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Magnetic Field and Feedback Control Effects in a Double Diffusive Binary Fluid Layer with Temperature Dependent Viscosity

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The effect of magnetic field and feedback control is studied in a double diffusive binary fluid on the stability of Rayleigh - Benard convection. The boundaries were set to represent the lower - upper boundaries as rigid - rigid, rigid - free or rigid - rigid. The lower boundary was set to be conducted to temperature and the upper boundary to be insulated to temperature. The critical stability parameters were obtained using linear stability analysis and the Galerkin method. The effect on the critical number of Rayleigh is also reported from the magnetic field, feedback control, Soret and Dufuor parameter.

Keywords: magnetic field; feedback control; double diffusive; stability; convection

I. INTRODUCTION

Convection in a two and multicomponent system where a horizontal fluid is heated from below has drawn attention in the fluid dynamic field as it may solve many problems arising in engineering field and in natural phenomena, such as in oceanography and astrophysics (Teamah 2008). When a more complex system is taken into account, two or more diffusive elements might exist in the system. For example, thermosolutal exists when two diffusive elements which are the temperature and salinity gradients compete with each other in a fluid. In a binary mixture, Soret (thermo-diffusion) and Dufour (diffusion-thermo) parameter exist in the system. Literature on double diffusive was started by Stommel *et al.* (1956) and later explained by Stern (1960) where they discover the salt fountain phenomenon that occurs when hot salt water lies above cold fresh water. Turner (1974) studied the double diffusive phenomena in lakes, stars, oceans and also in atmosphere.

Temperature dependent viscosity effect is when the viscosity varies exponentially with temperature. Few researchers have included the temperature dependent viscosity in their research. Hilt *et al.* (2014) did a research to include this effect in a binary mixture where they study on the separation ratio which is related to the Soret effect. Currently, Abidin *et al.* (2017) has built their interest to include temperature dependent viscosity in a binary fluid. Rodríguez & Brennecke (2006) studied the coupled temperature and composition dependence of both density and viscosity. The study was done experimentally in a binary mixture which is the water and ionic liquid. To our knowledge, there is still limited research done on the temperature dependent viscosity effect in a binary fluid whereas this effect is important to understand the instability of a convection (Ramírez & Sáez, 1990)

Convection can also be controlled by an external constraint such as the Lorentz force which exists due to the electron magnetic field. Chandrasekhar (1961) did a linear theory in a single component system where his research integrated the magnetic field. It showed that higher magnetic field will stabilize the system since the critical value of Rayleigh number will increase. Shivakumara *et al.*

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(2011) also integrate the magnetic field dependent viscosity parameter in a porous medium where their result shows that an increase of magnetic field dependent viscosity will delay the onset of ferromagnetic convection, but it shows no influence on the critical wave number.

The importance of understanding the feedback control is to stabilize nonstable states or maintaining a state of no-motion so that we can optimize any process involved. It may also help in gaining deeper insights into the dynamics of flow. Tang & Bau, (1998) and showed that it was possible to delay the critical Rayleigh number for the Rayleigh - Bénard convection. Hashim & Siri (2009) showed that feedback control could also delay the onset of Marangoni convection at the bottom with free - slip boundary conditions. In this study, we studied the effects of vertical magnetic field and feedback control in a double diffusive binary layer with temperature dependent viscosity. The linear stability theory is applied and the resulting problem of own value is numerically solved in order to obtain a detailed description of the marginal stability curves for Rayleigh convection.

II. MATHEMATICAL FORMULATION

We consider two horizontal layers of quiescent double diffusive binary fluid with thickness *d* heated from below and we examined the stability of the fluid with the existing of temperature dependent viscosity. The temperature difference between the lower and upper surfaces is represented by ΔT where the lower boundary temperature, T_l is higher than the upper boundary, T_u.

We choose a Cartesian coordinate system with *z* pointing upward and (*x*, *y*) in the horizontal direction at the lower boundary. The magnetic force has an opposite direction to the velocity. We assumed that the physical properties of the fluid are constant for a Boussinesq approximation except the viscosity, μ and density, ρ is taken in the form

$$\mu = \mu_0 exp\left[-\gamma\left(T - T_0\right)\right] \tag{1}$$

$$\rho = \rho_0\left[1 - \chi_t\left(T - T_0\right) + \chi_s\left(S - S_0\right)\right] \tag{2}$$

where μ_0, γ and ρ_0 are the reference values of the dynamic viscosity, mass diffusion constant and the density at the reference temperature, T_0 and the reference concentration, S_0. χ_t is the rate of change density with temperature and χ_s is the rate of change density with concentration. Let the solute concentrations to be taken as $S_0 + \Delta S$ at the lower boundary and S_0 at the upper boundary.

The derivation will start from four governing equations used for the Rayleigh-Benard convection following the analysis by Nield & Kuznetsov (2011), Nanjundappa *et al.* (2013) and Siddheshwar & Pranesh (2002).

$$\nabla \bullet \boldsymbol{v} = 0 \tag{3}$$

$$\rho_0\left[\frac{\partial \boldsymbol{v}}{\partial t} + \left(\boldsymbol{v}\bullet\nabla\right)\boldsymbol{v}\right] = -\nabla p + \nabla\left[\mu(\nabla\boldsymbol{v} + \nabla\boldsymbol{v}^T)\right] -\rho \boldsymbol{g} e_z + \mu_m\left(\boldsymbol{H}\bullet\nabla\right)\boldsymbol{H} \tag{4}$$

$$\left[\frac{\partial T}{\partial t} + \left(\boldsymbol{v}\bullet\nabla\right)T\right] = \kappa\nabla^2 T + D_{TS}\nabla^2 S \tag{5}$$

$$\left[\frac{\partial S}{\partial t} + \left(\boldsymbol{v}\bullet\nabla\right)S\right] = \kappa_s\nabla^2 S + D_{ST}\nabla^2 T \tag{6}$$

$$\left[\frac{\partial \boldsymbol{H}}{\partial t} + \left(\boldsymbol{v}\bullet\nabla\right)\boldsymbol{H}\right] = \left(\boldsymbol{H}\bullet\nabla\right)\boldsymbol{v} + \gamma_m\nabla^2\boldsymbol{H} \tag{7}$$

where the variables are represented as follows; $\boldsymbol{v} = \left(u, v, w\right)$ is the velocity, *p* is the pressure, $\boldsymbol{g}$ is the gravity, e_z is a unit vector in the *z*-direction, $\boldsymbol{H}$ is the magnetic field, κ is the thermal diffusivity, *S* is the solute concentration, κ_s is the solutal diffusivity, D_{ST} is the Soret diffusivity, D_{TS} is the Dufour diffusivity and lastly $\gamma_m = \frac{1}{\mu_m\sigma_m}$ is the magnetic viscosity where μ_m is the magnetic permeability and σ_m is the electrical conductivity. The fluid's basic state is quiescent and is given by:

$$\left(u,v,w\right) = \left(0,0,0\right), p = p_b\left(z\right), \rho = \rho_b\left(z\right),$$
$$S = S_b\left(z\right), T_b\left(z\right) = \left(\frac{T_l + T_u}{2}\right) - \left(\frac{T_l - T_u}{2}\right)\left(z - \frac{d}{2}\right)$$
$$\text{and } \boldsymbol{H}_b\left(z\right) = \boldsymbol{H}_o e_z. \tag{8}$$

where the subscript b denotes the basic state.

In this state, we perturb the system with perturbed variables (denoted by primes) in the following form:

$$u = 0 + u', v = 0 + v', w = 0 + w', T = T_b(z) + T', \\ p = p_b(z) + p', \rho = \rho_b(z) + \rho', S = S_b(z) + S', \\ \boldsymbol{H} = \boldsymbol{H}_b(z) + \boldsymbol{H}'. \quad (9)$$

Using the following definitions,

$$(x,y,z) = \frac{(x^*,y^*,z^*)}{d}, t = \frac{t^*\kappa}{d^2}, p = \frac{pd^2}{\nu\kappa\rho}, \\ (u,v,w) = \frac{d(u^*,v^*,w^*)}{\kappa}, T = \frac{T^* - T_0^*}{\Delta T^*}, \\ S = \frac{S^* - S_0^*}{\Delta S^*}, \bar{f}(z) = \frac{\mu(z)^*}{\mu_0}, \\ (H_x, H_y, H_z) = \frac{(H_x^*, H_y^*, H_z^*)}{H_0^*} \quad (10)$$

and by using equation (8) and equation (9), we obtain the non-dimensional equations:

$$\nabla \bullet \boldsymbol{v}' = 0 \quad (11)$$

$$\frac{1}{Pr}\frac{\partial \boldsymbol{v}'}{\partial t} = -\nabla p + \nabla^2 \boldsymbol{v}' - \rho g e_z + RaT'\hat{e}_z \\ + LeRsS'\hat{e}_z + Q\frac{Pr}{Pr_m}\frac{\partial \boldsymbol{H}'}{\partial z} \quad (12)$$

$$\frac{\partial T'}{\partial t} - w' = \nabla^2 T' + Df\nabla^2 S' \quad (13)$$

$$\frac{\partial S'}{\partial t} - w' = Le\nabla^2 S' + Sr\nabla^2 T' \quad (14)$$

$$\frac{\partial H'}{\partial t} = \frac{\partial w'}{\partial z}\hat{e}_z + \frac{Pr}{Pr_m}\nabla^2 \boldsymbol{H}' \quad (15)$$

$$\nabla \bullet \boldsymbol{H}' = 0 \quad (16)$$

where $Ra = \frac{\rho\chi_S g d^3 \Delta T}{\mu\kappa}$ is the Rayleigh number, $Rs = \frac{\rho\chi_S g d^3 \Delta S}{\mu\kappa_S}$ is the Solutal Rayleigh number, $Le = \frac{\kappa_S}{\kappa}$ is the Lewis number, $Sr = \frac{D_{ST}\Delta T}{\kappa\Delta S}$ is the Soret number, $Df = \frac{D_{TS}\Delta S}{\kappa\Delta T}$ is the Dufour number, $Pr = \frac{\mu}{\rho\kappa}$ is the Prandtl number, $Pr_m = \frac{\mu}{\rho\gamma_m}$ is the magnetic Prandtl number and $Q = \mu_m H_0^2 d^2 \rho / \mu\gamma_m$ is the Chandrasekhar number (asterisks dropped for easier reference).

The equation of the normal form is given by

$$(w', T', S', H') = [W(z), \Theta(z), \eta(z), H(z)] \\ exp[i(a_x x + a_y y)] \quad (17)$$

Substituting equation (17) after taking operators curl on equations (12) and (15) and later together with equations (11) and (16), we obtain the linearized form

$$\bar{f}(D^2 - a^2)^2 W + D^2\bar{f}(D^2 - a^2)W \\ +2D\bar{f}(D^2 - a^2)DW - a^2 Ra\Theta - Lea^2 Rs\eta \\ -H\frac{Pr}{Pr_m}(D - a)^2 DH = 0 \quad (18)$$

$$W + (D^2 - a^2)\Theta + Df(D^2 - a^2)\eta = 0 \quad (19)$$

$$W + Sr(D^2 - a^2)\Theta + Le(D^2 - a^2)\eta = 0 \quad (20)$$

$$(D^2 - a^2)H + \frac{Pr_m}{Pr}DW = 0 \quad (21)$$

where

$a = (a_x^2 + a_y^2)^{\frac{1}{2}}$ represents the wave number, $D = \frac{d}{dz}$ represents the differential operator and $\bar{f}(z) = exp\left[B\left(z - \frac{1}{2}\right) + \frac{(T_0 - T_a)}{\beta d}\right]$ where $B = \gamma\beta d$ is the dimensionless viscosity parameter and T_a represents the average temperature.

Further, it is worth mentioning that Equation (18) and equation (21) can be combined to give

$$\bar{f}(D^2 - a^2)^2 W + D^2\bar{f}(D^2 - a^2)W + 2D\bar{f} \\ (D^2 - a^2)DW - a^2 Ra\Theta - Lea^2 Rs\eta - HD^2 W = 0 \quad (22)$$

The boundary conditions for the temperature conditions were set to be conducting at the lower boundary, $\Theta = 0$ and insulating at the upper boundary, $D\Theta = 0$. For the velocity conditions, boundaries were set to be free-free, rigid-free or rigid-rigid representing the lower-upper boundaries. If boundary has a free-slip, $D^2W = 0$ and if rigid-slip, $DW = 0$. The uniform temperature boundary at the lower boundary, $z = 0$, is restored to include a gain K controller rule,

$$\Theta(0) + K\Theta(1) = 0 \quad (23)$$

The method used to find an approximate system solution is by Galerkin-type weighted residuals method where the series of basis function are

$$W = \sum_{p=1}^{N} A_p W_p, \Theta = \sum_{p=1}^{N} B_p \Theta_p, \eta = \sum_{p=1}^{N} C_p \eta_p \quad (24)$$

A_p, B_p and C_p are constants and the base functions W_p, Θ_p and η_p where $p = 1, 2, 3, \ldots, N$ will be chosen respective to the trial function satisfying the boundary conditions. Later, the Rayleigh number, Ra as the eigenvalue were obtained.

III. RESULTS AND DISCUSSIONS

In this research, we limit the temperature conditions to be conducting at the lower boundary and insulating at the upper boundary. Most past research investigates the case where both boundaries are conducted to temperature. Since no work has been done on different type of boundaries in double diffusive, we make an acceptable comparison with Nield & Kuznetsov (2011) results. Their research on double diffusive convection with both boundaries were set to be conducting to temperature and they obtained the critical Rayleigh number, Ra_c = 1140 which is 3.58% greater than the well-known exact value of 1100.65 in a regular fluid. In this research, we obtained Ra_c = 691.27 which is 3.33% greater than 669.001 obtained by Sparrow *et al.* (1964)in a regular fluid. We also represent the comparison of the critical Rayleigh number for different type of boundaries as shown in Table 1.

Figure 1 represents the Rayleigh number, Ra against wavenumber, a with values of vertical magnetic field, H = 1, 3 and 5. It is found that the marginal stability curves shift upwards as the vertical magnetic field, H increase. The rate of convection is delayed. The applied magnetic field impact in the Lorentz force and stabilize the system (Siddheshwar & Pranesh, 2002).

Table 1. The critical Rayleigh number for regular fluid and present study

Lower-upper boundaries	Regular fluid (Sparrow *et al.*, 1964)	Double Diffusive Binary Fluid (Present study)
	Ra_c	Ra_c
Free-free	-	404.64
Rigid-free	669.001	691.27
Rigid-rigid	-	1446.78

Figure 2 shows Rayleigh number's marginal stability curves, Ra with the wavenumber, a for the feedback control, K = 10, 20, 30 in three types of boundaries values. The other values were set to be H = 2, B = 2, Le = 100, Rs = 100, Sr = 0.5 and Df= 0.5. It shows that as K increases, Ra also increases, thereby stabilizing the system. Essentially, the controller modifies the dynamics of the system improving the disruption of fluid dissipation mechanisms (Tang &Bau, 1998).

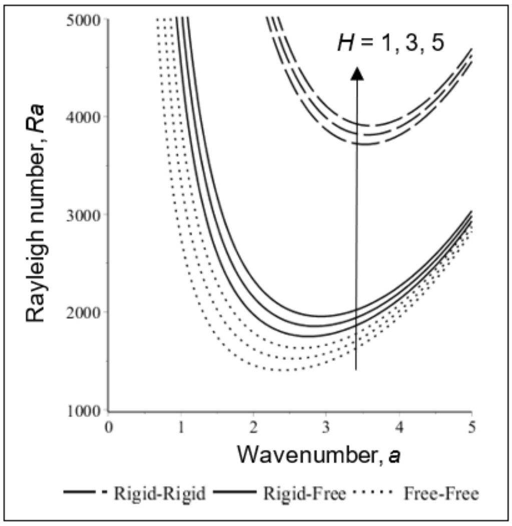

Figure 1. Ra vs. a for different values of H

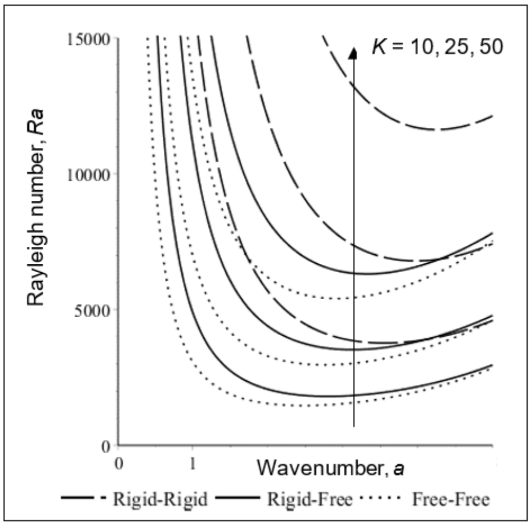

Figure 2. *Ra* vs. *a*for different values of *K*

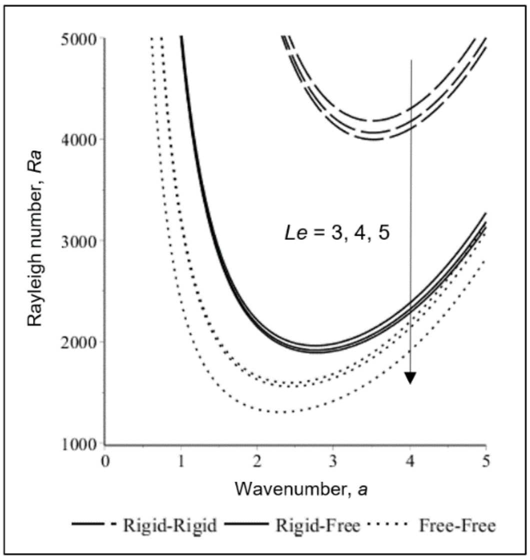

Figure 4. *Ra* vs. *a* for different values of *Le*

Figure 3 indicates the variations of Rayleigh number, *Ra*, with wavenumber, *a*, for different values of temperature dependent viscosity, $B = 1, 2, 2.5$. The marginal stability curves shift downwards when the value of *B* increases, which state that the temperature dependent viscosity destabilized for all wavenumber, *a*.

The illustration of the Lewis number, *Le* can be seen in Figure 4 when *Le* = 3, 4, 5. The Rayleigh number decreases as Lewis number increases and thus enhance the onset of convection in the system. Figure 5 shows that as the solutal Rayleigh number, *Rs* increases, the Rayleigh number also increases. In other words, an increase of *Rs* will stabilize the system.

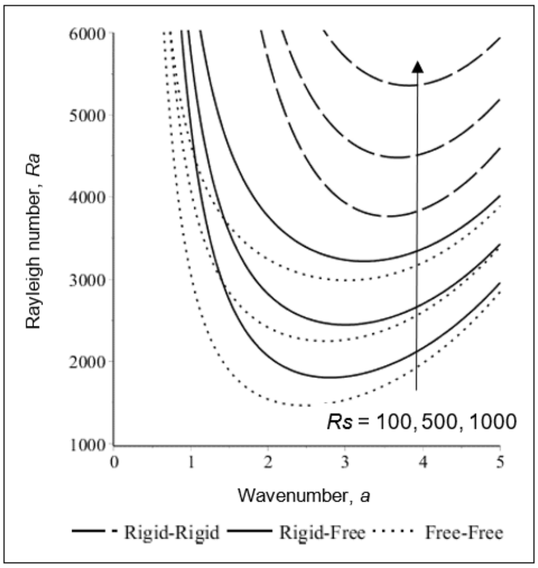

Figure 5. *Ra* vs. *a*for different values of *Rs*

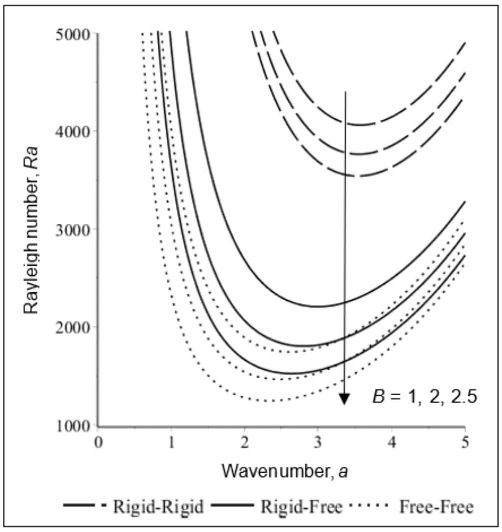

Figure 3. *Ra* vs. *a* for different values of *B*

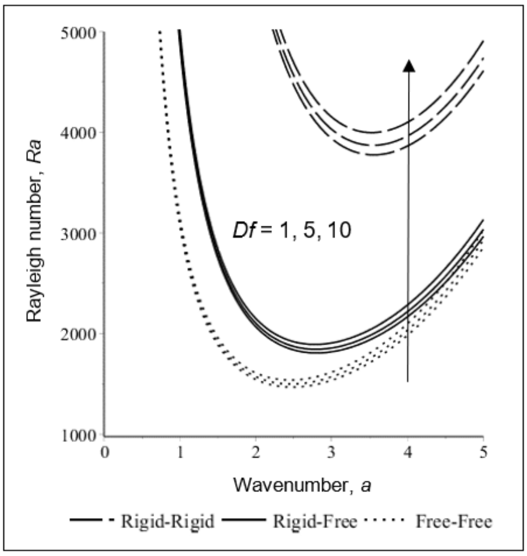

Figure 6. *Ra* vs. *a*for different values of *Df*

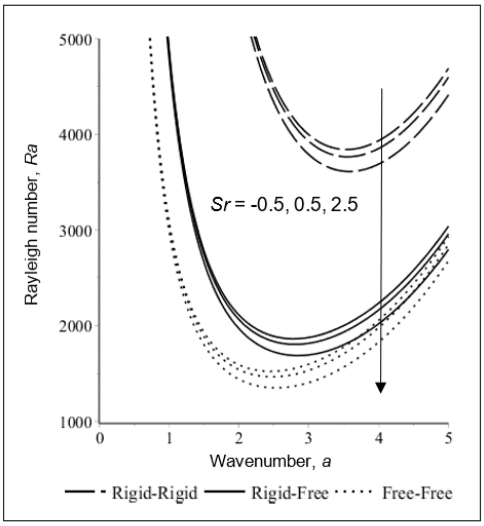

Figure 7. *Ra* vs. *a* for different values of *Sr*

Figure 6 shows that the marginal stability shift upwards when Dufour parameter increase. Dufour parameter which is the diffusion-thermo arises due to the concentration gradient. When the solute concentration increase, it drives the mass gradient of the system and delay the convection to make the system more stabilized. Figure 7 represents the other parameter that exists due to diffusion which is the Soret parameter. The Soret parameter or known as the thermo-diffusion arises due to temperature gradient whereas the temperature flux increases, it expedites the onset of convection to destabilize the system.

IV. SUMMARY

The magnetic field, feedback control and temperature dependent viscosity effect in a double diffusive binary is being examined in this research paper. The results show that the magnetic field, *H* and feedback control, *K* suppress the convection meanwhile the temperature dependent viscosity, *B* augments the convection. The stability of the convection due to the effects that exist in a double diffusive binary fluid which are the Lewis number, solutal Rayleigh number, Dufour and Soret number is also investigated. It is found that Lewis number, *Le* and Soret number, *Sr* destabilize the system, when both effects were increased. The other effects which are the Solutal Rayleigh number, *Rs* and Dufuor number, *Df*, make the system more stable when the values increase.

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