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Intuitionistic Fuzzy B-Spline Surface Modeling for 3-Dimensional Data Problem

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Geometric modeling for B-spline surface that involves intuitionistic data is very challenging to construct. In this paper, intuitionistic fuzzy B-spline for 3-dimensional data problem with intuitionistic fuzzy approach is introduced. Firstly, intuitionistic fuzzy control net is defined based on intuitionistic fuzzy concept. Later, the control net is blended with the B-spline basis function and the B-spline surface is visualized. A numerical example to generate the surface is also shown at the end of this paper.

Keywords: intuitionistic fuzzy B-spline; intuitionistic fuzzy control net; intuitionistic fuzzy set

I. INTRODUCTION

Intuitionistic fuzzy set is one of the possible generalizations from fuzzy set theory that were introduced in 1965 (Zadeh, 1965). Intuitionistic fuzzy set was first introduced by Atanassovin 1986 (Atanassov, 1986). Intuitionistic fuzzy set arise from the simultaneous consideration and existence of membership, non-membership and uncertainty function denoted by μ, ν and π respectively compared to fuzzy set that only deal with membership function (Atanassov, 1999; 2012). Presently, the research related to intuitionistic fuzzy set have been studied and successfully carried out in many fields include sciences and mathematics such as in similarity measure, decision making, image processing and much more (Atanassov & Gargov, 1989; Szmidt & Kacrzyk, 2000; Atanassov, 2000; Cornelis *et al.* 2004; Yuan *et al.* 2014; Bashir *et al.* 2015; Diaz *et al.* 2015; Rahman, 2016; Ngan, 2016; Wang & Chen, 2017; Hassaballah & Ghareeb, 2017).

The idea of intuitionistic fuzzy set appears to be useful in some applications. One of them is in the field of geometric modeling. Research of intuitionistic fuzzy set combined with geometric modeling have been done by Zulkifly and Wahab. They introduced an idea of intuitionistic fuzzy setin spline curve and surface which focused on Bézier spline where the curve and surface are blended with intuitionisticfuzzy control point (IFCP) (Zulkifly&Wahab, 2015). Wahab *et al.* (2016) discussed intuitionistic fuzzy Bézier model and generated intuitionistic fuzzy Bézier curve using interpolation method. They visualized intuitionistic fuzzy Bézier curve that consists of membership, non-membership and uncertainty curve by blending the Bernsteinpolynomial IFCP that have been defined. Later, Zulkifly & Wahab defined IFCP through intuitionistic fuzzy conceptwith some properties. They illustrated intuitionistic fuzzy bicubic Bézier surface through the approximation method byusing data point with intuitionistic features (Zulkifly&Wahab, 2018). By usingIFCP, they also generated cubic Bézier curve through interpolation method and intuitionistic fuzzy B-spline curve using approximation method (Wahab & Zulkifly, 2018a; 2018b).

Surface modelling is a method of mathematical representations construction in the form of geometry while intuitionistic fuzzy set is a mathematical representations that aimed at the concepts and techniques to tackle uncertainty data problem. The objective of this paper is to generate and visualize intuitionistic fuzzy B-spline surface through interpolation method by using IFCP that have been introduced in previous research. This paper is organized as follows. Section 1 present introductions and previous works that related to this research. Section 2 shows some definitions of intuitionistic fuzzy set and intuitionistic fuzzy point relation (IFPR) that will be used throughout this

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paper. In section 3, intuitionistic fuzzy control net (IFCN) is defined based on IFCP and intuitionistic fuzzy concept. Next, in section 4, intuitionistic fuzzy B-spline surface modelling is defined based on IFCN and intuitionistic fuzzy B-spline surface using 3-dimensional data problem is illustrated. A numerical example is also shown in this section. Finally, section 5 will conclude this research.

II. PRELIMINARIES

This section shows some basic definition of intuitionistic fuzzy set and intuitionistic fuzzy point relation.

Definition 1. Let a set *X* is fixed and let $A \subset X$ be a fixed set. An IFS A^* in *X* is an object of the following form (Atanassov, 1986):

$$A^* = \left\{ \left\langle x, \mu_A(x), \nu_A(x) \right\rangle \mid x \in X \right\} \qquad (1)$$

where functions $\mu_A : X \to [0,1]$ and $\nu_A : X \to [0,1]$ define the degree of membership and non-membership of the element $x \in X$ to the set *A*, respectively and for every $x \in X$, $0 \le \mu_A(x) + \nu_A(x) \le 1$. Obviously, the ordinary fuzzy set has the form $\left\{ \left\langle x, \mu_A(x), 1 - \mu_A(x) \right\rangle \mid x \in X \right\}$. If $\pi_A(x) = 1 - \left(\mu_A(x) + \nu_A(x) \right)$, then $\pi_A(x)$ is the degree of uncertainty or intuitionistic index of the membership of element $x \in X$ to set A^* where $0 \le \pi_A \le 1$.

IFPR is developed and introduced based on the concept of intuitionistic fuzzy set. Let *V*, *W* be a collection of universal space of points in the Euclidean space and $V, W \in \mathbb{R}^2$, then IFPR is defined as follows:

Definition 2. Let X, Y be a collection of universal space of points with non-empty set and $V, W, I \subseteq \mathbb{R} \times \mathbb{R} \times \mathbb{R}$, then IFPR is defined as

$$\begin{aligned} T^* = \Big\{ \Big\langle \left(v_i, w_j\right), \mu_T\left(v_i, w_j\right), \nu_T\left(v_i, w_j\right), \\ \pi_T\left(v_i, w_j\right) \Big\rangle \mid \Big(\mu_T\left(v_i, w_j\right), \nu_T\left(v_i, w_j\right) \\ , \pi_T\left(v_i, w_j\right) \Big) \in I \Big\} \end{aligned} \qquad (2)$$

where $\left(v_i, w_j\right)$ is an ordered pair of points and $\left(v_i, w_j\right) \in V \times W$. $\mu_T\left(v_i, w_j\right)$, $\nu_T\left(v_i, w_j\right)$ and $\pi_T\left(v_i, w_j\right)$ are the grade of membership, non-membership and uncertainty of the ordered pair of points respectively in $[0,1] \in I$. Furthermore, the condition $0 \le \mu_T\left(v_i, w_j\right) + \nu_T\left(v_i, w_j\right) \le 1$ is follows and the degree of uncertainty is denoted by

$$\pi_T\left(v_i, w_j\right) = 1 - \left(\mu_T\left(v_i, w_j\right) + \nu_T\left(v_i, w_j\right) \right) \qquad (3)$$

The IFPR is based on fuzzy point in the Euclidean space and the intuitionistic fuzzy point is in intuitionistic fuzzy set.

III. INTUITIONISTIC FUZZY CONTROL NET

The collection of all points or set of points that are used to determine the shape of a spline surface is called control net. The control net plays an important role in the process of generating, controlling and producing smooth surfaces. In this section, intuitionistic fuzzy control net (IFCN) is defined.

Definition 3. Let T^* be an IFPR, then intuitionistic fuzzy control point (IFCP) is defined as a set of points that indicates the positions and coordinates of a location and is denoted by

$$C^* = \left\{ C_1^*, C_2^*, \ldots, C_{n+1}^* \right\} \qquad (4)$$

Where *i* is one less than the number of points, *n*. Hence, from (2) and (4), IFCN can be defined as follows;

Definition 4. Let C^* be an IFCP, then IFCN can be defined as collection of points $n+1$ and $m+1$ for C^* in the direction of *u* and *v* respectively. IFCN is denoted by $C_{i,j}^*$ that indicates the positions and coordinates of a location to describe the surface with $C^* = \left\{ C_{i,j}^* \right\}_{i=1, j=1}^{n+1, m+1}$

and written as

$$C^*_{i,j} = \begin{bmatrix} C^*_{1,1} & C^*_{1,2} & \cdots & C^*_{1,j} \\ C^*_{2,1} & C^*_{2,2} & \cdots & C^*_{2,j} \\ \vdots & \vdots & \ddots & \vdots \\ C^*_{n+1,1} & C^*_{n+1,2} & \cdots & C^*_{n+1,m+1} \end{bmatrix} \quad (5)$$

In the next section, intuitionistic fuzzy B-spline surface modelling is introduced.

IV. INTUITIONISTIC FUZZY B-SPLINE SURFACE MODELING

The tensor product method is basically a bidirectional curve scheme that uses basic functions and geometric coefficients. Hence, intuitionistic fuzzy B-spline surface is defined as follows:

Definition 5. Let $C^*_{i,j}$ be IFCN where $C^* = \left\{C^*_{i,j}\right\}_{i=1,j=1}^{n+1,m+1}$, therefore tensor product for intuitionistic fuzzy B-spline surface is given by

$$S^*(u,v) = \sum_{i=1}^{n+1}\sum_{j=1}^{m+1} C^*_{i,j} N_i^k(u) M_j^\ell(v) \quad (6)$$

where $N_i^k(u)$ and $M_j^\ell(v)$ is B-spline basis functions respectively in *u* and *v* directions is written as follows:

$$N_i^1(u) = \begin{cases} 1 & if \quad u_i \le u < u_{i+1} \\ 0 & otherwise \end{cases}$$
$$N_i^k(u) = \frac{(u-u_i)}{u_{i+k-1}-u_i} N_i^{k-1}(u) + \frac{(u_{i+k}-u)}{u_{i+k}-u_{i+1}} N_{i+1}^{k-1}(u) \quad (7)$$

and

$$M_j^1(v) = \begin{cases} 1 & if \quad v_j \le v < v_{j+1} \\ 0 & otherwise \end{cases}$$
$$M_j^\ell(u) = \frac{(v-v_j)}{v_{j+\ell-1}-v_j} M_j^{\ell-1}(v) + \frac{(v_{j+\ell}-v)}{v_{j+\ell}-v_{j+1}} M_{j+1}^{\ell-1}(v) \quad (8)$$

with u_i and v_j is an element of a knot vector. An intuitionistic fuzzy B-spline surface in (6) consists of membership, non-membership and uncertainty surface and denoted as follows:

$$S^\mu(u,v) = \sum_{i=1}^{n+1}\sum_{j=1}^{m+1} C^\mu_{i,j} N_i^k(u) M_j^\ell(v) \quad (9)$$

$$S^\nu(u,v) = \sum_{i=1}^{n+1}\sum_{j=1}^{m+1} C^\nu_{i,j} N_i^k(u) M_j^\ell(v) \quad (10)$$

$$S^\pi(u,v) = \sum_{i=1}^{n+1}\sum_{j=1}^{m+1} C^\pi_{i,j} N_i^k(u) M_j^\ell(v) \quad (11)$$

To illustrate theintuitionistic fuzzy B-spline surface using interpolation method, let considered 4×4 IFCN, with degree of membership, non-membership and uncertainty written as follows:

$$\begin{bmatrix} C^*_{1,1} & C^*_{1,2} & C^*_{1,3} & C^*_{1,4} \\ C^*_{2,1} & C^*_{2,2} & C^*_{2,3} & C^*_{2,4} \\ C^*_{3,1} & C^*_{3,2} & C^*_{3,3} & C^*_{3,4} \\ C^*_{4,1} & C^*_{4,2} & C^*_{4,3} & C^*_{4,4} \end{bmatrix}$$

Each column with their respective value and 3-tuple degree $\langle \mu, \nu, \pi \rangle$ is given as;

$$\begin{bmatrix} C^*_{1,1} \\ C^*_{2,1} \\ C^*_{3,1} \\ C^*_{4,1} \end{bmatrix} = \begin{bmatrix} \langle(-17,17);0.3,0.6,0.1\rangle \\ \langle(-7,17);0.5,0.3,0.2\rangle \\ \langle(7,17);0.5,0.1,0.4\rangle \\ \langle(17,17);0.6,0.2,0.2\rangle \end{bmatrix}$$

$$\begin{bmatrix} C^*_{1,2} \\ C^*_{2,2} \\ C^*_{3,2} \\ C^*_{4,2} \end{bmatrix} = \begin{bmatrix} \langle(-17,7);0.8,0.2,0\rangle \\ \langle(-7,7);0.7,0.1,0.2\rangle \\ \langle(7,7);0.7,0.3,0\rangle \\ \langle(17,7);0.3,0.5,0.2\rangle \end{bmatrix}$$

$$\begin{bmatrix} C^*_{1,3} \\ C^*_{2,3} \\ C^*_{3,3} \\ C^*_{4,3} \end{bmatrix} = \begin{bmatrix} \langle(-17,-7);0.3,0.3,0.4\rangle \\ \langle(-7,-7);0.4,0.4,0.2\rangle \\ \langle(7,-7);0.4,0.6,0\rangle \\ \langle(17,-7);0.3,0.5,0.2\rangle \end{bmatrix}$$

$$\begin{bmatrix} C^*_{1,4} \\ C^*_{2,4} \\ C^*_{3,4} \\ C^*_{4,4} \end{bmatrix} = \begin{bmatrix} \langle(-17,-17);0.5,0.4,0.1\rangle \\ \langle(-7,-17);0.6,0.3,0.1\rangle \\ \langle(7,-17);0.4,0.2,0.4\rangle \\ \langle(17,-17);0.6,0.3,0.1\rangle \end{bmatrix}$$

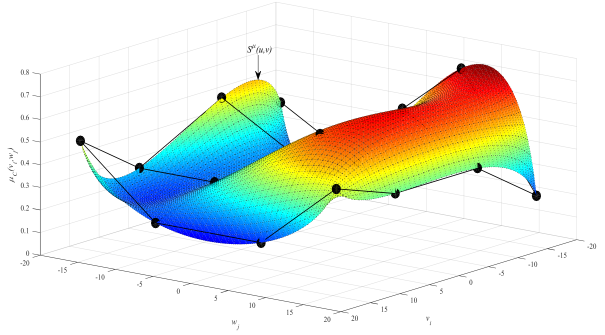

Figure 1. Intuitionistic fuzzy B-spline surface (membership)

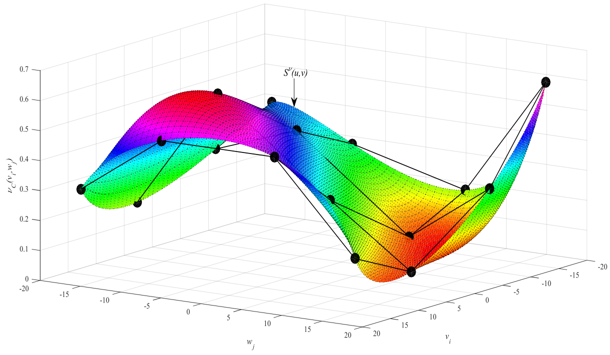

Figure 2. Intuitionistic fuzzy B-spline surface (non-membership)

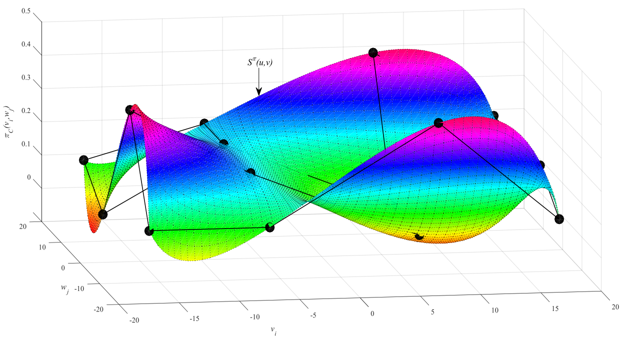

Figure 3. Intuitionistic fuzzy B-spline surface (uncertainty)

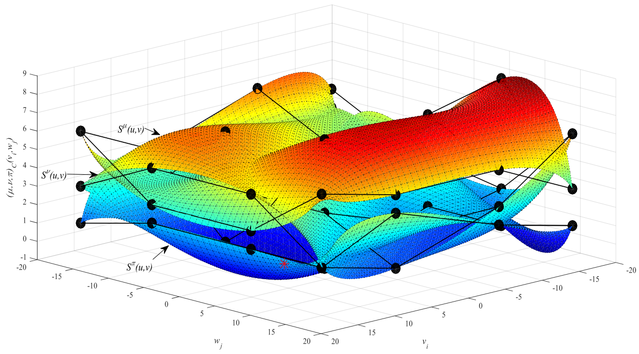

Figure 4. Intuitionistic fuzzy B-spline surface (membership, non-membership and uncertainty)

Figure 1 until Figure 4 illustrates intuitionistic fuzzy B-spline surfaces. Figure 1 is the membership fuzzy B-spline surface, Figure 2 is non-membership B-spline surface, Figure 3 is uncertainty fuzzy B-spline surface and finally Figure 4 is an intuitionistic fuzzy B-spline surface.

V. CONCLUSION

In this paper, intuitionistic fuzzy B-spline surface for 3-dimensional data problem is introduced and visualized by using interpolation method. Through intuitionistic fuzzy B-spline surface that have been introduced, problems involving uncertainty can be handled and illustrated smoothly. This model can be applied in image processing, underground economy, bathymetric data visualization and much more.

VI. REFERENCES

Atanassov, K. T. (1986). Intuitionistic Fuzzy Sets. Fuzzy Sets and Systems, 20(1), pp. 87-96.

Atanassov, K. T. & Gargov, G. (1989). Interval Valued Intuitionistic Fuzzy Sets. Fuzzy Sets and Systems, 31(3), pp. 343–349.

Atanassov, K. T. (1999). Intuitionistic Fuzzy Sets: Theory and Applications. New York: Physica-Verlag HD, pp. 1-9.

Atanassov, K. T. (2000). Two Theorems for Intuitionistic Fuzzy Sets. Fuzzy Sets and Systems, 110(2), pp. 267–269.

Atanassov, K. T. (2012). On Intuitionistic Fuzzy Sets Theory. New York: Springer-Verlag Berlin Heidelberg, pp. 1-12.

Bashir, Z., Rashid, T. & Zafar, S. (2015). Convergence of Intuitionistic FuzzySets. Chaos, Solitons & Fractals, 81 (Part A), pp. 11–19.

Cornelis, C., Deschrijver, G. & Kerre, E. E. (2004). Implication in Intuitionistic Fuzzy and Interval-Valued Fuzzy Set Theory: Construction, Classification, Application. International Journal of Approximate Reasoning, 35, pp. 55–95.

Diaz, S., Indurain, E., Janis, V. & Montes, S. (2015). Aggregation of Convex Intuitionistic Fuzzy Sets. Information Sciences, 308, pp. 61–71.

Hassaballah, M. & Ghareeb, A. (2017). A Framework for Objective Image Quality Measures Based on Intuitionistic Fuzzy Sets. Applied Soft Computing, 57, pp. 48–59.

Ngan, S-C. (2016). An Activation Detection Based Similarity Measure for Intuitionistic Fuzzy Sets. Expert Systems with Applications, 60, pp. 62–80.

Rahman, S. (2016) On Cuts of Atanassov's Intuitionistic Fuzzy Sets with Respect to Fuzzy Connectives. Information Sciences, 340–341, pp. 262–278.

Szmidt, E. & Kacrzyk, J. (2000). Distances between Intuitionistic Fuzzy Sets. Fuzzy Sets and Systems, 114(3), pp. 505–518.

Wahab, A. F., Zulkifly, M. I. E. & Husain, M. S. (2016). Bézier CurveModeling for Intuitionistic Fuzzy Data Problem. AIP Proceedings, 1750(1), 030047-1-030047-7.

Wahab, A. F. & Zulkifly, M. I. E. (2018a). Cubic Bezier Curve InterpolationBy Using Intuitionistic Fuzzy Control Point Relation. AIP Proceedings,1774(1), 020031.

Wahab, A. F. & Zulkifly, M. I. E. (2018b). Intuitionistic Fuzzy B-SplineCurve Approximation Model For Complex Uncertainty Data Problems. FarEast Journal of MathematicalSciences (FJMS), 106(1), pp. 217–235.

Wang, C-Y. & Chen, S-M. (2017). Multiple Attribute Decision MakingBased on Interval-Valued Intuitionistic Fuzzy Sets, Linear Programming Methodology, and the Extended TOPSIS Method.Information Sciences, 397–398, pp. 155–167.

Yuan, X-h., Li, H-x. & Zhang, C. (2014). The Theory of Intuitionistic Fuzzy Sets Based on the Intuitionistic Fuzzy Special Sets. Information Sciences, 277, pp. 284–298.

Zadeh, L. A. (1965). Fuzzy Sets. Information and Control, 8(3), pp. 338-353.

Zulkifly, M. I. E. & Wahab, A. F. (2015). Intuitionstic Fuzzy in Spline Curve/Surface. Malaysia Journal of Fundamental and Applied Sciences, 11(1), pp. 21–23.

Zulkifly, M. I. E. & Wahab, A. F. (2018). Intuitionistic Fuzzy Bicubic Bezier Surface Approximation. AIP Proceedings, 1974(1), 020064.