ASM Sc. J., 11, Special Issue 3, 2018 for SANREM, 48-52

Conceptual of Type-2 Fuzzy Geometric Modelling

Asdalifah Talibe[1], Rozaimi Zakaria[1*], Abdullah Bade[1] and Suzelawati Zenian[1]

[1]*Mathematics, Graphics and Visualization Research Group (M-GRAVS),*

Faculty of Science and Natural Resources, Universiti Malaysia Sabah,

Jalan UMS, 88400 Kota Kinabalu, Sabah, Malaysia.

Geometric modelling is a method of data representation illustrated through the formation of curves and surfaces in various forms. The construction of curves and surfaces is complicated when it comes to data that has complex uncertainty characteristics. Type-1 Fuzzy Set Theory (T1FST) is unable to define this complex uncertainty problem. To overcome this problem, Type-2 Fuzzy Set Theory (T2FST) is used due to its ability to define a higher level of uncertainty problem. In certain cases, both uncertainty and complex uncertainty data occur when there is a combination of degrees of ambiguity in a collection of data sets that would be modelled through the representation of curves and surfaces. Therefore, this paper will review some significant reason for implementation of T1FST and T2FST in geometric modelling. A review on type-1 and type-2 in fuzzy geometric modelling is also presented.

Keywords: Type-1 Fuzzy Set Theory (T_1FST), Type-2 Fuzzy Set Theory (T_2FST), geometric modelling.

I. INTRODUCTION

Geometric modelling deals with the construction and representation of free-form curves and surfaces. Research in this field is very useful because it can be applied to various forms of real-life data. However, errors or limitations in data collection may occur due to tools, human error, environmental factors and so forth. These kinds of data have uncertainty characteristics which cannot be used directly to form curves and surfaces model. To ensure that uncertainty data is usable, these data can be defined by T1FST. T1FST is introduced by Zadeh in 1965, which became the basic theory in solving uncertainty problems (Zadeh, 1965). From T1FST, Type-1 Fuzzy Number (T1FN) concept is established to solve the uncertainty data in the real number form. Thus, this finding becomes a basic solution in modelling uncertainty data and there are few studies on type-1 fuzzy geometric modelling has been conducted where B-spline, Bezier and NURBS functions are mostly used to produce geometric models (Abd. Fatah *et al.*, 2004; Abd. Fatah *et al.*, 2009; Abd. Fatah *et al.*, 2010; Rozaimi & Abd. Fatah, 2014).

*Corresponding author's e-mail: rozaimi@ums.edu.my

However, T1FST is unable to define on a higher level of uncertainty problem which known as complex uncertainty. Therefore, in 1975, Zadeh proposed T2FST which can define the complex uncertainty problem (Zadeh, 1975). Although Zadeh has introduced T2FST, Mendel and John come with an easy approach to understanding T2FST concept (Mendel & John, 2002). Since that, T2FST has been widely discussed by many researchers (Rozaimi & Abd. Fatah, 2013; Rozaimi *et al.*, 2013a; 2013b; 2013c). Some previous studies only focused on type-1 fuzzy models or type-2 fuzzy models which implement in real life data to form curves or surfaces (Gallo *et al.*, 2000; Anile *et al.*, 2000; Rozaimi & Abd. Fatah, 2012; Rozaimi *et al.*, 2014; Adesah *et al.*, 2017). Meanwhile, other researchers focusing on developing theories of fuzzy number such as Abd. Fatah and Rozaimi (2013a; 2013b) which developed eight theorems involving normal and perfectly normal type-2 triangular fuzzy number.

However, the case studies when both sets of data uncertainties and complex uncertainties in a database have not been done yet. Therefore, both the T1FST and T2FST should be used to ensure that uncertainty and complex uncertainty data can be modeled.

II. PRELIMINARIES

Fuzzy set theory introduced by Zadeh has become the main core in solving uncertainty problems. Fuzzy set also known as fuzzy number when it comes to define the set of real numbers that characterized by mean of membership functions. It gives information either "about" or "around" such number that has unique modal crisp value, convex, decreasing, increasing and left-right continuous (Abd. Fatah *et al.*, 2009). T1FST can be derived from T2FST when T2FST is being reduced to T1FST because the type-1 fuzzy set is in the left and the right footprint of T2FST. This section will present definition of T1FST, T1FN, T2FST and T2FN.

A. Type-1 Fuzzy Set Theory

Definition 1. A type-1 fuzzy set $\overleftrightarrow{A}$, which is a term of a single variable $x \in X$, with membership function (Zadeh 1965)

$$\overleftrightarrow{A} = \left\{ \left(x, \mu_{\overleftrightarrow{A}}(x) \right) \mid \forall x \in X \right\} \quad (1)$$

Definition 2. A fuzzy set $\overleftrightarrow{A}$ is called a fuzzy number (type-1) if the following conditions are fulfilled (Abd. Fatah et. al 2009).

i. There exist $x \in \mathbb{R}$, such that $\mu_{\overleftrightarrow{A}}(x) = 1$.

ii. For any $\alpha \in (0,1]$, the set $\left\{ x : \mu_{\overleftrightarrow{A}}(x) \geq \alpha \right\}$ is a closed interval denoted by $\left\langle \overleftrightarrow{A}_{\alpha}^{\leftarrow}, \overleftrightarrow{A}, \overleftrightarrow{A}_{\alpha}^{\rightarrow} \right\rangle$.

Definition 3. If the triangular fuzzy number represent as $\ddot{A}=(a,b,c)$ and $\ddot{A}_\alpha$ be a $\alpha-$ cut operation of triangular fuzzy number, then crisp interval by $\alpha-$cut operation is obtained as $\ddot{A}=\left[a^\alpha,c^\alpha\right]=\left[(d-a)\alpha+a,-(c-d)\alpha+c\right]$ (2) with $\alpha\in(0,1]$ where the membership function, $\mu_{\ddot{A}}(x)$ given by (figure 1) (Rozaimi & Abd. Fatah 2012)

$$\mu_{\ddot{A}}(x)=\begin{cases}0 & , x<a\\ \dfrac{x-a}{d-a} & , a\le x\le d\\ \dfrac{c-x}{c-d} & , d\le x\le c\\ 0 & , x>c\end{cases}$$

(3)

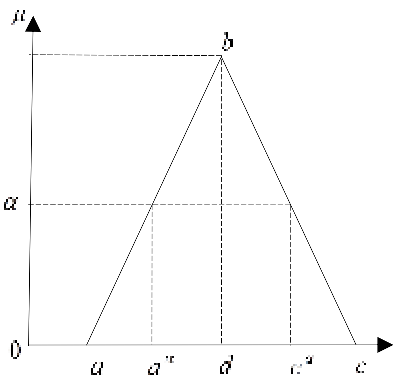

Figure 1. Triangular fuzzy number, $\ddot{A}=(a,b,c)$

B. Type-2 Fuzzy Set Theory

α

Definition 4. A type-2 fuzzy set (T2FS) denoted as $\ddot{\ddot{A}}$, is characterized by a type-2 membership function $\mu_{\ddot{\ddot{A}}}(x,u)$ where $\forall x\in X$ and $u\in U_x\subseteq[0,1]$, that is (Mendel 2001)

$$\ddot{\ddot{A}}=\left\{\left((x,u),\mu_{\ddot{\ddot{A}}}(x,u)\right)\mid\forall x\in X,\forall u\in U_x,\subseteq[0,1]\right\}$$

(4)

in which $0\le\mu_{\ddot{\ddot{A}}}(x,u)\le1$.

Definition 5. A T2FN is broadly defined as a T2FS that has a numerical domain. An interval T2FS is defined using the following four constraints, where $\ddot{\ddot{A}}=\left\{\left[a^\alpha,b^\alpha\right],\left[c^\alpha,d^\alpha\right]\right\}$, $\forall\alpha\in[0,1]$, $\forall a^\alpha,b^\alpha,c^\alpha,d^\alpha\in\mathbb{R}$ (figure 2) (Zimmermann 1985; Klir & Yuan 1995)

i. $a^\alpha\le b^\alpha\le c^\alpha\le d^\alpha$

ii. $\left[a^\alpha,d^\alpha\right]$ and $\left[b^\alpha,c^\alpha\right]$ generate a function that is convex and $\left[a^\alpha,d^\alpha\right]$ generate a function is normal.

iii. $\forall\alpha_1,\alpha_2\notin[0,1]:(\alpha_2>\alpha_1)\supset$ $\left(\left[a^{\alpha_1},c^{\alpha_1}\right]\supset\left[a^{\alpha_2},c^{\alpha_2}\right],\left[b^{\alpha_1},d^{\alpha_1}\right]\supset\left[b^{\alpha_2},d^{\alpha_2}\right]\right)$, for $c^{\alpha_2}\ge b^{\alpha_2}$.

iv. If the maximum of the membership function generated by $\left[b^\alpha,c^\alpha\right]$ is the level α_m, that is $\left[b^{\alpha_m},c^{\alpha_m}\right]$, then $\left[b^{\alpha_m},c^{\alpha_m}\right]\subset\left[a^{\alpha=1},d^{\alpha=1}\right]$.

x

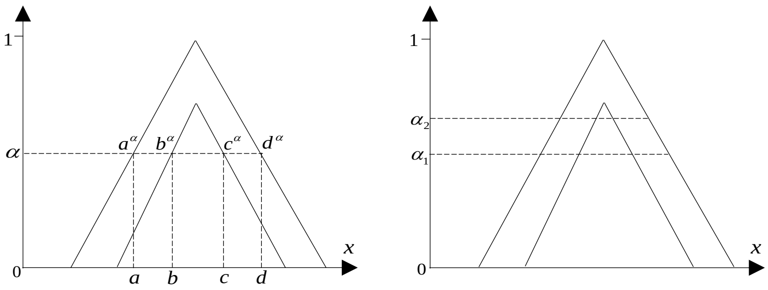

Figure 2. The interpretation of an interval T2FN

III. DISCUSSION

Based on the definitions of T1FST and T2FST, both theories deal with uncertainty and complex uncertainty. More specifically, in defining uncertainty and complex uncertainty data in order to allow the curve and surface function can be blended together to model those uncertainty data. The previous researchers had proven that the uncertainty and complex uncertainty data can be modeled through type-1 or type-2 fuzzy curve and surface function which the data forms are in type-1 and type-2 data respectively.

Although many studies have solved the problem of uncertainty and complex uncertainty through modeling, yet there has been no study to consider both uncertainty and complex uncertainty data. Therefore, the idea of this research is to propose a model of both uncertainty and complex uncertainty data by using only one fuzzy curve and surface function.

IV. CONCLUSIONS

As a conclusion, the ability of T1FST and T2FST to defined uncertainty and complex uncertainty data gives a good impact in geometric modelling field especially in forming perfect curves and surfaces. However, the study of curves and surface defined by using both type-1 and type-2 fuzzy set theory is only just the beginning. For further studies, we will implement fuzzy curves and surface model to illustrate both uncertainty and complex uncertainty data.

V. ACKNOWLEDGEMENTS

The authors would like to acknowledge Universiti Malaysia Sabah (UMSGreat GUG00088-SG-2-/2016) for their funding and providing the facilities to conduct this research.

[1] Abd. Fatah, W., Jamaludin, M.A., Ahmad, A.M. & Abu. Osman, M.T. (2004). Fuzzy Set In Geometric Modeling, Proceeding, International Conference on Computer Graphics, *Imaging and Visualization*, vol. 2004, pp. 227 - 232.

[2] Abd. Fatah, W., Jamaludin, M.A., Ahmad, A.M. (2009). Fuzzy Geometric Modeling, Proceeding, Sixth International Conference on Computer Graphics, *Imaging and Visualization* vol. 2009, pp. 276 - 280.

[3] Abd. Fatah, W., Jamaludin, M.A., Ahmad, A.M. & Abu. Osman, M.T. (2010). Penyelesaian Masalah Data Ketakpastian Menggunakan B-Splin Kabur, *Sains Malaysiana*, vol. 39, no. 4, pp. 661 - 670.

[4] Abd. Fatah, W. & Rozaimi, Z. (2013a). Generalized Normal Type-2 Triangular Fuzzy Number, Applied Mathematical Sciences, vol. 7, no. 45, pp. 2239 - 2252.

[5] Abd. Fatah, W. & Rozaimi Z. (2013b). On Perfectly Normal Type-2 Triangular Fuzzy Number, *Applied Mathematical Sciences*, vol. 7, no. 46, 2253 - 2263.

[6] Anile, A.M., Falcidieno, B., Gallo, G., Spagnuolo, M. & Spinello, S. (2000). Fuzzy Sets and Systems: Modeling Uncertain Data with Fuzzy B-Splines, *Fuzzy Sets and Systems*, vol. 113, pp. 397 – 410.

[7] Gallo, G., Spagnuolo, M. & Spinello, S. (2000). Fuzzy B-Splines A Surface Model Encapsulating Uncertainty, *Graphical Models*, vol. 62, 40 - 55.

[8] Klir G.J. & Yuan, B. (1995) Fuzzy Sets and Fuzzy Logic: Theory and Application. *Prentice Hall, New York*.

[9] Mendel, J.M. (2001) Uncertain Rule-Based Fuzzy Logic Systems: Introduction and New Directions. *Prentice Hall, New Jersey*.

[10] Mendel, J.M. & John, R.I.B. (2002). Type-2 Fuzzy Sets Made Simple, IEEE Transactions on Fuzzy Systems, vol. 10, no. 2, pp. 117 – 127.

[11] Rozaimi, Z. & Abd. Fatah, W. (2013). On Defining Complex Uncertainty Data Points by Type-2 Fuzzy Number: Two Specials Cases, *International Journal of Math*. Analysis, vol. 7, no. 26, pp. 1285 - 1300.

[12] Rozaimi, Z. & Abd. Fatah, W. (2012). Fuzzy B-spline Modeling of Uncertainty Data, *Applied Mathematical Sciences*, vol. 6, no. 140, 6971 - 6991.

[13] Rozaimi, Z. & Abd. Fatah, W.(2014). Pemodelan Titik Data Kabur Teritlak, *Sains Malaysiana*, vol. 43, no. 5, pp. 799 -805.

[14] Rozaimi. Z., Abd. Fatah, W., & Gobithaasan, R.U. (2013a) Type-2 fuzzy Bezier curve modelling, *AIP Conference Proceedings*, pp. 945 – 952.

[15] Rozaimi, Z., Abd. Fatah, W., & Gobithaasan, RU 2013b, Normal Type-2 Fuzzy Rational B-spline Curve, *Journal of Math. Analysis*, vol. 7, no. 16, pp. 789 – 806.

[16] Rozaimi, Z., Abd. Fatah, W., & Gobithaasan, R.U. (2013c) Perfectly Normal Type-2 Fuzzy Interpolation B-spline Curve, *Applied Mathematical Sciences*, vol. 7, no.21, pp. 1043 – 1055.

[17] Rozaimi, Z., Abd. Fatah, W., & Gobithaasan, R.U. (2014). Fuzzy B-Spline Surface Modeling, *Journal of Applied Mathematics*, vol. 2014, no. 1, pp. 1 - 8.

[18] Adesah, R.S., Zakaria, R., Wahab, A.F. & Talibe, A. (2017). Fuzzy Curve Model, IOP Conf. Series: *Journal of Physics: Conf. Series*, pp. 1 - 6.

[19] Zadeh, L. (1965). Fuzzy Sets, *Information and Control*, vol. 8, pp. 338 - 353.

[20] Zadeh, L. (1975). The Concept of a Linguistic Variable and its Application to Approximate Reasoning-I, *Information and Control*, vol. 8, pp. 199 - 249.

[21] Zimmermann, H.J. (1985). Fuzzy Set Theory and Its Applications. Kluwer Academic, USA.