-005\left(\frac{R}{d_{50}}\right)\left(\frac{v^2}{gy}\right)$
Model7f1LS	5	69.36	6	$0\left(\frac{\omega}{u^*}\right)^{0.5}+0.0022795\left(\frac{R}{d_{50}}\right)^{0.5}\left(\frac{v^2}{gy}\right)^{0.5}+-2.622e-005\left(\frac{R}{d_{50}}\right)\left(\frac{v^2}{gy}\right)$
Model7f2LS	3	69.36	6	$3.36883\text{e-}005\left(\frac{R}{d_{50}}\right)^{0.5}+-4.3848e-005\left(\frac{R}{d_{50}}\right)^{1.5}\left(\frac{v^2}{gy}\right)$

Model7f0LSN	4	72.99	6	$20.2894\frac{\left(\frac{vs}{\omega}\right)^{0.5}}{\left(\frac{R}{d_{50}}\right)\left(\frac{B}{y}\right)}+ 6.2504e\text{-}005\frac{1}{\left(\frac{v^2}{gy}\right)^{0.5}}$
Model7f1LSN	4	73.41	6	$20.2894\frac{\left(\frac{vs}{\omega}\right)^{0.5}}{\left(\frac{R}{d_{50}}\right)\left(\frac{B}{y}\right)}+ 6.2504e\text{-}005\frac{1}{\left(\frac{v^2}{gy}\right)^{0.5}}$
Model7f2LSN	8	76.30	6	$0.54817\frac{\left(\frac{\omega}{u^*}\right)\left(\frac{vs}{\omega}\right)}{\left(\frac{R}{d_{50}}\right)^{0.5}\left(\frac{B}{y}\right)^{0.5}\left(\frac{v^2}{gy}\right)^{0.5}}+ 1.8413e\text{-}007\frac{1}{\left(\frac{\omega}{u^*}\right)^{2}\left(\frac{vs}{\omega}\right)^{1.5}}+ 0.00029659\left(\frac{vs}{\omega}\right)^{0.5}$
Model8f0LS	5	70.11	4	$0\frac{1}{\left(\frac{u^*}{\omega_s}\right)^{0.5}} + -0.00022795\left(\frac{R}{d_{50}}\right)^{0.5}\left(\frac{v^2}{gy}\right)^{0.5} + -2.622e-005\left(\frac{R}{d_{50}}\right)\left(\frac{v^2}{gy}\right)$
Model8f1LS	5	69.94	4	$0\frac{1}{\left(\frac{u^*}{\omega_s}\right)^{0.5}} + -0.00022795\left(\frac{R}{d_{50}}\right)^{0.5}\left(\frac{v^2}{gy}\right)^{0.5} + -2.622e-005\left(\frac{R}{d_{50}}\right)\left(\frac{v^2}{gy}\right)$
Model8f2LS	5	71.10	4	$0\left(\frac{v^2}{gy}\right) + 3.1645e-005\left(\frac{R}{d_{50}}\right)^{0.5} + -9.0277e-006\left(\frac{R}{d_{50}}\right)\left(\frac{v^2}{gy}\right)$
Model8f0LSN	4	70.69	4	$8.3922\frac{\left(\frac{v^2}{gy}\right)^{1.5}}{\left(\frac{R}{d_{50}}\right)\left(\frac{B}{y}\right)^{0.5}}+ 7.6601e\text{-}005\frac{1}{\left(\frac{v^2}{gy}\right)^{0.5}}$
Model8f1LSN	4	71.10	4	$8.3922\frac{\left(\frac{v^2}{gy}\right)^{1.5}}{\left(\frac{R}{d_{50}}\right)\left(\frac{B}{y}\right)^{0.5}}+ 7.6601e\text{-}005\frac{1}{\left(\frac{v^2}{gy}\right)^{0.5}}$
Model8f2LSN	4	71.68	4	$8.3922\frac{\left(\frac{v^2}{gy}\right)^{1.5}}{\left(\frac{R}{d_{50}}\right)\left(\frac{B}{y}\right)^{0.5}}+ 7.6601e\text{-}005\frac{1}{\left(\frac{v^2}{gy}\right)^{0.5}}$
Model9f0LS	2	86.78	5	$0\frac{1}{\left(\frac{R}{d_{50}}\right)^{1.5}}+ 0.0068341\frac{1}{\left(\frac{R}{d_{50}}\right)^{0.5}}$
Model9f1LS	2	86.71	5	$0\frac{1}{\left(\frac{R}{d_{50}}\right)^{1.5}}+ 0.0068341\frac{1}{\left(\frac{R}{d_{50}}\right)^{0.5}}$
Model9f2LS	1	67.63	5	$0.00041211\left(\frac{B}{y}\right)^{0.5}$
Model9f0LSN	4	70.69	5	$8.3922\frac{\left(\frac{v^2}{gy}\right)^{1.5}}{\left(\frac{R}{d_{50}}\right)\left(\frac{B}{y}\right)^{0.5}}+ 7.6601e\text{-}005\frac{1}{\left(\frac{v^2}{gy}\right)^{0.5}}$
Model9f1LSN	4	71.10	5	$8.3922\frac{\left(\frac{v^2}{gy}\right)^{1.5}}{\left(\frac{R}{d_{50}}\right)\left(\frac{B}{y}\right)^{0.5}}+ 7.6601e\text{-}005\frac{1}{\left(\frac{v^2}{gy}\right)^{0.5}}$
Model9f2LSN	4	72.25	5	$8.3922\frac{\left(\frac{v^2}{gy}\right)^{1.5}}{\left(\frac{R}{d_{50}}\right)\left(\frac{B}{y}\right)^{0.5}}+ 7.6601e\text{-}005\frac{1}{\left(\frac{v^2}{gy}\right)^{0.5}}$
Model10f0LS	4	77.01	6	$2.8718\frac{1}{\left(\frac{R}{d_{50}}\right)^{1.5}}+ 0.31095\frac{1}{\left(\frac{R}{d_{50}}\right)} + 0\frac{1}{\left(\frac{u^*}{\omega_s}\right)^{1.5}\left(\frac{v^2}{gy}\right)^{2}}$
Model10f1LS	4	76.88	6	$2.8718\frac{1}{\left(\frac{R}{d_{50}}\right)^{1.5}}+ 0.31095\frac{1}{\left(\frac{R}{d_{50}}\right)} + 0\frac{1}{\left(\frac{u^*}{\omega_s}\right)^{1.5}\left(\frac{v^2}{gy}\right)^{2}}$
Model10f2LS	7	78.61	6	$0 \quad .005722\left(\frac{u^*}{v}\right)^{0.5}\left(\frac{v^2}{gy}\right)^{0.5} + -1.288e-005\left(\frac{R}{d_{50}}\right)\left(\frac{v^2}{gy}\right) + 8.7177e-011\frac{\left(\frac{R}{d_{50}}\right)^{1.5}}{\left(\frac{u^*}{v}\right)^{2}\left(\frac{v^2}{gy}\right)^{0.5}}$
Model10f0LSN	8	72.99	6	$0.10676\frac{\left(\frac{u^*}{v}\right)\left(\frac{v^2}{gy}\right)^{0.5}}{\left(\frac{R}{d_{50}}\right)^{0.5}}+ 3.9855e\text{-}007\frac{\left(\frac{u^*}{\omega_s}\right)}{\left(\frac{B}{y}\right)^{0.5}\left(\frac{u^*}{v}\right)^{2}\left(\frac{v^2}{gy}\right)^{1.5}}+ 0.00016018\left(\frac{u^*}{v}\right)^{1.5}$

Model10f1LSN	8	73.99	6	$0.10676\frac{\left(\frac{u^*}{v}\right)\left(\frac{v^2}{gy}\right)^{0.5}}{\left(\frac{R}{d_{50}}\right)^{0.5}}+3.9855\text{e-}007\frac{\left(\frac{u^*}{\omega_s}\right)}{\left(\frac{B}{y}\right)^{0.5}\left(\frac{u^*}{v}\right)^{2}\left(\frac{v^2}{gy}\right)^{1.5}}+0.00016018\left(\frac{u^*}{v}\right)^{1.5}$
Model10f2LSN	7	73.99	6	$4.6159\frac{\left(\frac{u^*}{v}\right)^{1.5}\left(\frac{v^2}{gy}\right)}{\left(\frac{R}{d_{50}}\right)^{0.5}\left(\frac{B}{y}\right)^{0.5}}+2.1553\text{e-}007\frac{\left(\frac{u^*}{\omega_s}\right)^{0.5}}{\left(\frac{u^*}{v}\right)^{1.5}\left(\frac{v^2}{gy}\right)^{1.5}}$
Model11f0LS	1	70.11	5	$0.00096449\left(\frac{u^*}{v}\right)^{0.5}$
Model11f1LS	1	69.94	5	$0.00096449\left(\frac{u^*}{v}\right)^{0.5}$
Model11f2LS ***	**4**	**100.00**	**5**	$0\frac{1}{\left(\frac{R}{d_{50}}\right)^{2}}+0.015724\frac{\left(\frac{u^*}{v}\right)^{0.5}}{\left(\frac{R}{d_{50}}\right)^{0.5}}+0\frac{1}{\left(\frac{u^*}{v}\right)^{1.5}}$
Model11f0LSN	9	74.71	5	$18.7575\frac{\left(\frac{vs}{\omega}\right)^{0.5}}{\left(\frac{R}{d_{50}}\right)\left(\frac{B}{y}\right)}+1.3436\text{e-}007\frac{\left(\frac{u^*}{\omega_s}\right)^{2}}{\left(\frac{B}{y}\right)\left(\frac{u^*}{v}\right)^{1.5}\left(\frac{vs}{\omega}\right)^{1.5}}+0.00018294\left(\frac{B}{y}\right)^{0.5}\left(\frac{u^*}{v}\right)$
Model11f1LSN	9	76.30	5	$18.7575\frac{\left(\frac{vs}{\omega}\right)^{0.5}}{\left(\frac{R}{d_{50}}\right)\left(\frac{B}{y}\right)}+1.3436\text{e-}007\frac{\left(\frac{u^*}{\omega_s}\right)^{2}}{\left(\frac{B}{y}\right)\left(\frac{u^*}{v}\right)^{1.5}\left(\frac{vs}{\omega}\right)^{1.5}}+0.00018294\left(\frac{B}{y}\right)^{0.5}\left(\frac{u^*}{v}\right)$
Model11f2LSN	6	74.57	5	$20.6487\frac{\left(\frac{vs}{\omega}\right)^{0.5}}{\left(\frac{R}{d_{50}}\right)\left(\frac{B}{y}\right)}+0.00078901\left(\frac{u^*}{v}\right)+1.4956\text{e-}007\frac{\left(\frac{u^*}{\omega_s}\right)^{2}}{\left(\frac{vs}{\omega}\right)^{1.5}}$
Model12f0LS	2	87.86	4	$0\frac{1}{\left(\frac{R}{d_{50}}\right)^{1.5}}+0.0068341\frac{1}{\left(\frac{R}{d_{50}}\right)^{0.5}}$
Model12f1LS	2	87.86	4	$0\frac{1}{\left(\frac{R}{d_{50}}\right)^{1.5}}+0.0068341\frac{1}{\left(\frac{R}{d_{50}}\right)^{0.5}}$
Model12f2LS	2	87.86	4	$0\frac{1}{\left(\frac{R}{d_{50}}\right)^{1.5}}+0.0068341\frac{1}{\left(\frac{R}{d_{50}}\right)^{0.5}}$
Model12f0LSN	5	70.11	4	$0.31667\frac{\left(\frac{vs}{\omega}\right)}{\left(\frac{R}{d_{50}}\right)^{0.5}\left(\frac{u^*}{\omega_s}\right)}+2.461\text{e-}007\frac{\left(\frac{u^*}{\omega_s}\right)^{2}}{\left(\frac{vs}{\omega}\right)^{1.5}}$
Model12f1LSN	5	70.52	4	$0.31667\frac{\left(\frac{vs}{\omega}\right)}{\left(\frac{R}{d_{50}}\right)^{0.5}\left(\frac{u^*}{\omega_s}\right)}+2.461\text{e-}007\frac{\left(\frac{u^*}{\omega_s}\right)^{2}}{\left(\frac{vs}{\omega}\right)^{1.5}}$
Model12f2LSN	4	68.21	4	$0.12605\frac{\left(\frac{vs}{\omega}\right)^{1.5}}{\left(\frac{u^*}{\omega_s}\right)^{1.5}}+2.7393\text{e-}007\frac{\left(\frac{u^*}{\omega_s}\right)^{2}}{\left(\frac{vs}{\omega}\right)^{1.5}}$
Model13f0LS	2	87.86	6	$0\frac{1}{\left(\frac{R}{d_{50}}\right)^{1.5}}+0.0068341\frac{1}{\left(\frac{R}{d_{50}}\right)^{0.5}}$
Model13f1LS	2	87.86	6	$0\frac{1}{\left(\frac{R}{d_{50}}\right)^{1.5}}+0.0068341\frac{1}{\left(\frac{R}{d_{50}}\right)^{0.5}}$
Model13f2LS	9	75.72	6	$0\quad .00471\left(\frac{u^*}{v}\right)^{0.5}\left(\frac{v^2}{gy}\right)^{0.5}+-1.1512e-006\left(\frac{R}{d_{50}}\right)^{1.5}\left(\frac{v^2}{gy}\right)^{1.5}+7.157e-013\frac{\left(\frac{R}{d_{50}}\right)^{1.5}\left(\frac{u^*}{\omega_s}\right)\left(\frac{v^2}{gy}\right)^{2}}{\left(\frac{u^*}{v}\right)^{1.5}\left(\frac{vs}{\omega}\right)^{2}}$
Model13f0LSN	7	71.84	6	$15.2937\frac{\left(\frac{v^2}{gy}\right)}{\left(\frac{R}{d_{50}}\right)\left(\frac{B}{y}\right)}+0.0077026\left(\frac{u^*}{v}\right)^{1.5}\left(\frac{v^2}{gy}\right)^{0.5}+2.0303\text{e-}007\frac{\left(\frac{u^*}{\omega_s}\right)^{2}}{\left(\frac{vs}{\omega}\right)^{1.5}}$
Model13f1LSN	7	72.83	6	$15.2937\frac{\left(\frac{v^2}{gy}\right)}{\left(\frac{R}{d_{50}}\right)\left(\frac{B}{y}\right)}+0.0077026\left(\frac{u^*}{v}\right)^{1.5}\left(\frac{v^2}{gy}\right)^{0.5}+2.0303\text{e-}007\frac{\left(\frac{u^*}{\omega_s}\right)^{2}}{\left(\frac{vs}{\omega}\right)^{1.5}}$

Model13f2LSN	6	73.99	6	$4.5415\frac{\left(\frac{u^*}{v}\right)^{1.5}\left(\frac{v^2}{gy}\right)}{\left(\frac{R}{d_{50}}\right)^{0.5}\left(\frac{B}{y}\right)^{0.5}}+ 2.4772e\text{-}007\frac{\left(\frac{u^*}{\omega_s}\right)^2}{\left(\frac{vs}{\omega}\right)^{1.5}}$

Note: * Best model**

Figure 1 shows performance of Group Model2f2LS (Model 1 until Model 7). Seven newly developed models have shown to predict the measured values within an acceptable limit. The model that best predicts the measured values is Model 6.

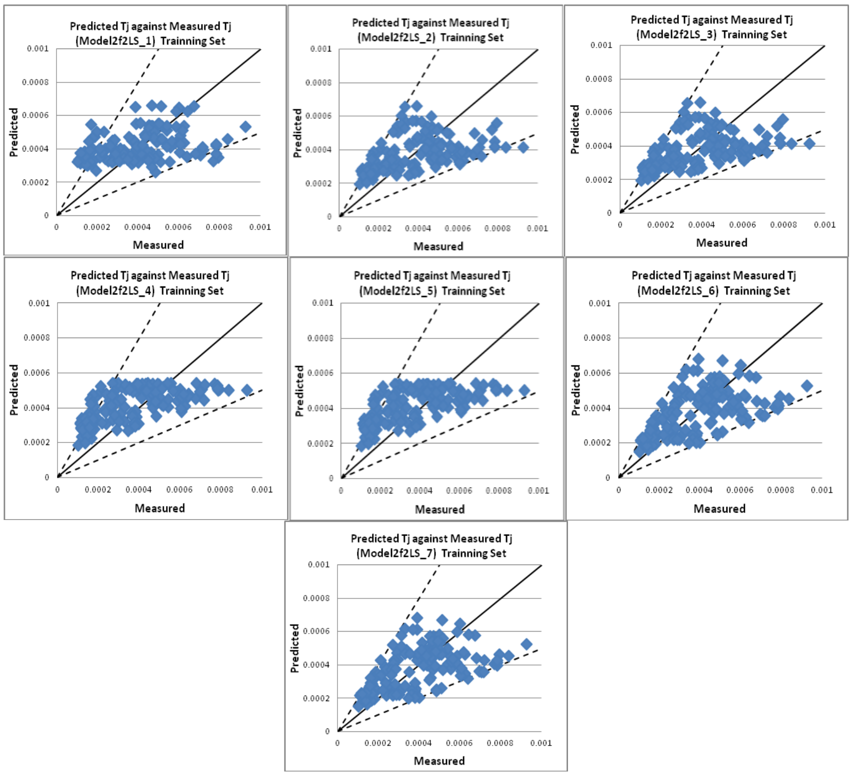

Figure 1. Model performance for Group Model2f2LS (Model 1 until Model 7)

Models in Figure 2 exhibit similar trends to Figure 1. Analyses suggest that Model 4 exhibit the best predictions compare to other 5 models.

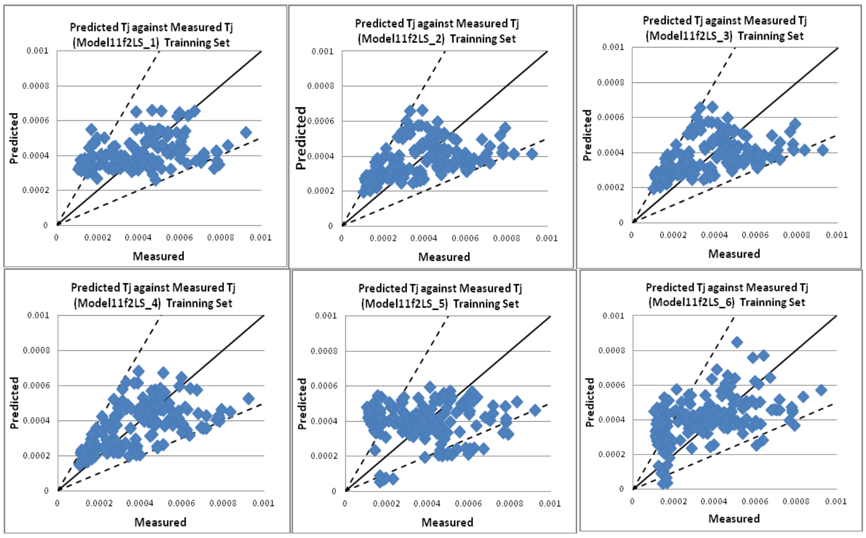

Figure 2. Model performance Group Model11f2LS (Model 1 until Model 6)

Results with data confirmation from Table 10 suggest that Model 6 (Model2f2LS) yield 100 percent accuracy in prediction. The exponential relations of Model 6 (Model2f2LS) and Model 4 (Model 11f2LS)are:

Model 6 (Model2f2LS)

$$\frac{Qt}{Q} = 0\frac{1}{\left(\frac{R}{d_{50}}\right)^2}+0.015724\frac{\left(\frac{u^*}{v}\right)^{0.5}}{\left(\frac{R}{d_{50}}\right)^{0.5}} + 0\frac{\left(\frac{R}{d_{50}}\right)^2}{\left(\frac{B}{y}\right)^2\left(\frac{u^*}{v}\right)^2} \quad (1)$$

Model 4 (Model 11f2LS)

$$\frac{Qt}{Q} = 0\frac{1}{\left(\frac{R}{d_{50}}\right)^2}+0.015724\frac{\left(\frac{u^*}{v}\right)^{0.5}}{\left(\frac{R}{d_{50}}\right)^{0.5}} + 0\frac{1}{\left(\frac{u^*}{v}\right)^{1.5}} \quad (2)$$

Both models 6 and 4 (Equations 1 and 2) suggest $\frac{U^*}{V}$ (ratio of shear velocity to flow velocity) and $\frac{R}{d_{50}}$ (ratio of hydraulic radius to mean sediment diameter) as the most significant and influential parameters. With the above discovery, the general expression for sediment concentration can be expressed as follows;

$$\frac{Qt}{Q} = 0.015724\,\frac{\left(U^*/V\right)^{0.5}}{\left(R/d_{50}\right)^{0.5}} \quad (3)$$

where; Qt is sediment total load (kg/s); V is flow velocity (m/s), d_{50} *is* median diameter of sediment load (m), Q is flow discharge (kg/s), U * is shear velocity (m/s) and R is hydraulic radius (m).

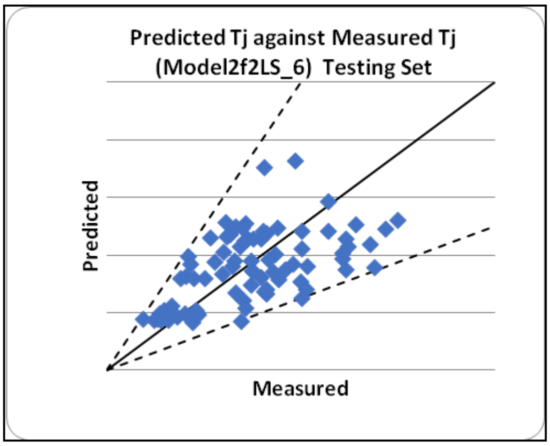

Figure 3. Graph of measured sediment total load versus predicted sediment total load using 82 testing data

IV. CONCLUSION

Analyses carried out on the model parameters have indicated that two variables namely $\frac{U^*}{V}$ (ratio of shear velocity to flow velocity) and $\frac{R}{d_{50}}$ (ratio of hydraulic radius to mean sediment diameter) to be the most significant and influential parameters. The above is confirmed by the performance of the model with 100 percent prediction accuracy. This new model is an improved model of Ariffin (2004), of which the latter has used four model parameters as predictors. In the improved model, analyses have confirmed that only two parameters could predict with greater accuracy the measured sediment load values.

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