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Gravity Acceleration Induced Mixed Convection Flow Pasta Vertical Channel with Newtonian Heating

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In this paper, the fully developed mixed convection flow induced by g-jitter in a vertical channel is studied. The mixed convection flow of a viscous fluid past through a vertical channel with Newtonian heating is considered. Using the flow assumptions, the governing equations are reduced into ordinary differential equations and solved analytically. This flow considered Newtonian heating of thermal boundary conditions. Dimensionless variables are used to reduce the governing equations along with imposed initial and boundary conditions into dimensionless forms. The resulting governing equations are solved using Fourier method. The graphical results are displayed to illustrate the influence of numerous embedded physical parameters that are mixed convection, frequency, thermal conjugate and temperature ratio. The behaviour of fluid flow and temperature distribution are investigated. The results show that as mixed convection, thermal conjugate, and temperature ratio increases, the velocity profiles increased.

Keywords: Mixed convection; g-jitter; Fourier method; Newtonian heating; vertical channel

I. INTRODUCTION

It is widely known that the development of human civilization extremely depends on the energy sources. Therefore, researchers and specialists are trying to develop the latest energy resources and energy inventions to employ the solar energy for the different heating procedure in technical and manufacturing industries that is appropriate, convenient and pleasant source (Hayat *et al.*, 2017). Fluid flow model is most attempting research that is required in our daily life. It contains the basic heat and mass transfer for some appropriate boundary layer condition with an addition of various kind of effect such as magnetohydrodynamic, porous, thermal radiation, chemical reaction and others.

In manufacturing industries, proper knowledge of heat transfer is crucial to attain the finest standard invention. There are commonly four types of heating process which one of them is Newtonian heating. It is occurred when the heat from the bounding surface shifting the heat to the moving fluid. The phenomenon of Newtonian heating has future significance in the petroleum industry, conjugate heat transport, heat exchangers, thermal energy storage, nuclear turbines and many more. The impact of Newtonian heating has been reported in numerical study of heat transfer on a stretching sheet by a finite-difference method called Keller-box method (Salleh *et al.*, 2010; Sarif *et al.*, 2013). It is shown that the Prandtl number affects the wall temperature, temperature profiles and the heat transfer coefficient.

Thus, in perspective of such applications some researchers have utilized the concept of Newtonian heating boundary condition under different physical aspects. Numerical investigation on fluid flow model considering the effect of thermal conjugate had been done by using different methods such as Keller-Box

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(Ullah *et al.*, 2017) and induced Runge-KuttaFehlberg integration scheme (Mehmood *et al.*, 2018) to solve the magnetohydrodynamic (MHD) Oldroyd-B fluid model.

Comprehensive reviews also have been conducted on analytical solution for double diffusive natural convection between parallel plates for the fully developed flow (Nelson & Wood, 1989). The present results were useful in understanding and correlating results for developing flows. This study allowed some guidelines to be developed for designing and operating thermal fluids systems for space experiments.

In spite of all studies, an analysis on the fully developed mixed convection flow between two parallel horizontal flat plates filled by a nanofluid containing both nanoparticles and gyrostatic microorganisms had been conducted (Xu & Pop, 2014). This problem used similarity reduction of the nonlinear equations and then being improved by homotopy analysis technique (HAM) with complicated boundary conditions for a better precision. The effects of the governing parameters on the temperature and the nanoparticle volume fraction are graphically presented and discussed.

Next, further research attempted using the same geometry and Newtonian heating effect but with different fluid that is Jeffery (Hayat *et al.*, 2016) and viscoelastic fluid (Nadeem *et al.*, 2017) together with magneto-hydrodynamic effect also have been done. The method used to solve the equation is homotopic method. Behaviour of velocity and temperature profiles was investigated using different range of parameter such as Prandtl number, skin friction and local Nusselt number.

Meanwhile, an investigation was conducted to study the effect of Newtonian heating on double diffusion in hydromagnetic flow for Casson fluid through a flat plate with effect of radiation and chemical reaction (Das *et al.*, 2015). This research using Laplace Transform technique to solve the governing equations. It was noticed the velocity, temperature and concentration of the fluid decreased when Casson parameter, chemical reaction parameter and Schmidt number increased. Next, it continued with the same problem but with different type of fluid that was Carreau liquid (Hayat *et al.*, 2017).

Recently, the chemical reaction effects on MHD stagnation point flow of Walters-B nanofluid with Newtonian heat and mass conditions was investigated. Thermophoresis models and Brownian motion were proposed in the temperature and concentration expressions. Heat and mass transfer with thermal radiation and chemical reaction over a stretching sheet being solved with homotopic method (Qayyum *et al.*, 2017).

The above literature survey indicates that less consideration has been given to mixed convection flows, heat and mass transfer towards vertical parallel plates with g-jitter effect. Further, the heat transfer through Newtonian heating in mixed convection flow is also less attended. Thus, the main objective of this research is to investigate the effect Newtonian heating towards mixed convection flow in a vertical channel. In addition, g-jitter effect is considered. Therefore, on the present study, this project mainly focused on the analytical study of the gravity acceleration induced mixed convection flow past a vertical channel with Newtonian heating.

Finally, the present research is produced to prevent such Newtonian heating circumstances occurred in some fundamental engineering machines as well as conjugate heat transfer nearby fins and heat exchanger. This is the motivation of this paper.

II. MATERIALS AND METHODS

The unsteady mixed convection flow pasta vertical channel with distance h is considered. The x-axis along the vertical channel in upward direction is taken with the gravity acceleration $g^*(t) = g_0 \sin(\omega t)$ where g_0 is the magnitude and ω is the g-jitter frequency that opposingthe fluid flow and y-axis is perpendicular to the channel. The channel is assumed to have steady temperatureT_0andconcentration C_0 with the linear velocity u_0 and pressure p_0 at time $t = 0$. The fluid velocity u, temperature T and concentration C are independent function of y and of time t.

Considering the fluid to be incompressible and viscous, using the regular boundary layer conditions and

Boussinesq approximation, the ordinary governing equations containing continuity, momentum and energy equations in the presence of g-jitter can be written as,

$$\nabla \cdot \boldsymbol{u} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \quad (1)$$

$$\rho \frac{\partial u}{\partial t} = -\frac{\partial p}{\partial x} + \mu \frac{\partial^2 u}{\partial y^2} + g^*(t)(\rho\beta_T)(T - T_0), \quad (2)$$

$$\frac{\partial T}{\partial t} = \frac{k}{(\rho c_p)} \frac{\partial^2 T}{\partial y^2} \quad (3)$$

employed by the initial and boundary conditions

$$t < 0: u = 0, T = T_0 \text{ for } 0 < y < h$$
$$t \geq 0: u = 0, \frac{\partial T}{\partial y} = -h_s T \quad \text{at } y = 0 \quad (4)$$
$$u = 0, T = T_2 \quad \text{at } y = h$$

where μ is the fluid dynamic viscosity, ρ is the density of the fluid, β_T is the volumetric coefficient of thermal expansion of the fluid, p is the pressure, k is the thermal conductivity, c_p is the fluid specific heat of constant pressure, h_s is the coefficient of heat transfer and T_2 is the temperature of the right wall.

To obtain the exact solutions, the dimensionless variables are introduced as

$$U = \frac{u}{u_0}, \tau = \frac{vt}{h^2}, P = \frac{hp}{\rho v u_0}, X = \frac{x}{h},$$
$$Y = \frac{y}{h}, \theta = \frac{T - T_0}{T_0}, g(\tau) = \frac{g^*}{g_0} \quad (5)$$

where v is the kinematic viscosity. Using (5), Eqns. (2) and (3) become

$$\frac{\partial U}{\partial \tau} = -\frac{\partial P}{\partial X} + \frac{\partial^2 U}{\partial Y^2} + \frac{Gr}{Re} \cdot g(\tau) \cdot \theta, \quad (6)$$

$$\frac{\partial \theta}{\partial \tau} = \frac{1}{Pr} \frac{\partial^2 \theta}{\partial Y^2}. \quad (7)$$

With Grashof number, Gr, Reynolds number, Re and Prandtl number, Pr are given by

$$Gr = \frac{g_0 \beta_T h^3 T_0}{v^2}, Re = \frac{u_0 h}{v}, Pr = \frac{v \rho c_p}{k}. \quad (8)$$

Then, conditions (4) become

$$\tau < 0: U = 0, \theta = 0 \text{ for } 0 < Y < 1$$
$$\tau \geq 0: U = 0, \frac{\partial \theta}{\partial Y} = -\gamma(\theta + 1) \quad \text{at } Y = 0 \quad (9)$$
$$U = 0, \theta = r_T \quad \text{at } Y = 1$$

where the parameter of non-dimensional wall temperature, r_T and thermal conjugate, γ are defined as

$$r_T = \frac{T_2 - T_0}{T_0}, \qquad \gamma = hh_s \quad (10)$$

Before solving Eqn. (6), first, Eqn. (7) need to be solved. Now, assume $\theta(Y, \tau) = P(Y)Q(\tau)$, using Fourier method, Eqn. (7) has the solution of

$$\theta(Y, \tau) = \frac{r_T\ (1 - \gamma Y) + \gamma(1 - Y)}{1 - \gamma}. \quad (11)$$

Next, Eqn. (6) is solved using the same method as (7). By assuming $U = e^{i\Omega\tau} \cdot \Phi(Y), P = e^{i\Omega\tau} \cdot F(X), g(\tau) = e^{i\Omega\tau}$ (Sharidan *et al.*, 2015) and substitute solution (11) into (6), obtained

$$i\Omega e^{i\Omega\tau}\Phi = -\frac{\partial F}{\partial X} e^{i\Omega\tau} + \Phi'' e^{i\Omega\tau} + \frac{Gr}{Re} e^{i\Omega\tau}. \left[\frac{r_T\ (1 - \gamma Y) + \gamma(1 - Y)}{1 - \gamma}\right] \quad (12)$$

with

$$\Phi(0) = 0, \Phi(1) = 0. \quad (13)$$

Further, the condition of conservation of mass is defined as

$$\int_0^1 \Phi(Y)\, dY = 1. \quad (14)$$

Hence, the general solution of $\Phi(Y)$in (10) becomes

$$\Phi(Y) = \frac{1}{\beta^2} \left\{ \left(\frac{\partial F}{\partial X} - \frac{Gr}{Re} \left[\frac{r_T + \gamma}{1 - \gamma} \right] \right) \frac{\sinh(\beta(1 - Y))}{\sinh \beta} + \left(\frac{\sinh \beta Y}{\sinh \beta} \left\{ \frac{\partial F}{\partial X} - \frac{Gr}{Re}(r_T) \right\} \right) + \frac{Gr}{Re} \left[\frac{r_T\ (1 - \gamma Y) + \gamma(1 - Y)}{1 - \gamma} \right] - \frac{\partial F}{\partial X} \right\} \quad (15)$$

where $\beta^2 = i\Omega$ with $\partial F / \partial X$ is given by

$$\frac{\partial F}{\partial X} = \frac{\beta^3 \sinh \beta}{2 \cosh \beta - \beta \sinh \beta - 2} + \frac{Gr}{Re} \left[\frac{2r_T + \gamma(1 - r_T)}{2(1 - \gamma)} \right]. \quad (16)$$

For validation purposes, the solutions of (11) and (15) are then being substituted back into Eqns. (6) and (7) and it is found that the left-hand side of the Eqns. (6) and (7) are equal to righthand side, respectively. Solutions (11) and (15) are also observed fully satisfied the boundary conditions (9). Therefore, it is verified that the solutions of temperature (11) and velocity profiles (15) are corrects.

III. RESULTS AND DISCUSSIONS

The solution for the velocity (15) is assumed to be imaginary because of the driving force of g-jitter where $g^*(t) = Imag(g_0 e^{i\omega t})$. Therefore, solution velocity (15) becomes

$$Imag(U = \Phi e^{i\Omega\tau}) = Real(\Phi) \sin(\Omega\tau) + Imag(\Phi) \cos(\Omega\tau) \quad (17)$$

Solutions of velocity (17) and temperature (11) then being evaluated in mathematical software named Matlab to plot the graph with some desirable values of the various embedded parameter.

Figure 1 demonstrates the effect of non-dimensional wall temperature, r_T on temperature distribution, θ. The values of r_T are taken in the range of $0 \le r_T \le 1$. From the graph, as r_T increases, θ also increases. This is because the value of r_T is directly proportional to the temperature, $\theta(T)$. This situation is due to as r_T increasing, more hot fluid is carried through the vertical channel due to the increasing fluid temperature which consequently results in higher wall temperature gradient. Meanwhile, in Figure 2 present the effect of thermal conjugate, γ on temperature profile. Asγ increases, θ also increases.

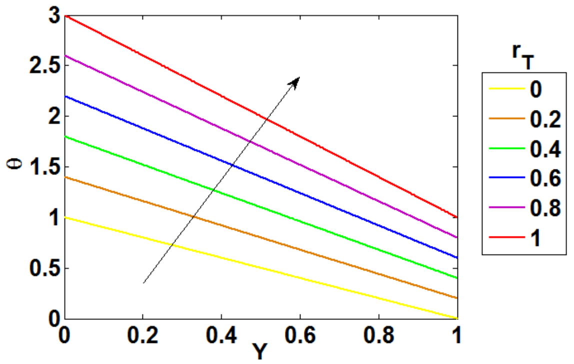

Figure 1. Temperature profiles for r_T when$\gamma = 0.5$

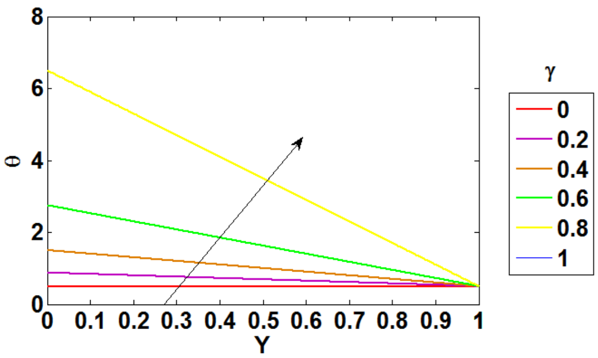

Figure 2. Temperature profiles for γ when$r_T = 0.5$

Figures 3 to 7 illustrate the velocity profiles of the consistent values of the γ, mixed convection parameter Gr/Re, frequency Ω and oscillation of g-jitter $\Omega\tau$ and r_T.

The analysis on the effect of γ and Gr/Re on the fluid flow are shown in Figures 3 and 4 respectively. It is found that the velocity of the fluid increases when Υ and Gr/Re increases. This is because the mixed convection raised the wall temperature angle which drives the fluid to become further from the surface resulting the high speed of flow. It enhances the heat transfer when they assist the forced flow.

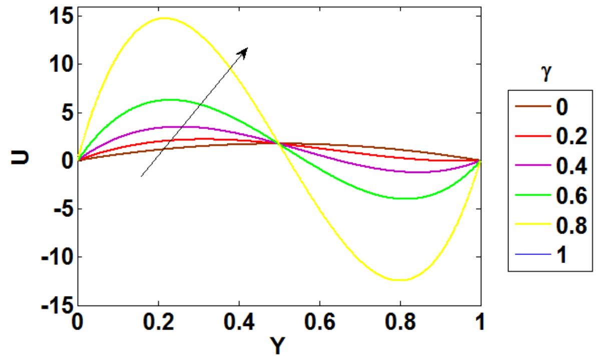

Figure 3. Velocity profiles for γ when

$$\Omega\tau = \frac{\pi}{2}, \Omega = 10, r_T = 0.5, \mathrm{Gr/Re} = 300$$

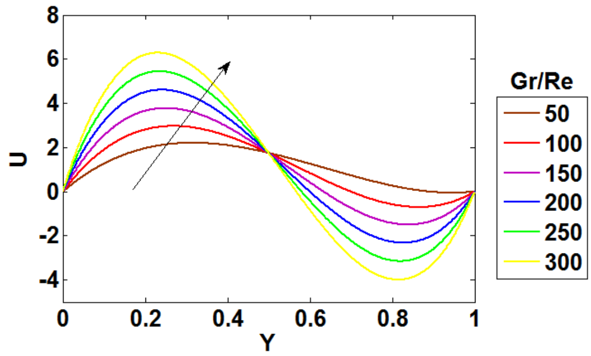

Figure 4. Velocity profiles for Gr/Re when

$$\Omega\tau = \frac{\pi}{2}, \Omega = 10, r_T = 0.5, \gamma = 0.6$$

Refer to Figure 5 to analyse the effect of frequency, Ω towards the behaviour of the fluid velocity, U. Observed that the velocity decelerated when Ω is increasing. Clearly, the fluid flow is less frequent as Ω increases. Therefore, the velocity is inversely proportional to the frequency. It means that the fluid flow decelerates with the increment of the frequency.

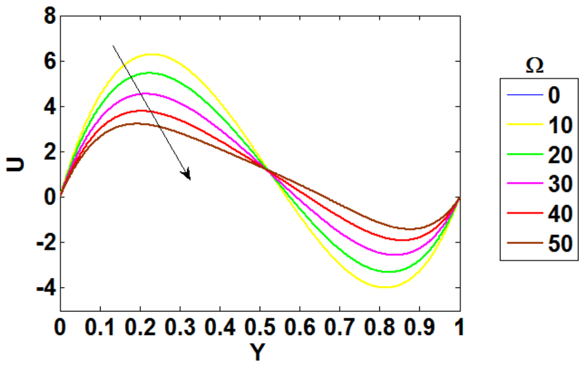

Figure 5. Velocity profiles for Ω when

$\Omega\tau = \frac{\pi}{2}, \gamma = 10, r_T = 0.5, \mathrm{Gr/Re} = 300$

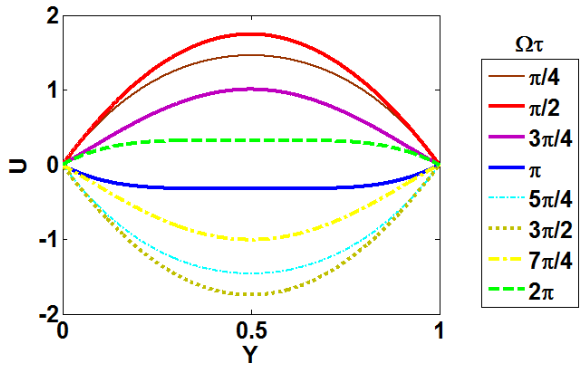

Figure 7. Velocity profiles for $\Omega\tau$ when

$r_T = 0.5, \Omega = 10, \gamma = 0, \mathrm{Gr/Re} = 300$

Meanwhile, for the comparison, different value of non-dimensional wall temperature, r_T is considered as shown in Figure 6. For the different values of r_T, the velocity profile increases as r_T increases. The results showed that the values of r_T affect the fluid flow as it also affects the fluid temperature. Physically, when the fluid making contact with plate, the heat is always transferred to the fluid, so the fluid temperature will also increases that enhance the velocity to accelerates.

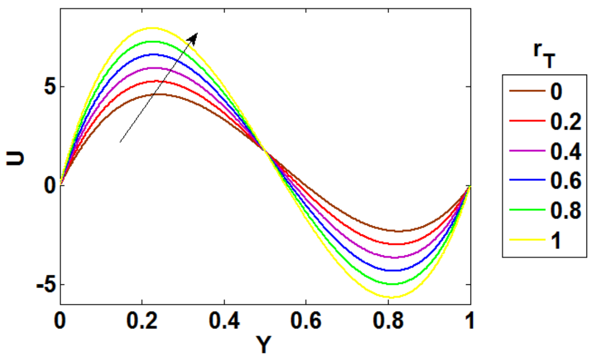

Figure 6. Velocity profiles for r_T when

$\Omega\tau = \frac{\pi}{2}, \Omega = 10, \gamma = 0.6, \mathrm{Gr/Re} = 300$

Finally, Figure 7 demonstrates the increasing values of Ωt that effect the velocity. It is seen that as Ωt increases, the fluid velocity fluctuated. The oscillating velocity has passed a quasi-steady state in a specific time about 2π and subsequently it fluctuates with fundamentally the same frequency of the g-jitter. This is due to the velocity becomes nonlinear, following around the identical scheme as g-jitter acceleration.

IV. RESULTS

This paper focused on heat transfer mixed convection flow in a vertical channel with Newtonian heating. The mechanism of the heat transfer was modelled in the existence of g-jitter effect. Therefore, following conclusions are made based on present research. Due to the increasing value of the wall temperature, r_T, and thermal conjugate, γ, the temperature, θ also increased. Moreover, the velocity, U decreased due to the frequent number of oscillations of the fluid parameter, Ω. Meanwhile, the velocity, U increases as $\mathrm{Gr/Re}$, γ and r_T increased.

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