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An Enhanced Mean-Extended Gini Model in Portfolio Optimization with Different Level of Risk Aversion

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The investors intend to minimize the risk and can achieve the expected rate of return in their investment. The mean-extended Gini model has been introduced in portfolio optimization to minimize the portfolio risk at the expected rate of return. The portfolio expected return is measured by the portfolio mean return while the portfolio risk is measured by the portfolio extended Gini. The investors show different level of risk aversion in the investment which are high risk aversion level and low risk aversion level. The extended Gini is the appropriate risk measure for the investors with different level of risk aversion in portfolio optimization. The cardinal constraint is an important constraint in portfolio optimization because the investors can determine the number of stocks to be invested in the optimal portfolio so it will be more practical for the investors. However, the cardinal constraint has not been considered in the existing mean-extended Gini model. The objective of this study is to propose an enhanced mean-extended Gini model by incorporating the cardinal constraint into the existing mean-extended Gini model. The enhanced mean-extended Gini model with cardinal constraint is employed in this study to construct the optimal portfolio for investors with the high-risk aversion level and low risk aversion level. The data of this study comprises the weekly returns of stocks that listed in Malaysian stock market. This study will give significant impact to the investors with high risk aversion level and low risk aversion level because they can minimize the portfolio risk at the expected rate of return as well as determine the number of stocks in their investment with the proposed enhanced mean-extended Gini model.

Keywords: risk aversion; mean return; cardinal constraint; optimal portfolio

I. INTRODUCTION

Risk and return are two important criteria to be considered in the issue of portfolio management. Portfolio management issues have been studied in the past research (Annuar *et al.* 1997; Jurczyk *et al.* 2016; Mohamed *et al.* 2009; Saiful *et al.* 2013; Saiful *et al.* 2014; Shinzato & Yasuda 2015). Investors wish to minimize the risk and can achieve the target rate of return. The investors exhibit different level of risk aversion in their investment. The high-risk aversion and low risk aversion investors intend to find the trade-off between the risk and return. The mean-extended Gini model has been studied by the past researchers in portfolio management (Butterworth & Holmes 2005; Lam & Lam 2016; Ringuest *et al.* 2004; Shalit & Greenberg 2013; Shalit & Yitzhaki 1989; Shalit & Yitzhaki 2005). The expected return of the investors is

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represented by the portfolio mean return while the extended Gini is used as the portfolio risk measure in this model. Extended Gini is the risk measure that can include the preference of investors towards risk. The mean-extended Gini model is appropriate for the high-risk aversion and low risk aversion investors in portfolio management because the investors can minimize the portfolio risk at the target rate of return. However, the cardinal constraint is not taken into consideration in the existing mean-extended Gini model. The investors can determine the number of stocks to be invested in the optimal portfolio with the cardinal constraint to make it more realistic and practical for investors (Beasley *et al.* 2003; Canakgoz & Beasley 2008; Guastaroba & Speranza 2012; Lam *et al.* 2014; Lam *et al.* 2015; Mezali & Beasley 2013). The investors will be able to manage the transaction cost by determining the number of stocks to be invested in the optimal portfolio with the cardinal constraint. Hence, the cardinal constraint is needed to be taken into consideration in the existing mean-extended Gini model. The objective of this paper is to propose an enhanced mean-extended Gini model by integrating the cardinal constraint into the existing model. The optimal portfolio for the high-risk aversion and low risk aversion investors are constructed in this study by using the enhanced mean-extended Gini model with cardinal constraint.

II. MATERIALS AND METHODS

The data of this study consists of the weekly returns of 20 stocks that listed in Malaysia stock market construction sector. The Bursa Malaysia construction sector is an important industry in contributing to the economy growth of Malaysia as construction industry is an economic investment and correlated with the economic development (Olanrewaju & Abdul-Aziz 2015).The period of this study covers from July 2011 until June 2016. An enhanced mean-extended Gini model is proposed in this study by integrating the cardinal constraint into the existing mean-extended Gini model to make it more practical and realistic for investors. The investors will be able to determine the number of stocks to be invested in the optimal portfolio with the cardinal constraint. Table 1 displays the summary statistics of the 20 stocks returns in this study which are mean (M), standard deviation (SD), skewness (S) and kurtosis (K).

Table 1. Summary statistics of the 20 stocks returns

Stock	M	SD	S	K
ASUPREM	0.0010	0.0918	0.1361	2.9697
AZRB	0.0009	0.0476	2.5456	14.0829
BENALEC	-0.0031	0.0457	0.2797	1.4396
BPURI	-0.0038	0.0372	0.3916	2.1582
CRESBLD	0.0014	0.0439	3.2929	24.7405
EKOVEST	-0.0006	0.0480	-3.7395	43.6247
FAJAR	-0.0019	0.0354	0.3483	1.8876
GADANG	0.0054	0.0498	1.6259	5.3104
GAMUDA	0.0012	0.0294	0.8359	5.7328
HSL	0.0005	0.0329	1.2212	6.1952
IJM	-0.0013	0.0405	-7.4293	92.6757
JAKS	0.0029	0.0580	0.9512	3.1958
KEURO	0.0005	0.0421	1.2848	6.8147
KIMLUN	0.0008	0.0400	0.7264	5.9849
MITRA	0.0041	0.0532	-0.7646	7.3628
MUDAJYA	-0.0041	0.0493	0.0537	5.3372
MUHIBAH	0.0034	0.0578	1.0180	5.4558
PRTASCO	0.0027	0.0402	0.7073	1.9684
PUNCAK	-0.0006	0.0669	0.5093	13.7999
WCT	-0.0018	0.0411	0.0878	2.6896

As reported in Table 1, the 20 stocks return show that the values of mean, standard deviation, skewness and kurtosis are different for each stock. GADANG gives the highest mean return value at 0.0054. Besides that, ASUPREM gives the highest standard deviation value at 0.0918. CRESBLD gives the highest skewness value at 3.2929 while IJM gives the highest kurtosis value at 92.6757. Investors prefer high skewness and low kurtosis value because it will reduce the probability of getting loss.

The mathematical model of the existing mean-extended Gini model is formulated as follows:

$$\text{Minimize } -\upsilon \sum_{i=1}^{n} w_i \operatorname{cov}\{x_i, [1-F_p(p)]^{\upsilon-1}\} \quad (1)$$

Subject to

$$E(p) = \sum_{i=1}^{n} w_i E(x_i) \quad (2)$$

$$\sum_{i=1}^{n} w_i = 1 \tag{3}$$

$$w_i \geq 0, i = 1,...,n \tag{4}$$

The mathematical model of the proposed enhanced mean-extended Gini model by incorporating the cardinal constraint is formulated as follows:

$$\text{Minimize } -\upsilon \sum_{i=1}^{n} w_i \operatorname{cov}\{x_i, [1-F_p(p)]^{\upsilon-1}\} \tag{5}$$

Subject to

$$E(p) = \sum_{i=1}^{n} w_i E(x_i) \tag{6}$$

$$\sum_{i=1}^{n} w_i = 1 \tag{7}$$

$$w_i \geq 0, i = 1,...,n \tag{8}$$

$$\sum_{i=1}^{n} z_i = K \tag{9}$$

$$L_i z_i \leq w_i \leq U_i z_i, \quad i = 1,...,n \tag{10}$$

$$z_i \in [0,1], \quad i = 1,...,n \tag{11}$$

where υ is a parameter determining the relative weight attributed to various portions of the probability distribution, $F_p(p)$ is the cumulative probability distribution of the portfolio returns p, w_i is the weight invested in asset i, x_i is the return of asset i, z_i is the variable that equals to 1 if the asset i is selected in the portfolio and equals to 0 otherwise, K is the number of assets to be selected in the portfolio, L_i is the lower bound of the investment proportion on asset $i$and U_i is the upper bound of the investment proportion on asset i. Objective function (1) and (5) define the portfolio extended Gini. Constraint (2) and (6) imply that the investors can achieve the expected rate of return. Constraint (3) and (7) imply that the sum of weights of the assets equals to one. Constraint (4) and (8) imply that the weights of all the assets are positive. Constraint (9) is the incorporated cardinal constraint to ensure that the number of assets to be selected in the portfolio equals to K. The variable z_i is introduced to indicate the asset selection problem with z_i = 1 indicates the ith asset is selected in the portfolio or otherwise z_i = 0 for the constraint (10) and (11). The optimal portfolio for the high risk aversion (υ = 6) and low risk aversion (υ = 2) investors are constructed by using the proposed enhanced mean-extended Gini model with cardinal constraint (5)-(11). Larger υ indicates higher risk aversion. The optimal portfolio composition for the high risk aversion and low risk aversion investors are compared in this study. Besides that, the optimal portfolio mean return and portfolio extended Gini for the high risk aversion and low risk aversion investorsare generated. The K number of stocks is set as 6 in this study as numerical illustration.

III. RESULTS AND DISCUSSIONS

The optimal portfolio composition in percentage for the high-risk aversion and low risk aversion investors are presented in Table 2.

Table 2. Optimal portfolio compositions in percentage

Stock	High risk aversion (%)	Low risk aversion (%)
ASUPREM	3.06	0.00
AZRB	0.00	0.00
BENALEC	0.00	0.00
BPURI	0.00	0.00
CRESBLD	0.00	11.22
EKOVEST	0.00	0.00
FAJAR	0.00	0.00
GADANG	10.09	0.00
GAMUDA	34.42	30.88
HSL	0.00	21.37
IJM	30.90	11.19
JAKS	0.00	0.00
KEURO	7.37	8.02
KIMLUN	0.00	0.00
MITRA	0.00	0.00
MUDAJYA	0.00	0.00
MUHIBAH	0.00	0.00

PRTASCO	14.15	17.31
PUNCAK	0.00	0.00
WCT	0.00	0.00

As shown in Table 2, the weight of stocks to be invested in the optimal portfolio for high risk aversion investors and low risk aversion investors are different. The optimal portfolio for high risk aversion investors consists of ASUPREM (3.06%), GADANG (10.09%), GAMUDA (34.42%), IJM (30.90%), KEURO (7.37%) and PRTASCO (14.15%). AZRB, BENALEC, BPURI, CRESBLD, EKOVEST, FAJAR, HSL, JAKS, KIMLUN, MITRA, MUDAJYA, MUHIBAH, PUNCAK and WCT are not selected to be invested in the optimal portfolio for high risk aversion investors because they give the value 0.00%. In contrast, the optimal portfolio for low risk aversion investors consists of CRESBLD (11.22%), GAMUDA (30.88%), HSL (21.37%), IJM (11.19%), KEURO (8.02%) and PRTASCO (17.31%). ASUPREM, AZRB, BENALEC, BPURI, EKOVEST, FAJAR, GADANG, JAKS, KIMLUN, MITRA, MUDAJYA, MUHIBAH, PUNCAK and WCT are not selected to be invested in the optimal portfolio for low risk aversion investors because they give the value 0.00%. It implies that different risk aversion level of investors will give different optimal portfolio composition.

Figure 1 and Figure 2 present the optimal portfolio composition for high risk aversion investors and low risk aversion investors respectively from highest to lowest percentage.

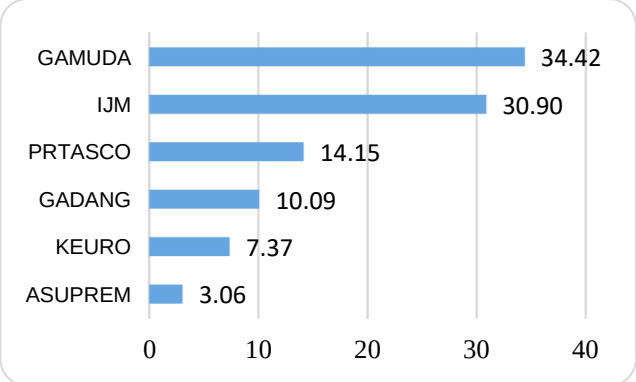

Figure 1. Optimal portfolio composition for high risk aversion investors (%)

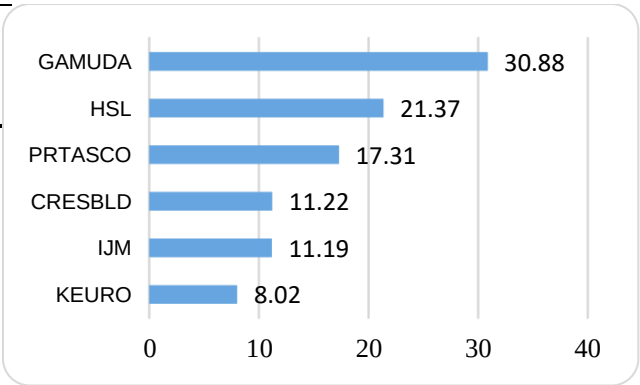

Figure 2. Optimal portfolio composition for low risk aversion investors (%)

Table 3 shows the summary statistics of the optimal portfolio for the high risk aversion and low risk aversion investors.

Table 3. Summary statistic of the optimal portfolio

Optimal portfolio	High risk aversion	Low risk aversion
Portfolio Mean Return	0.0010	0.0010
Portfolio Extended Gini	0.0281	0.0113

As reported in Table 3, the optimal portfolio for high risk aversion investors gives the portfolio mean return at 0.0010 and portfolio extended Gini at 0.0281. On the other hand, the optimal portfolio for low risk aversion investors gives the portfolio mean return at 0.0010 and portfolio extended Gini at 0.0113. It implies that the highrisk aversion and low risk aversion investors can minimize the portfolio risk as well as achieve the target rate of return and can determine the number of stocks to be invested in the optimal portfolio with the proposed enhanced mean-extended Gini model with cardinal constraint. The portfolio risk of the high-risk aversion and low risk aversion investors in their investment of stocks is measured by the portfolio extended Gini.

IV. CONCLUSION

In conclusion, an enhanced mean-extended Gini model is proposed and discussed in this study by incorporating the cardinal constraint into the existing mean-extended Gini model. The cardinal constraint will be more practical and

realistic for the investors in the portfolio management as they can determine the number of stocks to be invested in the optimal portfolio. The proposed model is appropriate in the investment risk management for the both high risk aversion and low risk aversion investors. This study is significant because the high-risk aversion and low risk aversion investors can minimize the portfolio risk at the target rate of return as well as determine the number of stocks in their investment with the proposed enhanced mean-extended Gini model with cardinal constraint. This research is the pioneer study in Malaysia. The future research of this study should be extended to the stock market of other countries for portfolio diversification.

V. ACKNOWLEDGEMENT

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