

g-Jitter Induced Free Convection Flow Near a Three-Dimensional Stagnation Point Region in Nanofluid with Heat Generation

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The free convection boundary layer flow near a three-dimensional stagnation point region is studied numerically with the implementation of nanofluid. The effect of gravity modulation and heat generation or absorption are considered while constant wall temperature boundary condition in the nanofluid are applied in this study. The mathematical formulation derived based on Tiwari and Das nanofluid model undertake boundary layer and Boussinesq approximations is then transform into non-dimensional governing equation using similarity transformation technique. Keller-box method is used to solve the non-dimensional governing equations. The velocity and temperature profile as well as skin friction and Nusset number near the stagnation point are presented graphically and discussed briefly under the influence of gravity modulation, curvature ratio, heat generation parameter and nanoparticles volume fraction. The result obtained show that the effect of heat generation gives rise to the skin frictions and Nusset number while additional of copper nanoparticles increase the thermal conductivity of the fluid.

Keywords: g-jitter; nanofluid; stagnation point; heat generation

I. INTRODUCTION

The outer space exploration starts when Russia launch an artificial satellite, Sputnik in 1957 then followed by human exploration and the first space (Catchpole, 2008). On the same time, an idea of producing a "perfect crystal" by implement zero gravity environment rise by most researchers which facing the variation of the impurity doping problem in semiconductor device manufacturing. However, from the conducted experiment, it is found that there are exist a small gravitational fluctuation were then called as g-jitter.

g-Jitter or periodical gravity modulation can be defined as an inertia effects due to quasi-steady, oscillatory or transient acceleration arising under microgravity environment (Nelson, 1994). Space research in fluid dynamics is conducted to enhance fundamental of understanding a fluids behavior under microgravity environment (Amin, 1988). Therefore, various practical and experimental study have been conducted on g-jitter effect

which exist in microgravity environment.

Most of experiments were conducted to investigate properties of buoyancy due to sedimentation which then defect the final product. Therefore, experiment is conducted under low gravity conditions where the effect of gravity is greatly reduced (DE Lombard *et al.*, 2005). Practical studies on g-jitter induced free convection flow in a sphere has been conducted focusing on growing crystal (Langbein, 1983).

As for theoretical studies on g-jitter, mathematical model was developed which applicable for most scenarios on material science testing within a space station. Researcher such as Amin (1998), Li (1996) Rees & Pop (2003) have studied theoretical fluid behavior induced by g-jitter on the boundary layer fluid flow. This study contributes in preparing actual experimental study for Newtonian and non-Newtonian fluid effected by g-jitter. A study was conducted on viscous fluid induced by g-jitter to investigate the effect of gravity modulation with magneto-convection in electrically conducting fluids with internal angular

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momentum (Siddheshwar & Pranesh, 2000). Furthermore, a research studying on effect of periodical gravity modulation induced by mixed convection to flow and heat transfer characteristics with stretching vertical surface in a viscous and incompressible fluid were also been conducted (Sharidan *et al.*, 2006). In addition, are search on viscous fluid were also been conducted with the presence of gravity modulation together with Darcy convection at the vertical through flow using a weak nonlinear stability analysis (Kiran, 2016).

As for the stagnation point problem, Rees and Pop investigate on the effect of fluctuating gravitational field near a forward stagnation point flow for two-dimensional symmetric body (Rees & Pop, 2001). The problem is then extended to examine the response of a non-linear system, such that consist one flow near the front stagnation point of a cylindrical surface embedded in a porous medium (Rees & Pop, 2001). On the other hand, for three dimensional cases of stagnation point flow, Sharidan *et al.* (2005) provide the numerical solution for a free convection boundary layer flow of a micropolar fluid near stagnation point region induced by g-jitter.

The effect of g-jitter is also been studied by most of the researcher with usage of nanofluid and its effect on boundary layer. The boundary layer problem of two-dimensional mixed convection nanofluid flow past a vertical permeable stretching sheet induced by g-jitter is studied (Rawi *et al.*, 2016). An extension study is conducted with modification on the geometry at boundary layer while the same effect was considered in studying the flow numerically. (Rawi *et al.*, 2017). For the non-Newtonian fluid, a study on unsteady mixed convection flow of Jeffrey fluid past an inclined stretching sheet with the presence of nanoparticles induced with g-jitter are also conducted numerically (Rawi *et al.*, 2018).

Motivated from the above literature, the objective of the present paper is to investigate the effect of g-jitter on a three-dimensional stagnation point region in the nanofluid with the present of heat generation. Nanofluid model proposed by Tiwari and Das applied in formulating the mathematical modelling presenting the problem (Tiwari & Das, 2007). Other parameter such as stagnation point parameter, g-jitter parameter and heat generation

parameter are also analyzed in this study and the analysis are presented graphically in term of profiles and physical properties of principal interest.

II. MATHEMATICAL FORMULATION

The problem was considered near a three-dimensional boundary layer flow of stagnation point region in the viscous and incompressible nanofluid induced by g-jitter and heat generation. Tiwari and Das nanofluid model were used to present the fundamental of the problem with copper nanoparticles and water as based fluid. The body of unsteady free convection flow was heated with uniform temperature, T_∞ . It is assumed that the uniform temperature of the body is suddenly changed from temperature at the wall, T_w to T_∞ .

A locally Cartesian with orthogonal system (x, y, z) is chosen where the origin is at the nodal point N, where the x – and y – coordinates are measured along the body surface, while the z – coordinate is measured normal to the body surface.

By considering the problem in the microgravity environment, the gravitational field depends on time takes the form,

$$g^*(t^*) = g_0[1 + \varepsilon \cos(\pi \omega t^*)]$$

where g_0 is the mean gravitational acceleration, ε is the scaling parameter amplitude of the gravity modulation, t^* is the time and ω is the frequency of oscillation of the g-jitter driven flow. Under the Boundary Layer and Boussinesq approximations, the boundary layer equations represent as,

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \quad (1)$$

$$\rho_{nf} \left(\frac{\partial u}{\partial t^*} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} \right) = \mu_{nf} \frac{\partial^2 u}{\partial z^2} + g^*(t^*)(\rho\beta)_{nf} ax(T - T_\infty) \quad (2)$$

$$\rho_{nf} \left(\frac{\partial v}{\partial t^*} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} \right) = \mu_{nf} \frac{\partial^2 v}{\partial z^2} + g^*(t^*)(\rho\beta)_{nf} by(T - T_\infty) \quad (3)$$

$$\frac{\partial T}{\partial t^*} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} + w \frac{\partial T}{\partial z} = \alpha_{nf} \frac{\partial^2 T}{\partial z^2} + \frac{Q_0}{(\rho C_p)_{nf}} (T - T_\infty) \quad (4)$$

subject to initial and boundary conditions

$$\begin{aligned} t^* < 0 : u = v = w = 0, T = T_\infty \\ \text{for any } x, y \text{ and } z \\ t^* \geq 0 : u = v = w = 0, T = T_w \\ \text{on } z = 0, x \geq 0, y \geq 0 \\ u = v = w = 0, T = T_\infty, \\ \text{on } x = 0, y \geq 0, z > 0 \\ u = v = w = 0, T = T_\infty, \\ \text{on } y = 0, x \geq 0, z > 0 \\ u = v = w = 0, T = T_\infty, \\ \text{as } z \rightarrow \infty, x \geq 0, y \geq 0 \end{aligned} \quad (5)$$

where u, v, w are the velocity components along x, y, z axes, T is temperature of the fluid, ρ_{nf} and μ_{nf} is the density and dynamic viscosity of nanofluid, β_{nf} is the thermal expansion of nanofluid and α_{nf} is the thermal diffusivity of nanofluid while C_p is the specific heat at the constant pressure. The parameters a and b are the principal curvatures at nodal point of the body measured in the planes $y = 0$ and $x = 0$ by holding the properties $|a| \geq |b|$ with $a > 0$. Correspond to the characteristic carried by the principle of curvature ratio at the nodal point, there are parameter c such that $c = b/a$ and c

is the curvature ratio. Practically, most of the shape lies between cylinder and sphere geometry where the curvature ratio values c , is between 0 and 1. The term $Q_0(T - T_\infty)$ is assumed to be the amount of heat generated or absorbed per unit volume.

From the Maxwell equation, the nanofluid constants are then defined as

$$\begin{aligned} \mu_{nf} &= \frac{\mu_f}{(1-\phi)^{2.5}} \\ \alpha_{nf} &= \frac{k_{nf}}{(\rho C_p)_{nf}} \\ \rho_{nf} &= (1-\phi)\rho_f + \phi\rho_s \\ (\rho\beta)_{nf} &= (1-\phi)(\rho\beta)_f + \phi(\rho\beta)_s \\ (\rho C_p)_{nf} &= (1-\phi)(\rho C_p)_f + \phi(\rho C_p)_s \\ \frac{k_{nf}}{k_f} &= \frac{(k_s + 2k_f) - 2\phi(k_f - k_s)}{(k_s + 2k_f) + \phi(k_f - k_s)} \end{aligned} \quad (6)$$

where ϕ is a nanoparticle volume fraction of the nanofluid, k is the thermal conductivity and the subscript f and s are for fluid and solid.

The complexity of the problem is reduced using similarity transformation technique. Equations (1) – (4) along with the initial and boundary conditions (5) will admit (Sharidan *et al.*, 2007)

$$\begin{aligned} \tau &= \Omega t \\ \eta &= Gr^{1/4} az \\ t &= \nu a^2 Gr^{1/2} t^* \\ u &= \nu a^2 x Gr^{1/2} f'(\tau, \eta) \\ v &= \nu a^2 y Gr^{1/2} h'(\tau, \eta) \\ w &= -\nu a Gr^{1/4} (f + h) \\ \theta(\tau, \eta) &= \frac{(T - T_\infty)}{(T_w - T_\infty)} \\ \Omega &= \frac{\omega}{\nu a^2 Gr^{1/2}} \\ g(t^*) &= \frac{g^*(t^*)}{g_0} \end{aligned} \quad (7)$$

where $Gr = g_0 \beta (T_w - T_\infty) / (a^3 \nu^2)$ is the Grash of number and the primes denote partial differential with respect to η . Here, ν is the kinematic viscosity while θ, Ω are the dimensionless variable for temperature and frequency of oscillation. The substitution of (6) and (7) into (1)-(4) produced following governing equation,

$$C_1 f''' + C_2 [(f+h)f'' - f'^2] + C_3 [1 + \varepsilon \cos(\pi\tau)] \theta = C_2 \Omega \frac{\partial f'}{\partial \tau} \quad (8)$$

$$C_1 h''' + C_2 [(f+h)h'' - h'^2] + C_3 c [1 + \varepsilon \cos(\pi\tau)] \theta = C_2 \Omega \frac{\partial h'}{\partial \tau} \quad (9)$$

$$\frac{C_4}{C_5 \text{Pr}} \theta'' + (f+h)\theta' + \frac{Q}{C_5} \theta = \Omega \frac{\partial \theta}{\partial \tau} \quad (10)$$

where

$$C_1 = \frac{1}{(1-\phi)^{2.5}},$$

$$C_2 = \left(1 - \phi + \frac{\phi \rho_s}{\rho_f} \right),$$

$$C_3 = \left(1 - \phi + \frac{\phi \rho \beta_s}{\rho \beta_f} \right),$$

$$C_4 = \frac{(k_s + 2k_f) - 2\phi(k_f - k_s)}{(k_s + 2k_f) + \phi(k_f - k_s)},$$

$$C_5 = 1 - \phi + \frac{\phi(\rho C_p)_s}{(\rho C_p)_f}$$

$$Q = \frac{Q_0}{C_p a^2 \mu_f Gr^{1/2}}$$

Pr is Prandtl number and Q is the heat generation parameter with the boundary conditions (5) become

$$\begin{aligned} f(\tau, 0) &= f'(\tau, 0) = 0, \\ h(\tau, 0) &= h'(\tau, 0) = 0, \\ \theta(\tau, 0) &= 1, \\ f' &\rightarrow 0 \quad h' \rightarrow 0 \quad \theta \rightarrow 0 \quad \text{as } \eta \rightarrow \infty \end{aligned} \quad (11)$$

III. RESULTS AND DISCUSSIONS

The non-dimensional governing equation together with its boundary and initial conditions are solve numerically using Keller box method and analyzed based on the velocity and temperature profiles together with the physical quantities which is skin frictions as well as the Nusselt number. The comparison results are shown in Table 1 with the chosen parameters $\phi = 0, Q = 0$ and $\text{Pr} = 0.72$ and it can be seen from this table that the present research has a very good agreement between the existing study which gives confidence in the numerical method employed.

Table 1. Comparison result of skin frictions and rate of heat transfer for final steady state.

	Admon (2011)		Present	
c	0	1	0	1
f''	0.8565	0.7657	0.8555	0.7645
h''	0	0.7602	0	0.7645
$-\theta'$	0.3735	0.4610	0.3741	0.4622

The results presented are marked as the velocity profiles in x -direction and y -direction as (a) and (b) while (c) represent the result of temperature profile. The physical quantities of principal interest such as skin friction on both directions are in Figures (d) and (e) while the analysis of Nusselt number is presented in Figure (f). The variation of velocity and temperature profiles together with skin frictions and Nusselt number are shown in Figure 1 respectively

for $c = 0, \phi = 0.05, \Omega = 0.2, \text{Pr} = 6.2, Q = 0.1$ with different values of ε . The result for the profiles as shown in (a) and (c) is seen to take place smoothly from the steady flow with some singularity happen on the flow for velocity profile. On the other hand, the skin frictions in (e) are found interesting to be discussed when it did not change in term of its magnitude as the ε and τ increase. The stagnation point parameter which is curvature ratio at $c = 0$ is found to be the factor effect the flow pattern. Curvature ratio $c = 0$ is correspond to the sphere surface that will lead to a plane stagnation case flow.

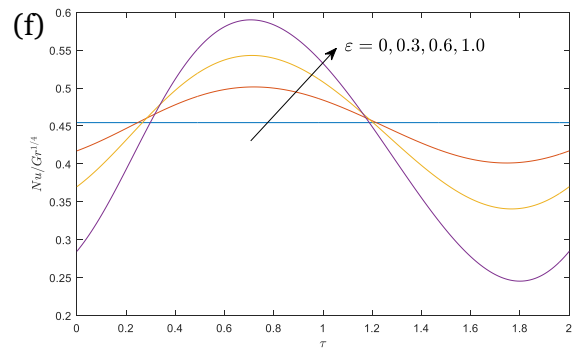
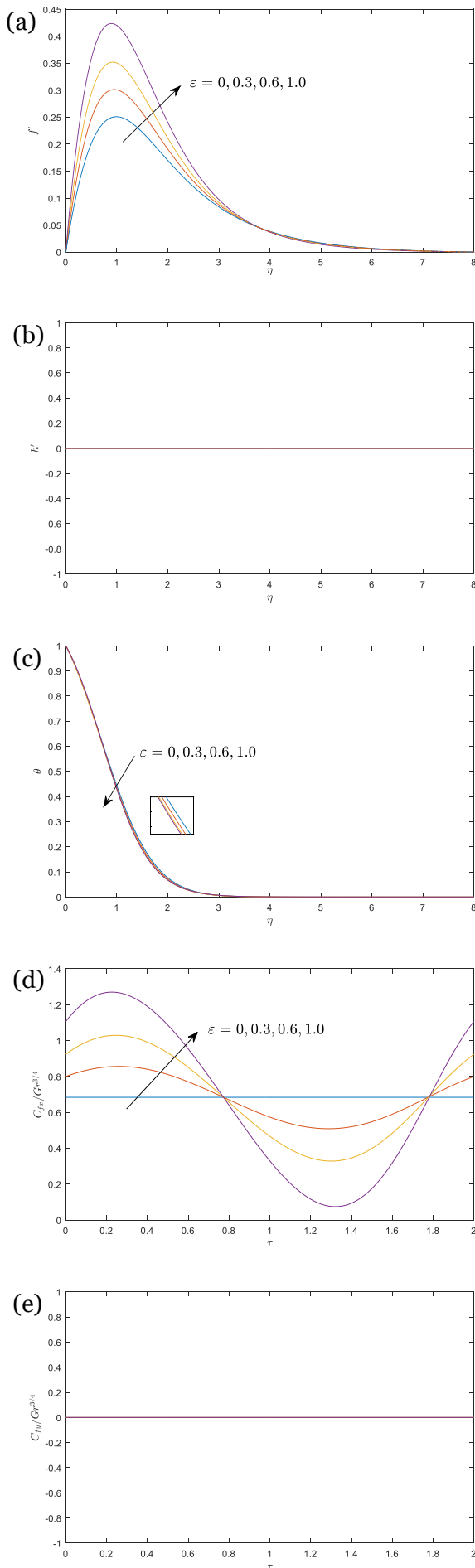
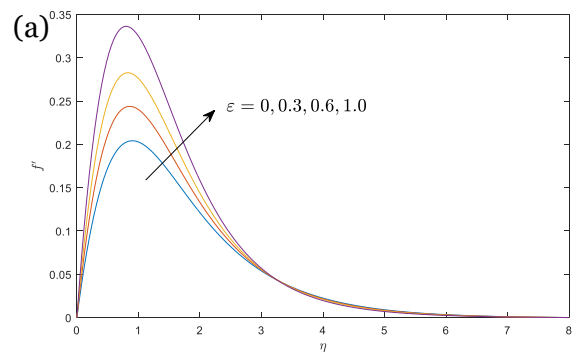


Figure 1. Velocity profile, temperature profile, skin frictions and Nusset number for $c = 0$, $\phi = 0.05$, $\Omega = 0.2$, $Q = 0.1$ with different values of ε

The velocity and temperature profiles together with the skin frictions and Nusset number with different values of c presented in Figure 2 with $c = 0.5$, $\phi = 0.05$, $\Omega = 0.2$ and $Q = 0.1$. From the figure, it is clearly seen that as ε increases, temperature profile in (c) is decrease and velocity profile for both direction in (a) and (b) proportional increase or decreases follow the direction for typical velocity profile for natural convection boundary layer flow. As for the physical quantities for the principal interest on (d), (e) and (f), both skin frictions and Nusset number are increase and decrease as the ε increases. From the steady state flow, $\varepsilon = 0$ for both cases, it is found that higher value of curvature ratio will provide a higher skin friction value on y -direction.



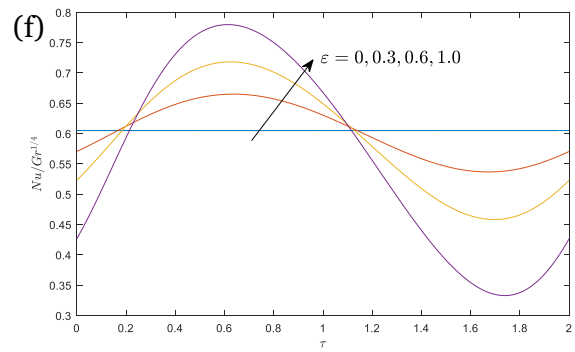
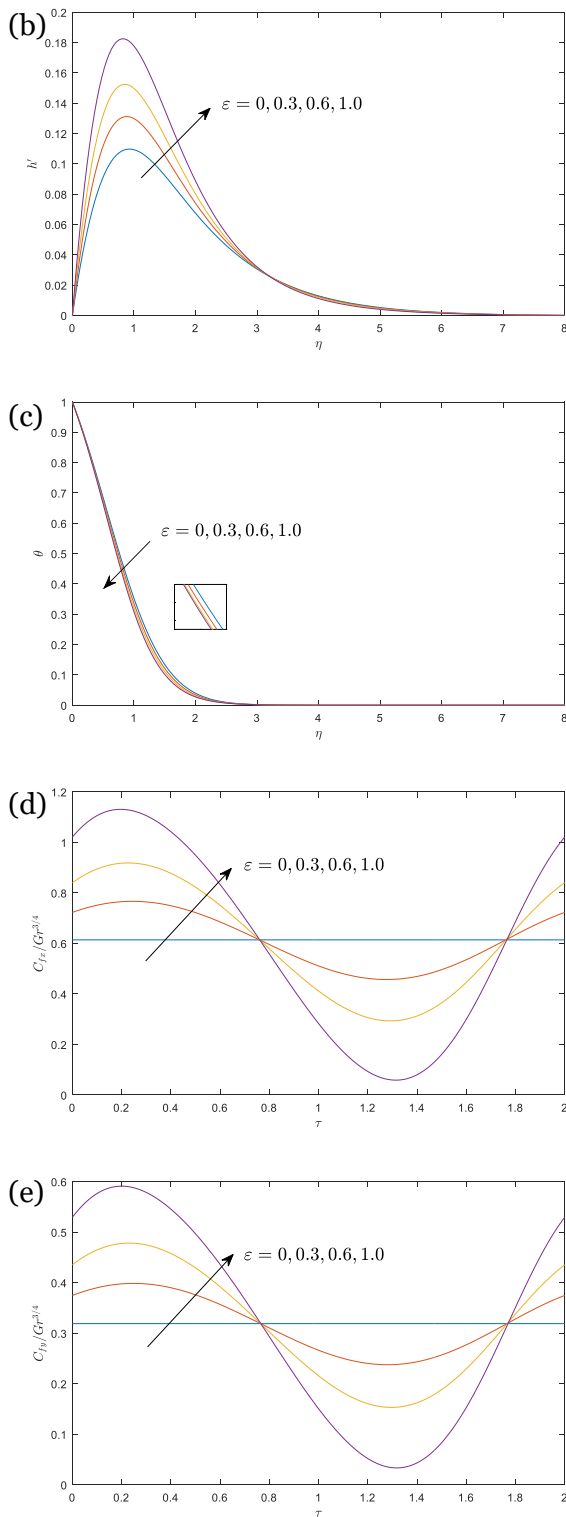


Figure 2. Velocity profile, temperature profile, skin frictions and Nusset number for

$c = 0.5$, $\phi = 0.05$, $\Omega = 0.2$, $Q = 0.1$ with different values of ε

Figure 3 illustrates the profiles and the physical quantities of principal interest that describe the transportation phenomena

happen in the flow with $c = 1.0$, $\phi = 0.05$, $\Omega = 0.2$, $Pr = 6.2$ and $Q = 0.1$.

Both profiles; flow and the heat coefficients in (a), (b) and (c) behave the same as before in Figure 1 and Figure 2 which follow the typical natural convection profiles. From the observation on the skin frictions for both directions in (a) and (b), each magnitude of the skin frictions are found to be the same in terms of magnitude. The cylindrical surface is represented with the curvature ratio lead to the special case flow known as axisymmetric stagnation case flow.

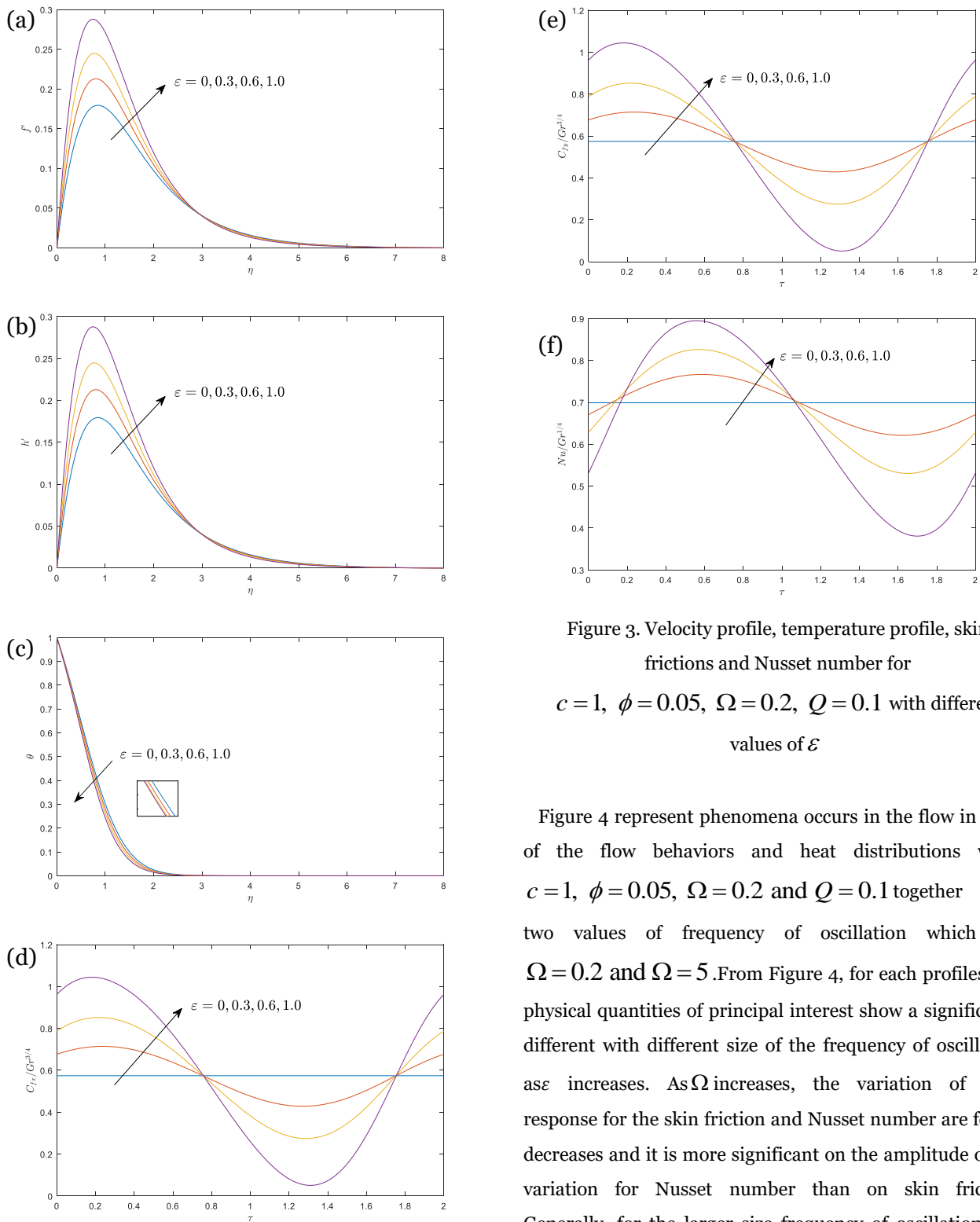


Figure 3. Velocity profile, temperature profile, skin frictions and Nusset number for $c = 1$, $\phi = 0.05$, $\Omega = 0.2$, $Q = 0.1$ with different values of ε

Figure 4 represent phenomena occurs in the flow in term of the flow behaviors and heat distributions when $c = 1$, $\phi = 0.05$, $\Omega = 0.2$ and $Q = 0.1$ together with two values of frequency of oscillation which are $\Omega = 0.2$ and $\Omega = 5$. From Figure 4, for each profiles and physical quantities of principal interest show a significance different with different size of the frequency of oscillation as ε increases. As Ω increases, the variation of peak response for the skin friction and Nusset number are found decreases and it is more significant on the amplitude of the variation for Nusset number than on skin friction. Generally, for the larger size frequency of oscillation, the convergence rate occurs faster than the smaller size of frequency of oscillation in the microgravity environment.

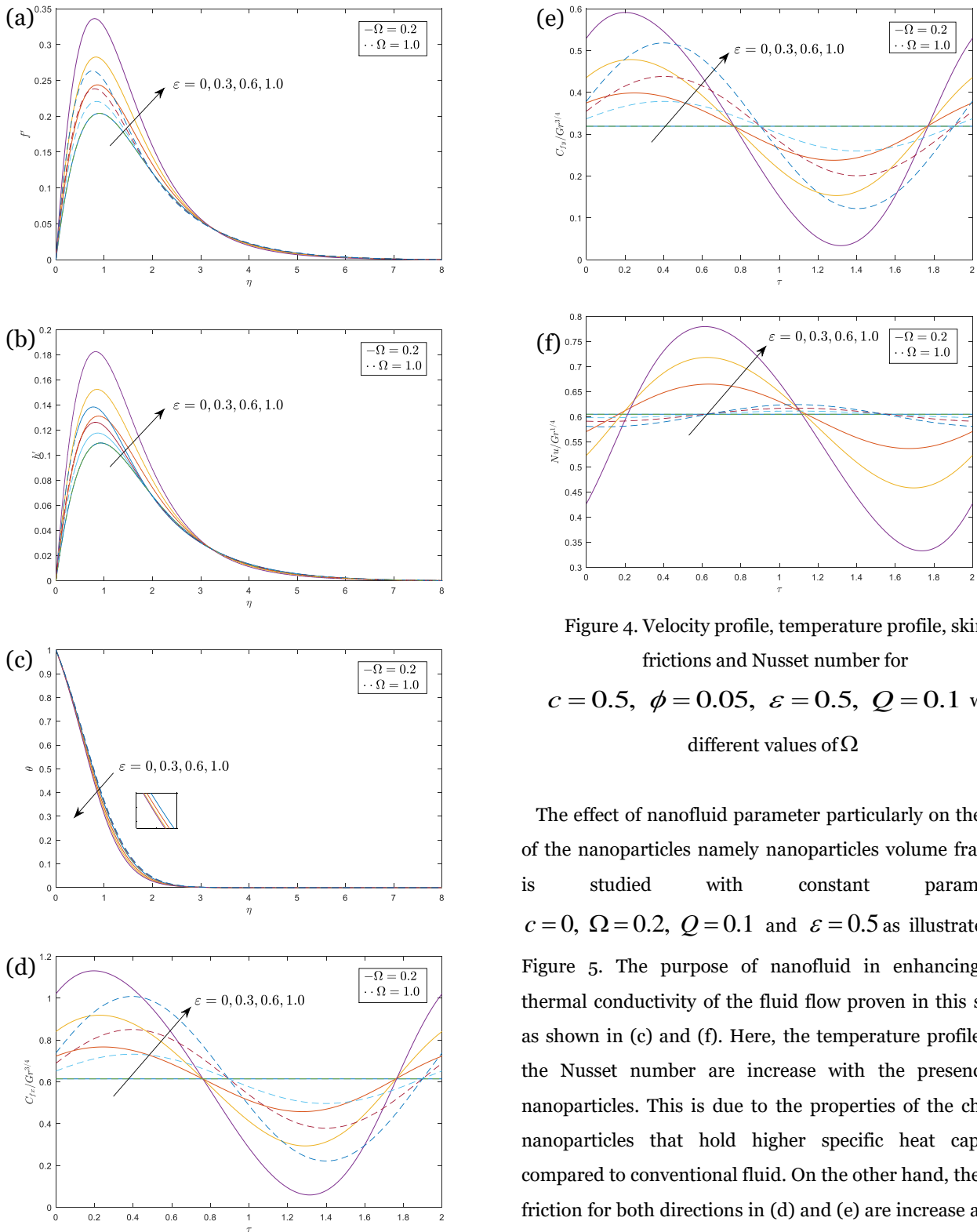


Figure 4. Velocity profile, temperature profile, skin

frictions and Nusset number for

$c = 0.5$, $\phi = 0.05$, $\varepsilon = 0.5$, $Q = 0.1$ with
different values of Ω

The effect of nanofluid parameter particularly on the size of the nanoparticles namely nanoparticles volume fraction is studied with constant parameters $c = 0$, $\Omega = 0.2$, $Q = 0.1$ and $\varepsilon = 0.5$ as illustrated in Figure 5. The purpose of nanofluid in enhancing the thermal conductivity of the fluid flow proven in this study as shown in (c) and (f). Here, the temperature profile and the Nusset number are increase with the presence of nanoparticles. This is due to the properties of the chosen nanoparticles that hold higher specific heat capacity compared to conventional fluid. On the other hand, the skin friction for both directions in (d) and (e) are increase as the nanoparticles volume friction increases. This is due to the additional of frictions on the surface of the plane that give resistance to the flow when the nanoparticles is added which also supported by the decreasing of the velocity profile as shown in (a) and (b).

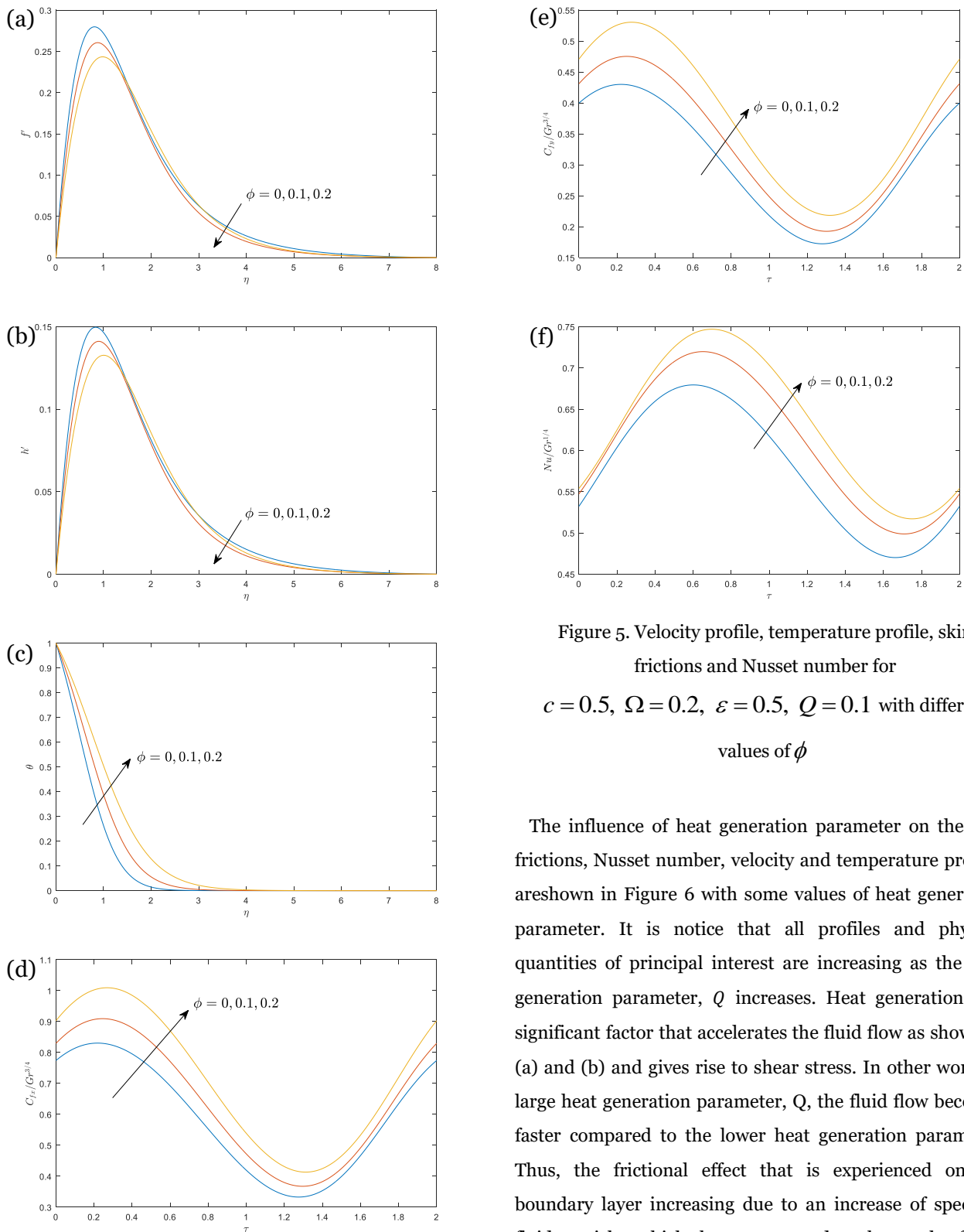


Figure 5. Velocity profile, temperature profile, skin frictions and Nusset number for $c = 0.5$, $\Omega = 0.2$, $\varepsilon = 0.5$, $Q = 0.1$ with different values of ϕ

The influence of heat generation parameter on the skin frictions, Nusset number, velocity and temperature profiles are shown in Figure 6 with some values of heat generation parameter. It is noticed that all profiles and physical quantities of principal interest are increasing as the heat generation parameter, Q increases. Heat generation is a significant factor that accelerates the fluid flow as shown in (a) and (b) and gives rise to shear stress. In other words, at large heat generation parameter, Q , the fluid flow becomes faster compared to the lower heat generation parameter. Thus, the frictional effect that is experienced on the boundary layer is increasing due to an increase of speed of fluid particles which then corresponds to the result of skin friction in (d) and (e).

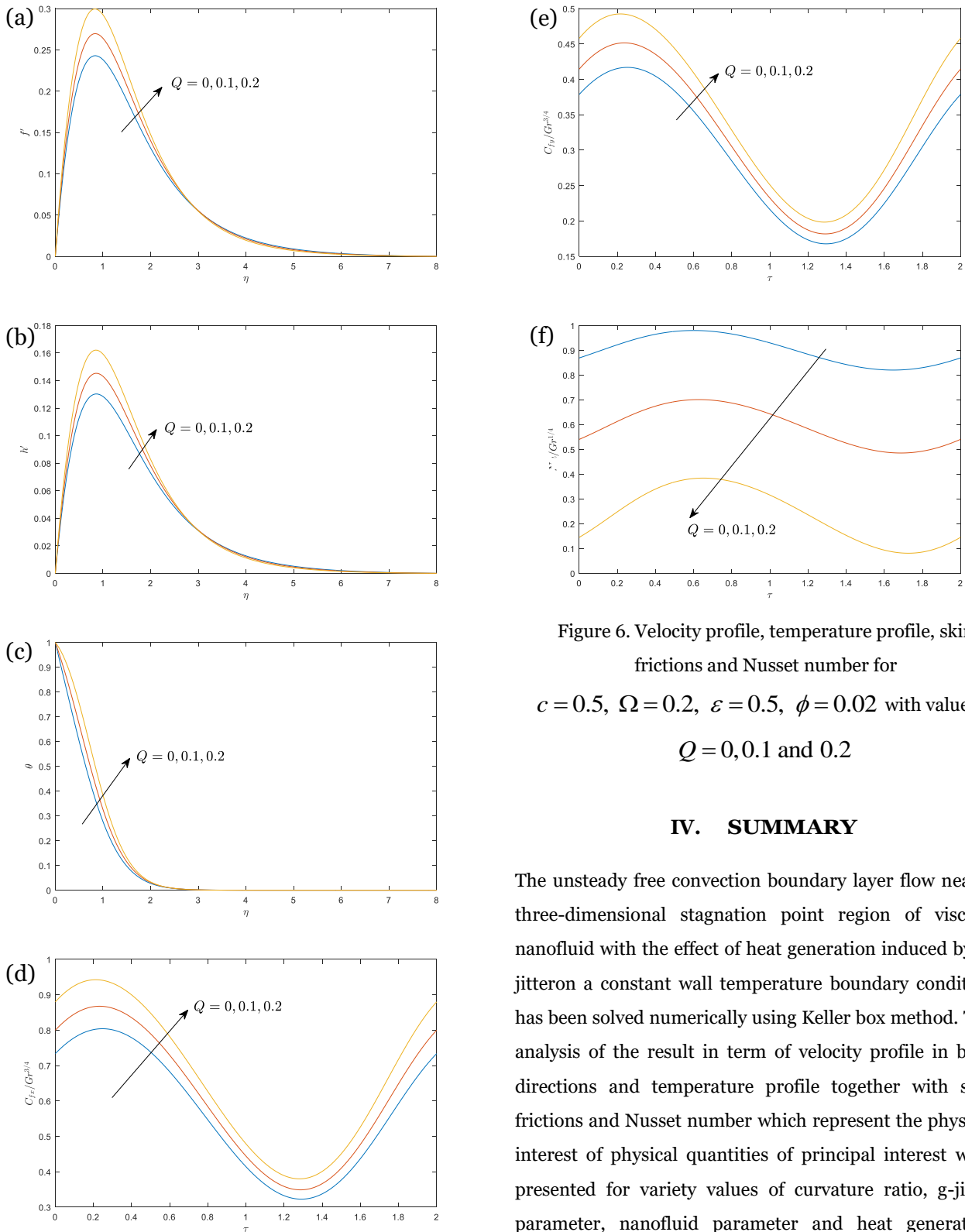


Figure 6. Velocity profile, temperature profile, skin frictions and Nusset number for $c = 0.5$, $\Omega = 0.2$, $\varepsilon = 0.5$, $\phi = 0.02$ with values of $Q = 0, 0.1$ and 0.2

IV. SUMMARY

The unsteady free convection boundary layer flow near a three-dimensional stagnation point region of viscous nanofluid with the effect of heat generation induced by g-jitteron a constant wall temperature boundary condition has been solved numerically using Keller box method. The analysis of the result in term of velocity profile in both directions and temperature profile together with skin frictions and Nusset number which represent the physical interest of physical quantities of principal interest were presented for variety values of curvature ratio, g-jitter parameter, nanofluid parameter and heat generation parameter. All results were presented graphically and discussed. As an extension from the previous research, heat generation effect in this problem seems to give a very significant result especially on the temperature transfer properties. Previous published result by Admon *et al.* (2011) also show a very good agreement with the present result.

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