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Urban Transit Frequency Setting using Multiple Tabu Search with Parameter Control

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Urban transit frequency setting is one of the multiobjective problems in public transportation system, which aims to find optimal time interval between subsequent buses along the routes. In this study, a Multiple Tabu Search (MTS) algorithm is employed to determine the bus frequency of the routes that minimize the number of buses, total waiting times and overcrowding simultaneously. The efficiency of the algorithm is tested on benchmark dataset by changing the value of the total domains. The chosen parameter gives considerable effect on the objective functions compared to other parameters such as the size of tabu list and the number of iterations. Using statistical hypotheses evaluation, the results indicate that the number of domains determines the quality of solutions for different instances of the problem. Additionally, the frequency setting problem is extended by revising the passenger assignment procedure and frequency optimization process with time-dependent demand in order to reflect a real-world scenario. The extended results from different size of routes are presented to show the effectiveness of the proposed algorithm.

Keywords: transit frequency setting; multiple tabu search; parameter tuning

I. INTRODUCTION

The continuous rise in private car ownership leads to several transportation problems such as air pollution, high carbon emission and traffic congestion. The improvement of public transportation system can increase its ridership by satisfying the transportation demand which consequently reduces the problems in a cost-effective manner using available resources. One of the important practices in urban public transit planning is to determine service frequency based on passenger demand, passenger waiting time, transit capacity and available resources. Since frequency setting is tackled at tactical planning level of urban transportation, the decision maker may needs to explore different variant of solutions from the conflicting objectives between the operators and the passengers. This leads to the formation of a multiobjective combinatorial optimization problem.

Multiple Tabu Search (MTS) is a metaheuristic method that proven to solve many NP-hard optimization problems. It employs adaptive memory properties to record attractive moves, apply responsive exploitation and exploration to avoid being trapped in local optimum and use multiple initial solutions to speed up the process for searching the best-known solution. A MTS algorithm includes a set of parameters such as tabu tenure, maximum iteration for intensification and termination as well as the number of initial solutions that need to be assigned appropriately to obtain desirable results. The importance of a parameter is usually depends on different versions of an algorithm that suit specific problem. The process of determining suitable values for the parameters during execution are time consuming and more challenging than before running the algorithm as it required automated scheme to control the parameters.

This paper is the extension of Uvaraja and Lee (2019) by evaluating the effectiveness of MTS algorithm with a parameter tuning. Section II describes the transportation related work for parameter control in TS algorithm.

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Section III explains the statistical test that will be applied for evaluating the MTS algorithm. A brief explanation on passenger assignment procedure and frequency optimization process are also presented. Section IV addresses the benchmark datasets and presents the computational results. Various number of domains for refining the algorithm are examined thoroughly in this section. Finally, Section V summaries the findings and concludes the paper.

II. LITERATURE REVIEW

The combination of different set of parameter values affects the rank of optimization algorithm in terms of computational time and the objective functions values. The parameter control in metaheuristic approaches can be conducted in several directions which are either fixed the parameter values before the execution or altering them during the process of optimization by dynamic and self-adaptive approaches (Xu *et al.*, 1998). The study of parameter control is still inadequate in urban transportation problem. Only a limited number of studies are conducted for parameter identifications in TS algorithm. Since the focus of this research is on the improvement of MTS algorithm, the studies of frequency optimization problem are omitted.

A local search approach based on network flow model and TS algorithm is proposed by Xu and Kelly (1996) to solve a vehicle routing problem. The capacity constraints are relaxed using penalty terms whose parameter values are altered according to time and search feedback. The total number of network moves is also changed dynamically throughout the iterations. A computational experience is conducted on a set of benchmark test problems and compare them with the best-known solutions in the literature. Based on the experiment, higher value for penalty term and network moves able to drive the search toward feasibility and diversity respectively.

A combined simulated annealing and TS strategy (SA-TABU) for network design problem ranging from 36 to 332 links is developed in Zeng (1998). A heuristic evaluation function (HEF) is used according to the characteristic of the problem and search strategy. The main features of SA-TABU are error variable of HEF, Markov chain length, temperature dropping rate and tabu list length. The sensitivity analysis conducted to find best parameter values for all the components showed that good solutions are recorded in relatively short computational times. Expanding approximately 10% of the links, produce high percentage improvement ranging from 73% to 97% for the five test networks.

The research by Gendreau *et al.* (1999) analysed user control parameters that require calibration such as neighbourhood size, tabu tenure and scaling parameter for diversification. A sensitivity analysis is performed by sequential process to find best possible values for solving heterogeneous fleet vehicle routing problem. After some experimentations, the adapted TS algorithm is able to find a good compromise results between the execution time and the solution quality by setting appropriate values for the parameters.

A simplex based TS algorithm is designed for capacitated network design (Crainic *et al.*, 2000). The experiment results show that the algorithm is robust with respect to the parameter values in 16 parameter combinations and 10 problem instances. The criteria including tabu tenure chosen from higher value interval, initial solution with the activation of tabu logic and three paths for each commodity performed better than other alternatives.

Most of the studies show that the determination of suitable parameter values for TS algorithm based on the problems involved is an important criteria to produce significant results. This type of experiment is highly conducted in early years in 90's by employing different strategies such as comparing the success rate of each parameter values. In recent years, the contribution of its methodological structure becomes the primary concern. Despite its effectiveness in numerous types of problems, the algorithm can be improved further by assigning appropriate values for the selected parameters that directly influence the algorithm.

III. MULTIPLE TABU SEARCH ALGORITHM FOR FREQUENCY OPTIMIZATION

In this section, the process for choosing the best value of a selected parameter and procedure of optimizing frequency are discussed.

A. Problem Formulation and Algorithm Development

The setting of transit frequency can be expressed as a bi-level process that consist of passenger assignment procedure and frequency optimization. This is an iterative procedure to achieve consistency in the route frequencies as both demand and frequency are dependent on each other. The assignment model takes initial setting of route frequencies and origin-destination demand as the input. Its output is the passengers demand flow for all the routes. The frequency share rule and multinomial logit model are adopted from Baaj and Mahmassani (1991) and Afandizadeh *et al.* (2013) respectively to determine the passenger route choice as these models are able to approximate better passenger's behaviour using the total travel time and have been employed extensively in the previous literature. Thus, the same procedures from the respective researchers are applied in this paper to allow for a fair comparison between the algorithms.

After generating the number of passengers travelling at each route, frequency of routes are determined using multiobjective optimization model with the aim of minimizing the number of buses, total waiting times and overcrowding. The first objective represents the operator cost such that higher frequency can directly affect the number of buses required. At the same time, the second and third objectives represent the preferences of passengers which also influenced by the frequency. Both of their preferences are contrary to each other where operators intended to reduce the total buses needed with lower frequency while passengers prefer to wait less at bus stops which require more bus frequency. The assignment strategy and model formulations are defined further in Uvaraja and Lee (2019).

The first model finds the suitable frequencies based on total passenger's demand of each route, similar to the approaches done in many literatures; the second extends the previous model by including timeslots to optimize routes frequencies according to time-dependent demands assumption that reflects the actual situation in real life. In other words, the passenger's demand obtained from the first level is divided into peak and off-peak hours throughout the time horizon studied. The demand on the peak hours is assumed to be doubled the demand on the off-peak hours.

For frequency optimization, the MTS algorithm is employed with appropriate parameter values as mentioned in Uvaraja and Lee (2019). The MTS algorithm is an iterative procedure that select the best move to find optimal solution from solution space beyond local optimality and implement intensification and diversification with adaptive memory structure to exploit and explore the search space respectively. The MTS algorithm works with several initial frequency that chosen from different subsets (domains) of the search space within the minimum frequency of 1 per hour and a maximum of 20 per hour. The neighborhood solutions are formed by adjusting the current solution based on a variable step size. To access a potential move, every solution in the neighborhood is evaluated for its dominance. The dominated solutions at every iteration are grouped together. The selected solution is kept in a tabu list for a number of iteration based on its size to avoid repeated moves. For this study, the tabu list size is set to be doubled the number of routes. When there is no acceptable solution available, intensification or diversification are conducted subject to the availability of the feasible solution in the intermediate memory structure. This process is repeated until there is no improvement in the best-known solution for a number of iterations for all the routes. As there are several solutions from different domains, the best result among the domains which minimize all the objectives is chosen.

B. Statistical Analysis

The direct comparison of the objective function values for different number of domains are not applicable because each of them do not completely dominate with one another. Therefore, a hypothesis test is applied to find the significant difference between each best solution for different number of domains. The purpose of this hypothesis testing is to show the existence of statistical significance and not caused by random variations. This testing is based on a null hypothesis which always assume that the variables have no effect on the results and alternative hypothesis which shows contradiction opinion with the previous one. A p-value is a calculated probability of acquiring a result at least as extreme given the null hypothesis is true while chi-square value is used to analyze Test of Independent for categorical variables.

At first, the chi-square value is calculated using equation (1) and the p-value is approximated. These statistical analysis are performed using Microsoft Excel 2010. By using the chi-square distribution table, the acceptance of the hypothesis is decided according to the degree of freedom (equation (3)) and the significant level. The observed values represent the objective function values, the number of rows indicate total number of domains which is 8 (3 – 10 domains) in this experiment and the columns show the number of objective functions. The expected values are computed based on the observed values using equation (2).

$$\chi^2 = \sum_{i=1}^{r}\sum_{j=1}^{c}\frac{\left(O_{i,j}-E_{i,j}\right)^2}{E_{i,j}}, \quad (1)$$

$$E_{ij} = \frac{\sum_{i=1}^{r} O_{ij} \times \sum_{j=1}^{c} O_{ij}}{\sum_{i=1}^{r}\sum_{j=1}^{c} O_{ij}}, \quad (2)$$

$$Degree\ of\ freedom = (r-1)\times(c-1), \quad (3)$$

where,

χ^2 = Chi-square probability

$O_{i,j}$ = observed values in the i-th row and j-th column

$E_{i,j}$ = expected values in the i-th row and j-th column

r = number of rows

c = number of columns

The dependency on the number of domains on the solution quality is analyzed from the values of every objective functions. The value of overcrowding is not included as it is always zero. The number of buses and the total waiting times are compared using p-value and chi-square value under the confidence level of 95%. The null hypothesis states that the capability of obtaining best solution is not depends on the number of domain used. The p-value must be higher than 0.05 (alpha) and the chi-square value should be lesser than 14 to accept the null hypothesis and conclude that there is no significant difference exist between the means. Conversely, if the statistical values are not within the limits, then the dependency to the total domains is assured.

IV. COMPUTATIONAL EXPERIMENT

In this section, two computational experiments are performed. First, the effect of number of domains on the solution quality in term of total buses and waiting times are investigated. The purpose of this experiment is to affirm the analysis conducted in Uvaraja and Lee (2019) that the total domains used will affect the performance of the algorithm. Then, the capability of MTS algorithm on the extended model is tested for higher number of routes using the accepted number of domains. The MTS algorithm with a parameter tuning is tested on Mandl's Swiss Network. The network is defined with 15 nodes, 21 undirected edges and 15570 passenger demands. The MTS algorithm is coded in ANSI-C language and executed on 2.30 GHz Intel® Core™ i3-2350M CPU with 2GB of RAM under Windows 7 operating system.

A. Analysis of Total Domains

In the MTS algorithm, the number of domains is used to divide the search space into several subsets. This parameter is chosen as it gives significant effect on the quality of solutions and computational times. Note that if the number of domains increases, the computational time also increases as the algorithm runs sequentially according to the domains. The number of domains within the interval of 3–10 is used to conduct the analysis. The total domain of 1 is not considered because the feature of MTS algorithm to start the search with multiple solutions is not utilized whereas the total domain of 2 might not always produce significant solutions as the division of the search space is not effective.

In the research of Uvaraja and Lee (2019), frequency optimization problem is studied with MTS algorithm using the similar route sets. They set the total domain of 10 as a parameter and found that the algorithm is able to produce superior solutions than the previous results for most of the route sets. Hence, the results from different parameter values in this research are compared with their solutions. If the number of buses required for each route set and the passengers total waiting times are reduced, then the improvement in the solution quality becomes apparent. As there are several solutions based on the number of domains used, the best solution among them that reduced the objective function values or closely related to the previously published results is selected. The comparison of solutions produced from various number of domains with the value from existing literature are excluded in this research as most of the solutions from every route set are equivalent to the previous results that generated using different algorithms. Therefore, the comparison does not give significant indication on the effectiveness of MTS algorithm.

Based on Table 1, the source of route sets are stated in the first column. The second column indicate the objective functions studied. Columns 3–10 record the values of respective measurements in column 2 such that the values in last column is obtained from Uvaraja and Lee (2019). For all the routes of size 4, the solutions from the total domains of 3–9 are comparable with the domain of 10. The number of buses increases as the total waiting times decreases and vice-verse. The results of Uvaraja and Lee (2019) dominate the solutions from the total domains of 9 for Mandl (1980) and of 4 and 8 for Chakroborty (2003). The total waiting times from the current experiment are higher although the total buses are greater or equal as compared to the previous results. On the other hand, the hypothesis test revealed that the number of domains does not affect the level of solutions for 4 routes as all the p-values are higher than 0.05 and the chi-square values are lower than 14 according to the distribution table.

Table 1. Comparison of results between numbers of domains for 4 routes

Source of route sets	**Objective Functions**	**Total domains**							
		3	**4**	**5**	**6**	**7**	**8**	**9**	**10**
[1]	Total buses	45	48	52	52	51	52	54	54
	Total waiting times	33220	30558	28538	28718	29088	28710	27727	27563
		p-value (0.6540) | Chi-square value (5.0490)							
[2]	Total buses	76	80	83	82	82	81	86	80
	Total waiting times	20382	19306	18605	18771	18820	19291	17930	19247
		p-value (0.8905) | Chi-square value (2.9396)							
[3]	Total buses	83	70	71	77	78	75	74	79
	Total waiting times	20809	24581	24282	22677	22237	23338	23928	22110
		p-value (0.4554) | Chi-square value (6.7494)							
[4]	Total buses	87	87	90	81	83	83	87	86
	Total waiting times	18749	19081	18479	20646	19980	20131	19014	19440
		p-value (0.8755) | Chi-square value (3.1009)							
[5]	Total buses	92	87	90	92	91	94	94	86
	Total waiting times	17987	18578	18004	17541	17956	17376	17288	18767
		p-value (0.9341) | Chi-square value (2.4050)							
[6] - passenger	Total buses	83	86	89	93	92	92	94	88
	Total waiting times	20686	19801	19164	18374	18670	18555	18201	19489
		p-value (0.7118) | Chi-square value (4.5736)							
[6] - operator	Total buses	67	66	60	57	61	65	66	54
	Total waiting times	20046	20247	22371	23623	21786	20737	20219	24711
		p-value (0.1716) | Chi-square value (10.3123)							
[7]	Total buses	75	67	70	75	77	68	72	76
	Total waiting times	21663	23561	22431	21088	20587	23341	22004	20780
		p-value (0.6178) | Chi-square value (5.3458)							
[8]	Total buses	80	89	89	91	94	95	83	86
	Total waiting times	22457	20583	20376	20073	19604	19274	22139	21095
		p-value (0.1128) | Chi-square value (11.6464)							

Note:

[1]:Mandl (1980); [2]:Chakroborty (2003); [3]:Mumford (2013); [4]:Chew et al. (2013); [5]:Nikolić and Teodorović (2013);

[6]:Nikolić and Teodorović (2014); [7]:Arbex and da Cunha (2015); [8]:Buba and Lee (2018); [9]:Baaj and Mahmassani (1991); [10]:Shih and Mahmassani (1994); [11]:Bagloee and Ceder (2011)

For 5 routes (see, Table 2), the solutions from every total domains are equivalent to each other except for the total domains of 7 and 9 where the total waiting times is reduced for the former solutions with equal number of buses. When compared with the solutions from Uvaraja and Lee (2019), the number of buses are greater with lesser total waiting times. The values of objective functions are significantly different in between the values of total domains which reflects its dependency. On the other hand, the number of domains does not affect the performance of MTS algorithm for some of the route sets of size 6 without including the route sets from Arbex and da Cunha (2015), Chew *et al.* (2013), Nikolić and Teodorović (2013) and Buba and Lee (2018). For these route sets, there are at least one parameter value that considerably different from the usual pattern. For examples, 77 buses (total domain of 3) from Chew *et al.* (2013), 80 buses (total domain of 5) from Nikolić and Teodorović (2013), 77 buses (total domain of 8) from Arbex and da Cunha (2015) and 75 buses (total domain of 3) from Buba and Lee (2018) are highly differ with the respective solutions from the total domain of 10. The statistical test also show that the number of domains affects the quality of solutions. Other route sets produce comparable solutions that are close to each other that prevent their dependency to the number of domains.

Table 2. Comparison of results between numbers of domains for 5 and 6 routes

Source of route sets	**Objective Functions**	**Total domains**							
		3	**4**	**5**	**6**	**7**	**8**	**9**	**10**
		5 routes							
[7]	Total buses	87	71	79	73	74	69	74	64
	Total waiting times	19275	22856	21468	22696	22472	24326	22518	26262
		p-value (0.01550) | Chi-square value (17.3039)							
		6 routes							
[9]	Total buses	74	80	76	79	79	81	83	76
	Total waiting times	20181	18680	19647	18883	18713	18328	17911	19558
		p-value (0.8590) | Chi-square value (3.2690)							
[10]	Total buses	72	74	78	81	80	79	83	82
	Total waiting times	22547	21342	20883	20097	20281	20527	19686	19869
		p-value (0.7010) | Chi-square value (4.6631)							
[3]	Total buses	83	97	85	93	99	87	97	88
	Total waiting times	19869	16641	19361	17515	16201	18582	16709	18433
		p-value (0.0728) | Chi-square value (12.9706)							
[4]	Total buses	77	99	109	94	100	106	101	101
	Total waiting times	22902	17603	15789	18305	17142	16291	17028	17457
		p-value (1.39×10^{-4}) | Chi-square value (29.1021)							
[5]	Total buses	90	97	80	92	82	89	100	100
	Total waiting times	20761	18485	22597	19147	21726	20200	18040	18147
		p-value (0.0129) | Chi-square value (17.7984)							
[6] - passenger	Total buses	103	84	91	98	101	102	94	98
	Total waiting times	18941	22898	21501	19850	19325	19001	20545	19697
		p-value (0.1153) | Chi-square value (11.5248)							
[6] - operator	Total buses	76	60	62	68	69	63	66	61
	Total waiting times	20780	26518	25595	23199	23164	25020	24198	25946
		p-value (0.0931) | Chi-square value (12.2334)							
[7]	Total buses	65	58	71	64	71	77	73	63
	Total waiting times	25545	28947	24182	26461	24098	22224	23330	26728

		p-value (0.0384) | Chi-square value (14.8200)							
[8]	Total buses	75	92	80	91	93	81	92	85
	Total waiting times	26554	21437	24580	21536	21157	24768	21310	23091
		p-value (0.0099) | Chi-square value (18.5115)							

For 7 routes (see, Table 3), 5 out of 8 route sets able to maintain their independency towards the total domain. The route sets of Buba and Lee (2018), Mumford (2013) and Nikolić and Teodorović (2014) show their reliance on total domains based on the lower p-values and higher chi-square values. Likewise, for 8 routes (see, Table 4), the results for Chew *et al.* (2013), Nikolić and Teodorović (2013), Buba and Lee (2018) and Mumford (2013) depend on the number of domain used. Overall, the quality of solutions from every parameter values from 3 to 9 are quite similar to the solution from total domains of 10. Some of the solutions from this study are dominated by the solution published in Uvaraja and Lee (2019), by producing higher total waiting times with the same number of buses. Apart from that, the objective functions values for the route sets of Shih and Mahmassani (1994) from the total domain of 8 is better than the output from the total domain of 10 with lesser waiting times.

Table 3. Comparison of results between numbers of domains for 7 routes

Source of route sets	**Objective Functions**	**Total domains**							
		3	**4**	**5**	**6**	**7**	**8**	**9**	**10**
[9]	Total buses	62	64	65	66	69	71	70	71
	Total waiting times	26872	26327	26036	25885	24563	23945	24051	23901
		p-value (0.7087) | Chi-square value (4.6000)							
[3]	Total buses	94	79	94	84	91	89	97	101
	Total waiting times	18399	21300	17785	20048	18570	18765	17227	16446
		p-value (0.0254) | Chi-square value (15.9142)							
[4]	Total buses	90	101	111	98	106	94	106	108
	Total waiting times	20433	18279	16782	18816	17290	19274	17271	17077
		p-value (0.0530) | Chi-square value (13.9000)							
[5]	Total buses	89	76	85	73	87	81	74	82
	Total waiting times	19620	23682	21382	24563	20177	21672	24085	21458
		p-value (0.0525) | Chi-square value (13.9275)							
[6] - passenger	Total buses	79	93	97	89	97	86	93	96
	Total waiting times	24535	20994	19709	21538	19853	22193	20668	20169
		p-value (0.0913) | Chi-square value (12.2951)							
[6] - operator	Total buses	74	58	71	63	72	69	74	63
	Total waiting times	29028	37295	30000	33827	29497	30725	28741	33700
		p-value (0.0342) | Chi-square value (15.1400)							
[7]	Total buses	77	67	77	70	63	75	65	76
	Total waiting times	21947	25717	22527	24193	26637	22392	26312	21955
		p-value (0.0680) | Chi-square value (13.1742)							
[8]	Total buses	93	77	91	83	95	83	95	90
	Total waiting times	24506	30942	25836	27807	24639	27315	25150	25584
		p-value (0.0486) | Chi-square value (14.1512)							

For the route sizes from 9–12 as shown in Tables 5 and 6, solutions from all the route sets are highly dependent on the total domains. This is because the solutions between the total domains are highly varies to each other. Besides, solutions from the total domain of 3 to 9 are equivalent to the solution from the total domain of 10. Most of them increases the number of buses while decreasing the total waiting times.

Generally, the dependency of the solution towards number of domains increases as the route size becomes larger. It can be seen clearly that most of the solutions are analogous to each other. The objective functions values are always conflicting such that the increase in number of buses attributes to the reduction in total waiting times and vice versa. This experiment proves that the result produced by Uvaraja and Lee (2019) that total domain of 10 yield acceptable solutions for all the route sets.

Table 4. Comparison of results between numbers of domains for 8 routes

Source of route sets	**Objective Functions**	**Total domains**							
		3	**4**	**5**	**6**	**7**	**8**	**9**	**10**
[9]	Total buses	58	70	77	66	71	72	69	72
	Total waiting times	27266	22229	20626	23966	22363	23023	23125	22200
		p-value (0.1001) | Chi-square value (3.2690)							
[10]	Total buses	58	67	55	64	68	62	67	62
	Total waiting times	25600	21668	25856	22382	21149	23027	21379	23227
		p-value (0.2023) | Chi-square value (9.7640)							
[3]	Total buses	95	83	102	88	81	92	89	96
	Total waiting times	18863	22018	17972	20582	21807	19227	20539	18510
		p-value (0.0409) | Chi-square value (14.6382)							
[4]	Total buses	91	73	90	77	91	83	76	86
	Total waiting times	23287	28055	24292	27402	22937	24798	27957	23843
		p-value (0.0203) | Chi-square value (16.5868)							
[5]	Total buses	96	81	104	88	78	97	86	96
	Total waiting times	19532	23302	18165	21347	23977	19413	21787	19512
		p-value (0.0009) | Chi-square value (24.3600)							
[6] - passenger	Total buses	99	84	97	91	103	93	84	95
	Total waiting times	20411	23681	21168	21947	19190	21566	23398	20804
		p-value (0.0711) | Chi-square value (12.9853)							
[6] - operator	Total buses	70	60	69	77	68	60	70	65
	Total waiting times	23535	28320	23620	22043	24412	27641	23910	25479
		p-value (0.0656) | Chi-square value (13.2803)							
[7]	Total buses	72	74	76	69	75	70	62	71
	Total waiting times	23072	23261	21960	25083	22647	24133	27411	23803
		p-value (0.3119) | Chi-square value (8.2395)							
[8]	Total buses	90	79	106	112	100	92	105	93
	Total waiting times	28240	29658	22158	21534	23254	25846	22439	25083
		p-value (3.43×10^{-6}) | Chi-square value (37.7138)							

Table 5. Comparison of results between numbers of domains for 9, 10, and 11 routes

Source of route sets	**Objective Functions**	**Total domains**							
		3	**4**	**5**	**6**	**7**	**8**	**9**	**10**
[7]	**9 routes**								
	Total buses	72	62	71	70	59	70	61	58
	Total waiting times	27738	31618	27166	28671	34001	28914	32775	34381
		p-value (0.0312) | Chi-square value (15.4026)							
	10 routes								
	Total buses	106	80	70	94	80	74	87	81
	Total waiting times	18293	24373	28055	20285	24090	26009	22066	23903
		p-value (7.88×10^{-8}) | Chi-square value (46.2327)							

	11 routes							
Total buses	95	72	96	78	72	60	62	67
Total waiting times	19509	25221	18885	23673	25257	30237	30009	26863
	p-value (2.69×10^{-13}) | Chi-square value (73.6541)							

Table 6. Comparison of results between numbers of domains for 12 routes

Source of route sets	Objective Functions	Total domains							
		3	**4**	**5**	**6**	**7**	**8**	**9**	**10**
[11]	Total buses	91	80	66	83	77	83	78	73
	Total waiting times	19511	20780	25625	20031	21649	20203	21259	22520
		p-value (0.0162) | Chi-square value (17.1845)							
[6] - passenger	Total buses	97	79	97	86	77	95	88	95
	Total waiting times	20910	29941	25181	27303	29892	24734	27178	24675
		p-value (6.15×10^{-3}) | Chi-square value (25.5122)							
[6] - operator	Total buses	89	70	58	80	67	62	61	52
	Total waiting times	20949	26274	29243	22746	27905	28916	29665	33828
		p-value (1.17×10^{-9}) | Chi-square value (55.5247)							
[7]	Total buses	48	95	71	62	84	74	68	62
	Total waiting times	43269	20854	27891	32334	23020	26169	29226	31162
		p-value (4.21×10^{-17}) | Chi-square value (92.2755)							
[8]	Total buses	138	108	88	71	73	87	81	72
	Total waiting times	21453	28214	35844	42043	32989	37630	38458	42468
		p-value (7.57×10^{-37}) | Chi-square value (186.6765)							

B. *Results of Extended Model*

In Uvaraja and Lee (2019), the route set of size 4 from Buba and Lee (2018) are experimented. In this study, the investigation is continued for 6–12 routes using the extended model. The algorithm is run for 10 times and the average solution for every domain are tabulated. As there is no previous solution in the literature, comparison is not possible for all the routes.

Based on the Table 7, the number of buses increase and the total waiting times decrease as the domains become higher. This is due to the range of frequency allocated for every domain also increases from 27 to 369 for the time period of 18 hours. Moreover, the first two domains show overcrowding which caused by inadequate number of buses to fetch all the passengers at a timeslot of the routes. The number of buses and total waiting times are greater as compared to the solutions in Tables 1– 6 although same data sets and setting for the MTS algorithm are employed. For instance, the number of buses and total waiting times for the standard model without timeslot are 85 buses and 23091 munites respectively for 6 routes from Buba and Lee (2018) as shown in Table 2. According to Table 7, the objective values from domain 6 for 6 routes are 121 buses and 44379 minutes of waiting times. The increase in total buses and waiting times are caused by the addition of layover time and dwell time into the bus route trip and the difference in frequency for various timeslots that changes the time interval between consecutive buses. Overall, the number of buses increase when the size of route sets also escalate. The total waiting times and overcrowding are not directly influenced by the increase in number of routes as they are calculated based on the routes frequency which is in the same range for each domain regardless of the number of routes.

Among the 10 solutions, the values from domains 3 to 10 are more acceptable as there is no overcrowding. Since this is a multiobjective problem, the best solution within the domains are determined based on the objectives to be achieved. If the preference of operator is considered, then the solution with less number of buses is preferable. Otherwise, if the passenger's point of view is measured, the solution with lower total waiting times is chosen.

V. CONCLUSIONS

In this paper, we conducted an analysis with varies parameter values to evaluate the efficiency of MTS algorithm for urban transit frequency optimization problem. A statistical test is employed to check the dependency of the number of domains on the solution quality. Besides, in order to extend the computational study of frequency optimization problem, MTS algorithm is tested with larger route sets from Mumford (2013). The results of this study leads to the following remarks: (1) the values of total buses and waiting times are subject to the number of domains and its dependency increases as the route size becomes higher; (2) larger number of domains must be assigned to obtain better solutions that reduce both of the objective values and therefore the number of domains of 10 is most preferred as it is able to produce better solutions for most of the route sets than other number of domains; (3) feasible solutions are obtained for higher number of route sets with sizes 6, 7, 8, and 12 that indicates the effectiveness of MTS algorithm. Based on the experiments, the performance of MTS algorithm for frequency optimization problem is highly influenced by the number of domains although it gives comparable solutions as compared to other methods.

Table 7. Results obtained for extended model

Domain	Number of buses				Total waiting times (min)				Overcrowding			
	[a]	[b]	[c]	[d]	[a]	[b]	[c]	[d]	[a]	[b]	[c]	[d]
1	24	30	30	41	224684	245168	200065	194664	6392	8738	2705	2848
2	44	57	57	71	118787	124059	105327	105710	392	935	126	442
3	62	78	83	103	84876	89896	72591	72522	0	0	0	0
4	83	102	110	137	65045	69740	56400	55727	0	0	0	0
5	102	126	132	167	52542	55994	47086	46191	0	0	0	0
6	121	150	160	200	44379	46954	38414	38094	0	0	0	0
7	139	176	183	233	38777	40432	33845	32796	0	0	0	0
8	159	199	208	263	34177	35873	30037	29186	0	0	0	0
9	181	226	235	296	29825	31399	26416	25895	0	0	0	0
10	197	296	258	325	27540	28863	24042	23682	0	0	0	0

Note: [a]: 6 routes; [b]: 7 routes; [c]: 8 routes; [d]: 12 routes

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