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Integrated-Uncertainty Fuzzy Bezier Curve Modelling

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This paper discusses on the construction of an integrated-uncertainty fuzzy Bezier curve model. In this model, a combination of both type-1 fuzzy set theory (T1FST) and type-2 fuzzy set theory (T2FST) were used to define the integrated-uncertainty data. Integrated-uncertainty data consisting of uncertainty data together with complex uncertainty data that occur when the combination of degrees ambiguity is inherent in a collection of data sets that want to be modelled. Therefore, the fuzzification, type-reduction, and defuzzification methods were three processes needed to get the crisp integrated-uncertainty fuzzy Bezier curve. The result of this study is shown in this paper.

Keywords: Integrated-uncertainty; Type-1 Fuzzy Set Theory (T1FST); Type-2 Fuzzy Set Theory (T2FST); Bezier Curve

I. INTRODUCTION

The uncertainty issue is being widely discussed in many fields. In geometric modelling, uncertainty in data collection is the main problem to form a smooth curve and surface model. To overcome this uncertainty problem, in 1965 Zadeh introduced T1FST which became the significant solution in dealing with the uncertainty problem. However, the uncertainty level can be higher than usual, and it is known as complex uncertainty where T1FST is unable to define this type of uncertainty. Therefore, Zadeh (1975) comes with T2FST which can define the complex uncertainty problem.

The type-1 fuzzy number (T1FN) and type-2 fuzzy number (T2FN) also been introduced and had been used for modelling the uncertainty data in the real number form (Zadeh, 1965; 1975). In 2002, Mendel and John came with a simplified and easy approach to understanding T2FST concept.

Since the establishment of T1FST and T2FST, there are few studies conducted in type-1 or type-2 fuzzy geometric modelling using various curves and surfaces functions such as Bezier, B-spline and NURBS (Abd. Fatah *et al.* 2009; Abd. Fatah *et al.* 2010; Gallo *et al.* 2000; Anile *et al.* 2000; Rozaimi & Abd. Fatah, 2012; Rozaimi & Abd. Fatah, 2013; Rozaimi *et al.* 2013a; 2013b; 2013c; Abd. Fatah & Rozaimi, 2013a;2013b; Rozaimi & Abd. Fatah, 2014; Rozaimi *et al.* 2014; Adesah *et al.* 2017; Abd Fatah & Mohammad Izzat, 2017).

However, uncertainty and complex uncertainty both can occur when there is a combination of degrees ambiguity that exists in a collection of data sets. Therefore, both T1FST and T2FST should be used together to define the integrated-uncertainty data so that this data can be modelled through approximation Bezier curve function. The process starts with the fuzzification method using alpha-cut operation and then followed by type-reduction method. The type-reduction method only allows type-2 fuzzy data to go through this process where this process reduced type-2 fuzzy data to type-1 fuzzy data in order to allow the defuzzification process. Defuzzification is the last process to get crisp fuzzy solution of integrated-uncertainty data.

II. METHOD

This section will show some concepts and definitions of T1FST and T2FST.

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Definition 1. (Zadeh, 1965) A type-1 fuzzy set, $\overset{\leftrightarrow}{A}$ which is a term of a single variable $x \in X$, with membership function

$$\overset{\leftrightarrow}{A} = \left\{ \left(x, \mu_{\overset{\leftrightarrow}{A}}(x) \right) \mid \forall x \in X \right\} \quad (1)$$

Definition 2. (Rozaimi & Abd. Fatah, 2012) If the triangular T1FN represents as $\overset{\leftrightarrow}{A} = (k, l, m)$ and $\overset{\leftrightarrow}{A}_\alpha$ be a $\alpha -$cut operation of a triangular fuzzy number, then crisp interval by $\alpha-$cut operation is obtained as $\overset{\leftrightarrow}{A} = \left[k^\alpha, m^\alpha \right] = \left[(n-k)\alpha + k, -(m-n)\alpha + m \right]$ with $\alpha \in (0,1]$ where the membership function is (figure 1)

$$\mu_{\overset{\leftrightarrow}{A}}(x) = \begin{cases} 0 & , x < k \\ \dfrac{x-k}{n-k} & , k \le x \le n \\ \dfrac{m-x}{m-n} & , n \le x \le m \\ 0 & , x > c \end{cases} \quad (2)$$

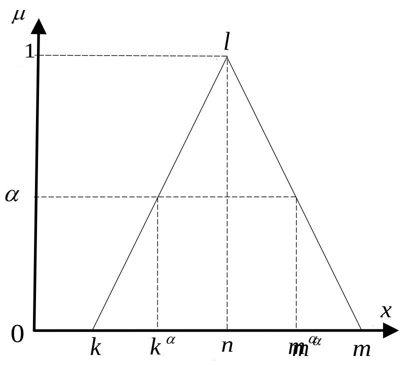

Figure 1. Triangular fuzzy number, $\overset{\leftrightarrow}{A}$

Definition 3. (Mendel, 2001) A type-2 fuzzy set (T2FS), $\overset{\leftrightarrow}{\overset{\leftrightarrow}{A}}$ is characterized by a type-2 membership function $\mu_{\overset{\leftrightarrow}{\overset{\leftrightarrow}{A}}}(x,u)$ that is

$$\overset{\leftrightarrow}{\overset{\leftrightarrow}{A}} = \left\{ \left((x,u), \mu_{\overset{\leftrightarrow}{\overset{\leftrightarrow}{A}}}(x,u) \right) \mid \forall x \in X, \forall u \in U_x, \subseteq [0,1] \right\} \quad (3)$$

where $0 \le \mu_{\overset{\leftrightarrow}{\overset{\leftrightarrow}{A}}}(x,u) \le 1$.

Definition 4. (Aguero, 2007) A T2FN is defined as a T2FS that has a numerical domain. An interval T2FS is defined using the following four constraints, where $\overset{\leftrightarrow}{\overset{\leftrightarrow}{A}} = \left\{ \left[p^\alpha, q^\alpha \right], \left[r^\alpha, s^\alpha \right] \right\}, \ \forall \alpha \in [0,1]$, $\forall p^\alpha, q^\alpha, r^\alpha, s^\alpha \in \mathbb{R}$ (figure 2)

i. $p^\alpha \le q^\alpha \le r^\alpha \le s^\alpha$

ii. $\left[p^\alpha, s^\alpha \right]$ and $\left[q^\alpha, r^\alpha \right]$ generate a convex function and $\left[p^\alpha, s^\alpha \right]$ generate a normal function.

iii. $\forall \alpha_1, \alpha_2 \notin [0,1] : (\alpha_2 > \alpha_1) \supset \left(\left[p^{\alpha_1}, r^{\alpha_1} \right] \supset \left[p^{\alpha_2}, r^{\alpha_2} \right], \left[q^{\alpha_1}, s^{\alpha_1} \right] \supset \left[q^{\alpha_2}, s^{\alpha_2} \right] \right)$, for $r^{\alpha_2} \ge q^{\alpha_2}$.

iv. If the maximum of the membership function generated by $\left[q^\alpha, r^\alpha \right]$ is the level α_m, that is $\left[q^{\alpha_m}, r^{\alpha_m} \right]$, then $\left[q^{\alpha_m}, r^{\alpha_m} \right] \subset \left[p^{\alpha=1}, s^{\alpha=1} \right]$.

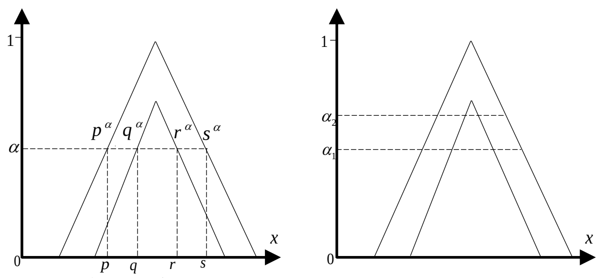

Figure 2. An interval T2FN

Based on the previous definition, we can now define the integrated-uncertainty fuzzy control points (IUFCPs) as ${}^{C}\overset{\leftrightarrow}{\overset{\leftrightarrow}{P}}_i$ where C refers to the combination of type-1 and type-2 fuzzy data.

Definition 5. Let ${}^{C}\overset{\leftrightarrow}{\overset{\leftrightarrow}{P}}_i$ be the IUFCPs. Then, ${}^{C}\overset{\leftrightarrow}{\overset{\leftrightarrow}{P}}_{i_\alpha}$ are the fuzzification processof IUFCPsthat trough alpha-cut operation with $\alpha \in [0,1]$.

$$
\begin{aligned}
{}^{C}\overset{\leftrightarrow}{\vec{P}}_{i_\alpha} &= \left\langle {}^{C}\overset{\leftrightarrow}{\vec{P}}{}^{\leftarrow}_{i_\alpha}, {}^{C}P_i, {}^{C}\overset{\leftrightarrow}{\vec{P}}{}^{\rightarrow}_{i_\alpha} \right\rangle \\
&= \left\langle \left\langle {}^{C}\overset{\leftarrow}{\overset{\leftrightarrow}{\vec{P}}}{}^{\leftarrow}_{i_\alpha}; {}^{C}\overset{\leftrightarrow}{\vec{P}}{}^{\leftarrow}_{i_\alpha}; {}^{C}\overset{\rightarrow}{\overset{\leftrightarrow}{\vec{P}}}{}^{\leftarrow}_{i_\alpha} \right\rangle, {}^{C}P_i, \left\langle {}^{C}\overset{\leftarrow}{\overset{\leftrightarrow}{\vec{P}}}{}^{\rightarrow}_{i_\alpha}; {}^{C}\overset{\leftrightarrow}{\vec{P}}{}^{\rightarrow}_{i_\alpha}; {}^{C}\overset{\rightarrow}{\overset{\leftrightarrow}{\vec{P}}}{}^{\rightarrow}_{i_\alpha} \right\rangle \right\rangle \qquad (4) \\
&= \left\langle \left({}^{C}P_i - \left\langle {}^{C}\overset{\leftarrow}{\overset{\leftrightarrow}{\vec{P}}}{}^{\leftarrow}_{i_\alpha}; {}^{C}\overset{\leftrightarrow}{\vec{P}}{}^{\leftarrow}_{i_\alpha}; {}^{C}\overset{\rightarrow}{\overset{\leftrightarrow}{\vec{P}}}{}^{\leftarrow}_{i_\alpha} \right\rangle \right)\alpha + \left\langle {}^{C}\overset{\leftarrow}{\overset{\leftrightarrow}{\vec{P}}}{}^{\leftarrow}_{i_\alpha}; {}^{C}\overset{\leftrightarrow}{\vec{P}}{}^{\leftarrow}_{i_\alpha}; {}^{C}\overset{\rightarrow}{\overset{\leftrightarrow}{\vec{P}}}{}^{\leftarrow}_{i_\alpha} \right\rangle, \right. \\
&\qquad \left. {}^{C}P_i, \; -\left(\left\langle {}^{C}\overset{\leftarrow}{\overset{\leftrightarrow}{\vec{P}}}{}^{\rightarrow}_{i_\alpha}; {}^{C}\overset{\leftrightarrow}{\vec{P}}{}^{\rightarrow}_{i_\alpha}; {}^{C}\overset{\rightarrow}{\overset{\leftrightarrow}{\vec{P}}}{}^{\rightarrow}_{i_\alpha} \right\rangle - {}^{C}P_i \right)\alpha + \left\langle {}^{C}\overset{\leftarrow}{\overset{\leftrightarrow}{\vec{P}}}{}^{\rightarrow}_{i_\alpha}; {}^{C}\overset{\leftrightarrow}{\vec{P}}{}^{\rightarrow}_{i_\alpha}; {}^{C}\overset{\rightarrow}{\overset{\leftrightarrow}{\vec{P}}}{}^{\rightarrow}_{i_\alpha} \right\rangle \right\rangle
\end{aligned}
$$

where ${}^{C}P_i$ is a crisp number, ${}^{C}\bar{\overset{\leftrightarrow}{P}}{}^{\leftarrow}_{i_\alpha}$ is left fuzzification of IUFCPs with ${}^{C}\overset{\leftarrow}{\overset{\leftrightarrow}{\vec{P}}}{}^{\leftarrow}_{i_\alpha}$ (left-left) and ${}^{C}\overset{\rightarrow}{\overset{\leftrightarrow}{\vec{P}}}{}^{\leftarrow}_{i_\alpha}$ (left-right) fuzzification. ${}^{C}\bar{\overset{\leftrightarrow}{P}}{}^{\rightarrow}_{i_\alpha}$ is right fuzzification of IUFCPs with ${}^{C}\overset{\leftarrow}{\overset{\leftrightarrow}{\vec{P}}}{}^{\rightarrow}_{i_\alpha}$ (right-left) and ${}^{C}\overset{\rightarrow}{\overset{\leftrightarrow}{\vec{P}}}{}^{\rightarrow}_{i_\alpha}$ (right-right) fuzzification.

Definition 6. Let ${}^{C}\overset{\leftrightarrow}{\vec{P}}_{i_\alpha}$ be the IUFCPs after fuzzification. Then, ${}^{C}\bar{\overset{\leftrightarrow}{P}}_{i_\alpha}$ are the type-reduction of ${}^{C}\overset{\leftrightarrow}{\vec{P}}_{i_\alpha}$ is given as

$$
\begin{aligned}
{}^{C}\bar{\overset{\leftrightarrow}{P}}_{i_\alpha} &= \left\{ {}^{C}\bar{\overset{\leftrightarrow}{P}}{}^{\leftarrow}_{i_\alpha}, {}^{C}P_i, {}^{C}\bar{\overset{\leftrightarrow}{P}}{}^{\rightarrow}_{i_\alpha} \right\} \\
&= \begin{cases} \dfrac{1}{3} \displaystyle\sum_{i=0,1,\ldots,n} \left\langle {}^{C}\overset{\leftarrow}{\overset{\leftrightarrow}{\vec{P}}}{}^{\leftarrow}_{i_\alpha} + {}^{C}\overset{\leftrightarrow}{\vec{P}}{}^{\leftarrow}_{i_\alpha} + {}^{C}\overset{\rightarrow}{\overset{\leftrightarrow}{\vec{P}}}{}^{\leftarrow}_{i_\alpha} \right\rangle, {}^{C}P_i, \\ \dfrac{1}{3} \displaystyle\sum_{i=0,1,\ldots,n} \left\langle {}^{C}\overset{\leftarrow}{\overset{\leftrightarrow}{\vec{P}}}{}^{\rightarrow}_{i_\alpha} + {}^{C}\overset{\leftrightarrow}{\vec{P}}{}^{\rightarrow}_{i_\alpha} + {}^{C}\overset{\rightarrow}{\overset{\leftrightarrow}{\vec{P}}}{}^{\rightarrow}_{i_\alpha} \right\rangle \end{cases} \qquad (5)
\end{aligned}
$$

where ${}^{C}P_i$ is a crisp number, ${}^{C}\bar{\overset{\leftrightarrow}{P}}{}^{\leftarrow}_{i_\alpha}$ is left type-reduction of $\alpha-$IUFCPs, and ${}^{C}\bar{\overset{\leftrightarrow}{P}}{}^{\rightarrow}_{i_\alpha}$ is right type-reduction of $\alpha-$ IUFCPs.

Definition 7. Let ${}^{C}\bar{\overset{\leftrightarrow}{P}}_{i_\alpha}$ be the IUFCPs after fuzzification and type-reduction process. Then ${}^{C}\bar{\bar{P}}_{\alpha}$ aredefuzzification process of ${}^{C}\bar{\overset{\leftrightarrow}{P}}_{i_\alpha}$ for every ${}^{C}\bar{\overset{\leftrightarrow}{P}}_{i_\alpha} \in {}^{C}\bar{\overset{\leftrightarrow}{P}}_{i}$, $i = 0,1,\ldots,n$ where

$$
{}^{C}\bar{\bar{\overset{\leftrightarrow}{P}}}_{i_\alpha} = \frac{1}{3} \sum_{i=0,1,\ldots,n} \left\langle {}^{C}\bar{\overset{\leftrightarrow}{P}}{}^{\leftarrow}_{i_\alpha}, {}^{C}P_i, {}^{C}\bar{\overset{\leftrightarrow}{P}}{}^{\rightarrow}_{i_\alpha} \right\rangle \qquad (6)
$$

Definition 8. (Abd. Fatah *et al.* 2009) Let $n+1$ fuzzy control points $\vec{P}_i$, then the fuzzy Bezier curve is defined as

$$
\vec{B}(t) = \sum_{i=0}^{n} B_i^n(t) \vec{P}_i \qquad (7)
$$

where $B_i^n(t) = \begin{pmatrix} n \\ i \end{pmatrix} t^i (1-t)^{n-i}$ is the Bernstein polynomials of degree n.

III. RESULT AND DISCUSSION

After defining integrated-uncertainty data using T1FST and T2FST, and follows by fuzzification, type-reduction and defuzzification processes, then all these processes can be illustratedusing approximation fuzzy Bezier curve function. Thus, the integrated-uncertainty fuzzy Bezier curve model (IUFBC) can be expressed as in equation 8.

$$
{}^{C}\overset{\leftrightarrow}{\vec{B}}(t) = \sum_{i=0}^{n} B_i^n(t)\, {}^{C}\overset{\leftrightarrow}{\vec{P}}_i \qquad (8)
$$

where ${}^{C}\overset{\leftrightarrow}{\vec{P}}_i$ is IUFCPs where C refers to the combination of type-1 and type-2 fuzzy data.

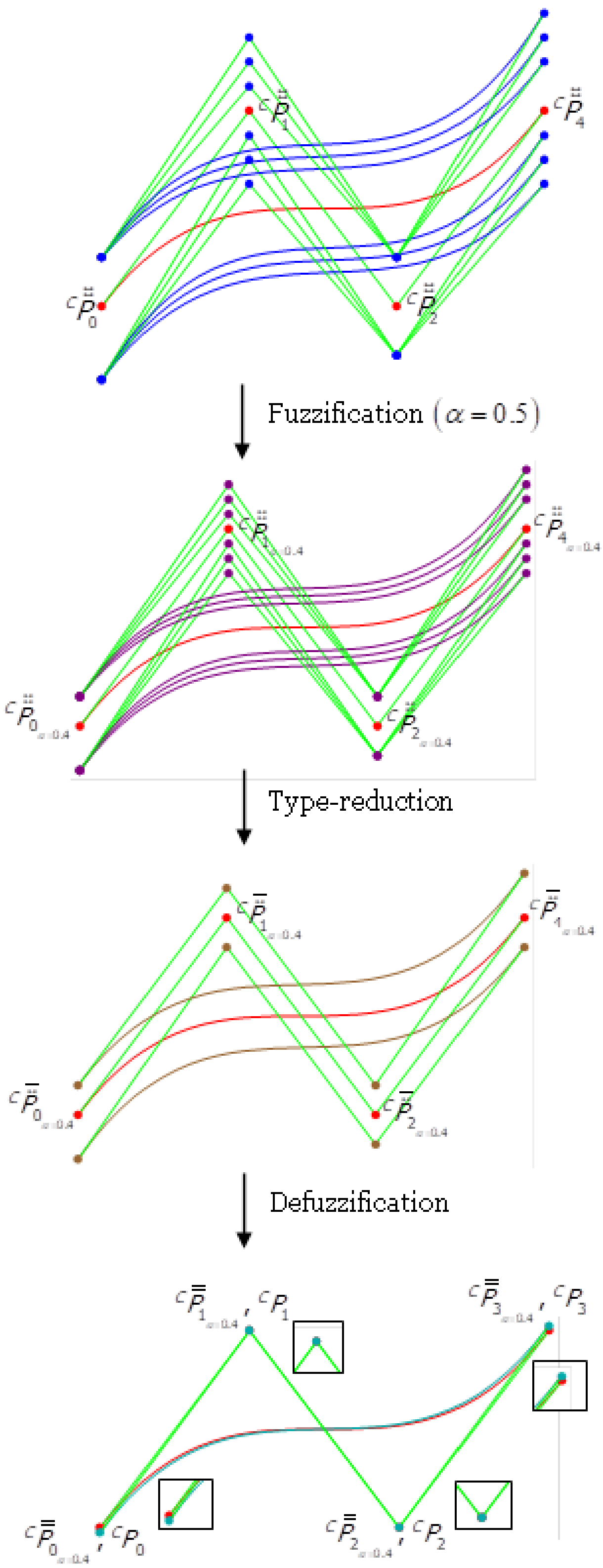

Figure 3. The process of modelling integrated-uncertainty data to get a crisp solution

Figure 3 shows the process of modelling integrated-uncertainty fuzzy control points (IUFCPs) to obtain crisp IUFCPs as a final solution where the last process was the modelling of the crisp IUFBC (light bluecurve/ points), together with crisp Bezier curve (red curve/ points).

This formation of the curve shows that the combination of type-1 and type-2 fuzzy data (integrated-uncertainty data) was able to be modelled and can be blended with fuzzy Bezier curve function to form IUFBC model.

IV. CONCLUSION

In this paper, we presented the modelling ofintegrated-uncertainty data. Integrated-uncertainty data were defined using both T1FST and T2FST, which definitely can define uncertainty and complex uncertainty data based on earlier research. Three processes, fuzzification, type-reduction and defuzzification methods were applied to get the crisp integrated-uncertainty data as a final solution.

Bezier curve function was chosen as a fundamental in modelling the integrated-uncertainty data. For future research, we will test and modelled the integrated-uncertainty data using Bezier surface function.

V. ACKNOWLEDGEMENT

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VI. REFERENCES

Abd. Fatah, Wahab., Jamaludin, Md. Ali. & Ahmad, Abd. Majid. (2009). Fuzzy Geometric Modeling, Proceeding, Sixth International Conference on Computer Graphics, Imaging and Visualization 2009, pp. 276 - 280.

Abd. Fatah, Wahab., Jamaludin, Md. Ali, Ahmad, Abd. Majid & Abu Osman, Md. Tap. (2010). Penyelesaian Masalah Data Ketakpastian Menggunakan B-Splin Kabur, Sains Malaysiana, vol. 39, no. 4, pp. 661 - 670.

Abd. Fatah, Wahab & Rozaimi, Zakaria. (2013a). Generalized Normal Type-2 Triangular Fuzzy Number, Applied Mathematical Sciences, vol. 7, no. 45, pp. 2239 – 2252.

Abd. Fatah, Wahab & Rozaimi, Zakaria. (2013b). on Perfectly Normal Type-2 Triangular Fuzzy Number, Applied Mathematical Sciences, vol. 7, no. 46, pp. 2253 - 2263.

Abd. Fatah, Wahab & Mohammad Izat Emir, Zulkifly. (2017). A New Fuzzy Bezier Curve Modeling by Using Fuzzy Control Point Relation, Applied Mathematical Sciences, vol. 11, no. 1, pp. 39 – 57.

Adesah, R. S., Zakaria, R., Wahab, A. F. & Talibe, A. (2017). Fuzzy Curve Model, IOP Conf. Series: Journal of Physics: Conf. Series, pp. 1 - 6.

Aguero, J. R. & Vargas, A. (2007). Calculating Functions of Interval Type-2 Fuzzy Numbers for Fault Current Analysis. IEEE Transactions on Fuzzy Systems, vol. 15, no. 1, pp. 31 - 40.

Anile, A. M., Falcidieno, B., Gallo, G., Spagnuolo, M. & Spinello, S. (2000). Fuzzy Sets and Systems: Modeling Uncertain Data with Fuzzy B-Splines, Fuzzy Sets and Systems, vol. 113, pp. 397 – 410.

Gallo, G., Spagnuolo, M. & Spinello, S. (2000). Fuzzy B-Splines A Surface Model Encapsulating Uncertainty, Graphical Models, vol. 62, pp. 40 - 55.

Mendel, J. M. (2001). Uncertain Rule-Based Fuzzy Logic Systems: Introduction and New Directions. Prentice Hall, New Jersey.

Mendel, J. M. & John, R. I. B. (2002). Type-2 Fuzzy Sets Made Simple, IEEE Transactions on Fuzzy Systems, vol. 10, no. 2, pp. 117 – 127.

Rozaimi, Zakaria. & Abd Fatah, Wahab. (2013). on Defining Complex Uncertainty Data Points by Type-2 Fuzzy Number: Two Specials Cases, International Journal of Math. Analysis, vol. 7, no. 26, pp. 1285 - 1300.

Rozaimi, Zakaria. & Abd. Fatah, Wahab. (2012). Fuzzy B-spline Modeling of Uncertainty Data, Applied Mathematical Sciences, vol. 6, no. 140, pp. 6971 - 6991.

Rozaimi, Zakaria. & Abd. Fatah, Wahab. (2014). Pemodelan Titik Data Kabur Teritlak, Sains Malaysiana, vol. 43, no. 5, pp. 799 -805.

Rozaimi, Zakaria., Abd Fatah, Wahab. & Gobithaasan, R. U. (2013a). Type-2 fuzzy Bezier curve modelling, AIP Conference Proceedings, pp. 945 – 952.

Rozaimi, Zakaria., Abd Fatah, Wahab. & Gobithaasan, R. U. (2013b). Normal Type-2 Fuzzy Rational B-spline Curve, Journal of Math. Analysis, vol. 7, no. 16, pp. 789 – 806.

Rozaimi, Zakaria., Abd Fatah, Wahab. & Gobithaasan, R. U. (2013c) Perfectly Normal Type-2 Fuzzy Interpolation B-spline Curve, Applied Mathematical Sciences, vol. 7, no.21, pp. 1043 – 1055.

Rozaimi, Zakaria., Abd Fatah, Wahab. & Gobithaasan, R. U. (2014). Fuzzy B-Spline Surface Modeling, Journal of Applied Mathematics, vol. 2014, no. 1, pp. 1 - 8.

Zadeh, L. (1965). Fuzzy Sets, Information and Control, vol. 8, pp. 338 - 353.

Zadeh, L. (1975). The Concept of a Linguistic Variable and its Application to Approximate Reasoning-I, Information and Control, vol. 8, pp. 199 - 249.