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Kernel Estimation in Line Transect Sampling for Parametric Model

Gamil A.A. Saeed[1], Noryanti Muhammad[1*] and Wan Nur Syahidah Wan Yusoff[1]

[1]*Centre for Mathematical Sciences, College of Computing and Applied Sciences,*
Universiti Malaysia Pahang, 26300, Gambang, Pahang, Malaysia

Line transect sampling is a common method used in ecology for sampling the sample required. It is an important procedure for estimating the population density of objects in interested study area. There are two main ways to estimate the population density which are parametric and nonparametric estimation methods. In this paper, we present kernel method to propose new estimator of the propose population density. Kernel estimation method is used due to avoid the assumption about the shape of the unknown detectable functions. We investigate the performance of the new estimator using simulation study and compared with the existing estimators. Based on the simulation study, the results show that the proposed estimator preforms better than other well-known estimators.

Keywords: line transect sampling; kernel method; nonparametric estimation method; parametric model

I. INTRODUCTION

There are many ways to estimate the population abundance, one of the methods is line transect sampling. In line transect sampling technique, the density (D) of the objects on interested area depends on the measured distances between the detected objects and the line of transect, L. Two key assumptions of line transect distances sampling (DS) are all objects on the line are certainty detected and the objects must be detected at their original location. If these two assumptions are valid, the DS gives unbiased estimators of the population Density (Buckland *et al.*, 2001). These assumptions lead us to define the important concept in the DS, also called a detection function. Detection function plays the central role to the line transect sampling technique which can be defined as

$$g(y) = P(\text{observing object / its perpendicular distances } y \text{ from center line}). \quad (1)$$

It is reasonable to assume that the detectability function g(y) is nonincreasing function on $[0;\infty)$, that means the detection function have a monotone decreasing curve, the probability of detection should close to one as distance from the line increases from zero, that means the detection function satisfied shoulder property which is $(g(0) = 1)$. Let $Y_{1,\dots,}Y_n$ donated to the nonpooled sighting perpendicular distances which selected randomly and independently from the transect strips, with common density function $f(y)$, defined on $[0;w]$, $f(y)$ was considered by Burnham and Anderson (1976) as

$$f(y) = \frac{g(y)}{\int_0^w g(t)dt}\ ; 0 \le y \le w. \quad (2)$$

By assuming that all objects on the line and have perfect probability, Burnham and Anderson (1976) showed that the density D of the objects in surveyed area related to the probability density function (pdf) $f(y)$ which evaluated at $y = 0$ as

$$D = \frac{E(n)f(0)}{2L} \quad (3)$$

where $E(n)$ is the expected value of sighted objects. Since D depends to $f(0)$, the density D estimated by $\hat{f}(0)$. Based on Burnham and Anderson (1976) and Buckland *et al.*, (1993; 2015), the density D can be estimated by

$$\hat{D} = \frac{n}{2L}\hat{f}(0) \quad (4)$$

*Corresponding author's e-mail: noryanti@ump.edu.my

Let $Y_{1,\ldots,}Y_n$ be a set of perpendicular distances which are usually assumed to be a random sample, having a density function $f(y;\ \theta)$ depends on unknown parameter θ, where θ may one parameter or vector of parameters. Since the $f(0)$ is function of the parameter θ therefore, the estimate of θ lead us to estimate $\hat{f}(0) = \hat{f}(0;\ \hat{\theta})$. The exponential detection function is presented by Gates *et al.* (1968) given as

$$g(x;\theta) = e^{-y/\theta};\ y \geq 0\ ,\theta > 0 \tag{5}$$

with corresponding pdf,

$$f(x;\theta) = \frac{1}{\theta}\ e^{-y/\theta};\ y \geq 0\ ,\theta > 0 \tag{6}$$

The maximum likelihood estimator (MLE) of $f(0)$ is,

$$\hat{f}_{MLE}(0) = \frac{1}{\bar{Y}}, \tag{7}$$

where $\bar{Y}$ is the sample mean. It is important to refer that the negative exponential (NE) model does not satisfy $\acute{f}(0) = 0$ while half normal (HN) model satisfies $\acute{f}(0) = 0$. This means half normal detection deals with the property of the shoulder. In contrast, the NE detection $g(y)$ does not achieve the shoulder condition. Hemingway (1971) suggested the half normal model with pdf

$$f(y;\sigma^2) = \sqrt{\frac{2}{\pi\sigma^2}}\ e^{-y^2/2\sigma^2};y \geq 0\ ,\sigma^2 > 0\ , \tag{8}$$

and the half normal detection function is,

$$g(y;\sigma^2) = e^{-y^2/2\sigma^2};y \geq 0\ ,\sigma^2 > 0\ . \tag{9}$$

The main estimator to estimate σ^2 for density in equation (8) is $\frac{1}{n}\sum_{i=1}^{n} y_i^2$, given by using MLE to estimate $f(0)$. Since $f(0) = \sqrt{\frac{2}{\pi\sigma^2}}$, the MLE of $f(0)$ is given by

$$\hat{f}_{MLE}(0) = \sqrt{\frac{2}{\pi}}\left(\frac{1}{n}\sum_{i=1}^{n} y_i^2\right)^{-1/2} \tag{10}$$

II. MATERIALS AND METHOD

Let $Y_{1,\ldots,}Y_n$ random variables of size n representing the perpendicular distances have probability density function (pdf) $f(y)$, and independent and identically distributed (iid) have a detection function $g(y)$. Saeed *et al.*, (2017) introduced $g(y;\ \sigma^2)$ which is given as

$$g(y;\sigma^2) = \left(2 - e^{-y^2/2\sigma^2}\right)e^{-y^2/2\sigma^2};y \geq 0\ ,\sigma^2 > 0\ , \tag{11}$$

and the first derivative of $g(y;\ \sigma^2)$ is given as

$$g'(y;\sigma^2) = \frac{2}{\sigma^2}\ ye^{-y^2/2\sigma^2}\left(e^{-y^2/2\sigma^2} - 1\right). \tag{12}$$

For all $y \geq 0;\ \sigma^2 > 0$, we can easily observe that $0 \leq e^{-y^2/2\sigma^2} \leq 1$, then equation (12) can be shown that $g'(y;\sigma^2) < 0$, therefore, the detection function $g(y;\ \sigma^2)$ is monotone decreasing function on $[0;\infty)$. In addition, $g(y;\ \sigma^2)$ continuous function satisfies the condition $g'(0;\ \sigma^2) = 1$, which means that the probability of sighted object on the line equals to one. Figure 1 shows the shapes of the detection function for certain values of σ^2. Moreover, $f(y;\ \sigma^2)$ is continuous function and proportional to the $g(y;\ \sigma^2)$, $f(y;\ \sigma^2)$ is decreasing function and related with $g(y;\ \sigma^2)$ as

$$g(y;\sigma^2) = \mu f(y;\ \sigma^2), \tag{13}$$

where

$$\mu = \int_0^\infty g(y;\sigma^2)dy. \tag{14}$$

Then, the corresponding pdf of $g(y;\sigma^2)$, given by Saeed *et al.*, (2017) as

$$g(y;\sigma^2) = f(y;\ \sigma^2)\left(2 - e^{-y^2/2\sigma^2}\right)e^{-y^2/2\sigma^2};y \geq 0\ ,\sigma^2 > 0 \tag{15}$$

By solving the integration in equation (14), the $f(0;\ \sigma^2)$ is given as (Saeed *et al.*, 2017)

$$f(0;\ \sigma^2) = \frac{2}{(2\sqrt{2}-1)\sqrt{\pi\sigma^2}} \tag{16}$$

Equation (16) shows that the $f(0;\ \sigma^2)$ is a function with the parameter σ^2. Therefore, it is enough to estimate σ^2 for estimating $f(0;\ \sigma^2)$. The considered model in equation (15) was studied by Noryanti *et al.* (2018), the maximum likelihood (MLE) method is used to estimate the proposed estimator, the performance of the proposed estimator is evaluated by simulation study. In next section, for this purpose, kernel method with the proposed model are used to compute the smoothing parameter and studied the proposed estimator performance.

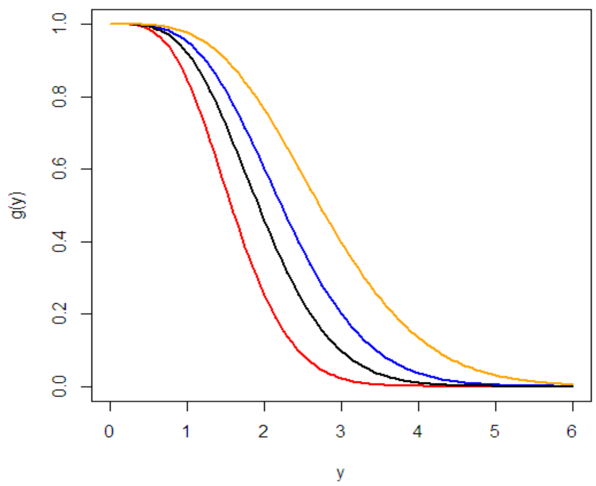

Figure 1. The detection function $g(y)$ graph of the proposed model for different parameter σ^2

III. KERNAL ESTIMATORR

Consider $Y_{1,\dots,}Y_n$ be random sample of size n, distributed from continuous probability density function (pdf) $f(y)$. According to in Silverman (1986) and (2018), the kernel estimate $\hat{f}(y)$ of $f(y)$ supported on $[0;\infty)$, is given by

$$\hat{f}(x) = \frac{1}{nh}\sum_{i=1}^{n}\left(K\left(\frac{y-Y_i}{h}\right) + K\left(\left(\frac{y+Y_i}{h}\right)\right)\right);\ y \geq 0\ , \tag{17}$$

where h is the bandwidth, which controls the smoothness of the fitted function shape and K is a kernel function, assumes to be symmetric function and satisfies the following

$$\int_{-\infty}^{\infty} K(t)dt = 1\,,\int_{-\infty}^{\infty} tK(t)dt = 0\,,\int_{-\infty}^{\infty} t^2K(t)dt = c \neq 0\ . \tag{18}$$

the standard kernel estimate $\hat{f}(0)$ of $f(0)$ is given by Chen (1996) as,

$$\hat{f}(0) = \frac{2}{nh}\sum_{i=1}^{n} K\left(\frac{Y_i}{h}\right). \tag{19}$$

By assuming $nh \to \infty$ as $n \to \infty$, the bias and variance of $\hat{f}_k(0)$ can be represented as

$$Bias\left(\hat{f}_k(0)\right) = 2hf'(0)\int_0^{\infty} uk(u)du\ + h^2f''(0)\int_0^{\infty} u^2k(u)du + o(h^2) \tag{20}$$

and

$$Var\left(\hat{f}_k(0)\right) = \frac{4f(0)}{nh}\int_0^{\infty} K^2(u)du + o(n^{-1}h^{-1}) \tag{21}$$

Depending on the assumption that the shoulder condition is true, and the function $f(0)$ has continuous derivative. Thus, lead us to introduce the following estimator.

$$f(0) = \hat{f}_k(0) - h^2 f''(0)\int_0^{\infty} u^2K(u)du\ . \tag{22}$$

The $f''(0)$ is the second derivative of $f(y)$ at $y = 0$. Based on equation (22) and for underlying $f(y)$ which satisfies the shoulder condition, the bias of $\hat{f}(0)$ is given as

$$Bias\left(\hat{f}(0)\right) = \frac{2h^3f^{(3)}(0)}{6}\int_0^{\infty} u^3K(u)du + \frac{2h^4f^{(4)}(0)}{24}\int_0^{\infty} u^4K(u)du\ + O(h^5)\,. \tag{23}$$

and the variance of $\hat{f}(0)$ is

$$Var\left(\hat{f}(0)\right) = \frac{4}{nh}f(0)\int_0^{\infty}\left(K(u)\right)^2du + o(n^{-1}) \tag{24}$$

From equations (23), (24) and the assumption that

$$f'(0) = f'''(0) = 0\,,$$

the asymptotic mean square error (AMSE) can be written as

$$AMSE\left(\hat{f}(0)\right) = \frac{4}{nh}f(0)\int_0^{\infty}\left(K(u)\right)^2du + \frac{2h^8}{576}\left(f^{(4)}(0)\int_0^{\infty} u^4K(u)du\right)^2 \tag{25}$$

The key aspect in nonparametric kernel method the value h which plays the major milestone in the performance of $\hat{f}_k(0)$. A large smoothing parameter h leads to an estimate with small variance and large bias while a small h produces a large variance and small bias. Then, the minimisation of the AMSE in equation (25) leads to compute the optimal value of h, which is given by

$$h = \left(\frac{72f(0)\int_0^{\infty}\left(K(u)\right)^2du}{\left(f^{(4)}(0)\int_0^{\infty} u^4K(u)du\right)^2}\right)^{1/9} n^{-1/9}\ . \tag{26}$$

The smoothing parameter h in equation (26) depends on $f(0)\,, f^{(4)}(0)$ and the kernel function $K(u)$. The negative exponential, half normal models and the proposed model in equation (15) are used as references to compute the smoothing parameter h. In this paper for all models, we use standard normal $N(0;\,1)$ as a kernel function. The optimal values h for the different estimators have been computed and give $h_1 = 1.18229\,\bar{Y}n^{-1/9}$ for $\hat{f}_{1;k}(0)$, $h_2 =$

$0.949713Tn^{-1/9}$ for $\hat{f}_{2;k}(0)$ and $h_3 = 0{:}904949\bar{Y}n^{-1/9}$ for the proposed estimator $\hat{f}_{3;k}(0)$, respectively, where $\bar{Y}$ is sample mean and $T = \sqrt{\frac{1}{n}\sum_{i=1}^{n} y_i^2}$.

IV. SIMULATION RESULT AND DISCUSSION

A simulation study is performed in order to investigate the performances of the considered estimators in Section 3. The data is simulated using one of commonly models used in line transect which is Hazard-Rate (HR) model (Hemingway (1971) given as

$$f(x) = \frac{1}{\Gamma(1-1/\beta)}\left(1 - e^{-x^{-\beta}}\right), \quad x \geq 0\,, \beta \geq 1. \qquad (27)$$

For this purpose, we used HR model to generate 400 samples of sizes $n = 50, 100$ and $n = 200$ of perpendicular distances data set. Four HR models is selected with parameter values β and the corresponding truncated value w are given as $(\beta; w) = \{(1.5, 20), (2, 12), (2.5, 8);\ (3, 6)\}$. The Relative Mean Error (RME) and Relative Bias (RB) be estimated to evaluate the performance of the estimators $\hat{f}_{1;k}(0)$, $\hat{f}_{2;k}(0)$, and $\hat{f}_{1;k}(0)$. The RME and RB is given by

$$RME = \frac{\sqrt{MSE(\hat{f}(0))}}{f(0)}, \qquad (28)$$

and

$$RB = \frac{E(\hat{f}(0)) - f(0)}{f(0)}. \qquad (29)$$

The results of RME and RB are summarised in Table 1.

Table 1. Simulated values of RB and RME for the different estimators.

n	w	β		$\hat{f}_{1;k}(0)$	$\hat{f}_{2;k}(0)$	$\hat{f}_{3;k}(0)$
50			RB	-0.40688	-0.50084	-0.30589
			RME	0.42316	0.50911	0.33455
100	1.5	20	RB	-0.38122	-0.38122	-0.27824
			RME	0.39244	0.48645	0.29807
200			RB	-0.35529	-0.45676	-0.25086
			RME	0.36101	0.45947	0.26129
50			RB	-0.26845	-0.35539	-0.17278
			RME	0.29379	0.37279	0.21703
100	2	12	RB	-0.24750	-0.33672	-0.15292
			RME	0.26519	0.34771	0.18513
200			RB	-0.22132	-0.31187	-0.12957
			RME	0.23039	0.31683	0.14770
50			RB	-0.14835	-0.20491	-0.06831
			RME	0.18622	0.23302	0.14057
100	2.5	8	RB	-0.13294	-0.18801	-0.05846
			RME	0.15791	0.20408	0.11091
200			RB	-0.10605	-0.15896	-0.03845
			RME	0.12484	0.17095	0.08168
50			RB	-0.09536	-0.12424	-0.03392
			RME	0.14688	0.16463	0.13001
100	3	6	RB	-0.07614	-0.10291	-0.02344
			RME	0.10982	0.12760	0.09334
200			RB	-0.05828	-0.08309	-0.01275
			RME	0.08253	0.09942	0.06988

Table 1 shows that the results of RB and RME in of the three considered estimators which demonstrated in Section 3. The estimator $\hat{f}_{1;k}(0)$ is performed better than $\hat{f}_{2;k}(0)$ for all considered cases regardless of sample size n. For each sample size and for all considered cases, the estimator $\hat{f}_{1;k}(0)$ provides the smallest value of RME and RB comparing to the estimators $\hat{f}_{1;k}(0)$ and $\hat{f}_{1;k}(0)$. Indeed, the significant result is that the performance of proposed estimator $\hat{f}_{3;k}(0)$is outperformed well-known considered estimators.

Another noticeable results, the values of RME decrease as the sample size increases for all estimators which indicates that the consistency of the proposed estimators for $f(0)$. This result can be shown in Figures 2 and 3.

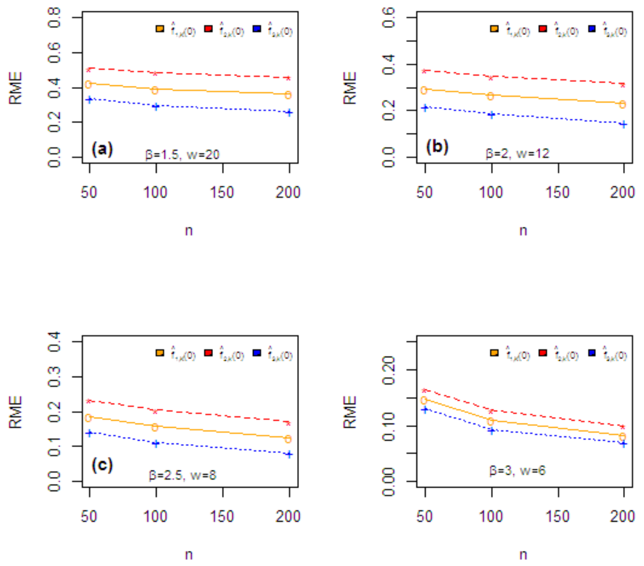

Figure 2. RME values for the different estimators

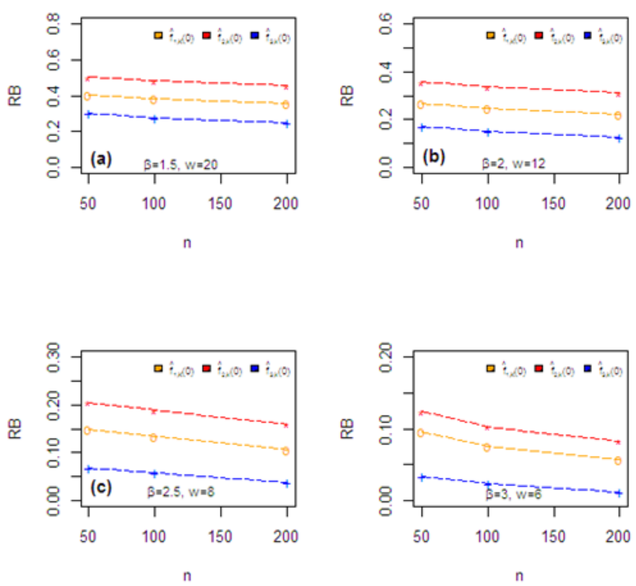

Figure 3. RB values for the different estimators

V. CONCLUSION

In this paper, parametric model is used to construct the nonparametric kernel estimator $f(0)$. Moreover, the smoothing parameter of the kernel estimator is computed for considered estimators, which plays a major milestone in the performance of the kernel estimator. Simulation study is constructed to compare the performance of the proposed estimator with other existing estimators. The simulated results of RB and RME indicate that the proposed estimator is performed better than other estimators considered. In general, the proposed model be recommended to estimate $f(0)$ and then to estimate the population density D.

VI. ACKNOWLEDGEMENT

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