

# Overhead Reduction by Channel Estimation Using Linear Interpolation for SC-FDE Transmission

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We propose a new transmission structure for single carrier – frequency domain equalization (SC-FDE) to reduce pilot overhead. The proposed structure transmits multiple data frames between two pilot frames. At the receiver, two channel estimates for the pilot frames are obtained first, and the channels for the data frames are found by linear interpolation of the two channel estimates from the pilots. Computer simulation shows that the proposed structure reduces the pilot overhead significantly while reliable channel estimation and equalization are possible in time-varying fading channels.

**Keywords:** equalization; channel estimation; linear interpolation; pilot overhead; SC-FDE

## I. INTRODUCTION

The orthogonal frequency division multiplexing (OFDM) transmission frequently used in a broadband wireless communication system involves dividing the entire frequency band into numerous narrowband subchannels and simultaneously transmitting data in parallel (Nee & Prasad 2000). In OFDM transmission, we generate quadrature amplitude modulation (QAM) and phase shift keying (PSK) modulation signals, convert the modulated signal via inverse fast Fourier transform (IFFT), and finally, transmit this converted signal. OFDM is used as a transmission method in many wideband radio communication systems such as IEEE 802.11 wireless LAN, fourth-generation wireless network long-term evolution (LTE) systems because it is suitable for high-speed data transfer (Abdelrahman *et al.*, 2015; Abed *et al.*, 2012; 3GPP TS 36.201, 2007). Moreover, when

compensating for channel distortion due to frequency selective fading channels, the processes of equalization and computation are relatively simple since frequency-domain equalization can be used. For multi-path fading channels with long delay spread, frequency-domain equalization is much simpler than time-domain equalization. However, OFDM signals have large peak-to-average power ratio (PAPR), and OFDM transmission is sensitive to interchannel interference (ICI) owing to carrier frequency offsets (Karam & Sari, 1989; Moose, 1994; Wang & Giannakis, 2000). The high PAPR lowers the transmission power efficiency and increases the nonlinear distortion caused by high-power amplifiers, making it difficult to apply OFDM for battery-powered communication equipment such as cell phones and military communication devices (O'Neill & Lopes, 1995; Tellado, 2000).

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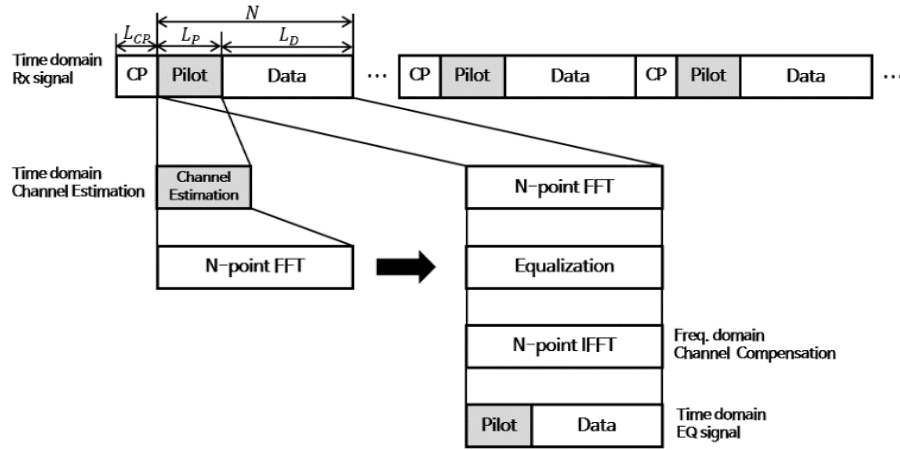


Figure 1. Conventional SC-FDE transmission structure

Recently, high-speed data communication affected by such problems has been actively researched, and new transmission techniques for broadband wireless communication systems have been proposed. Among them, single carrier - frequency-domain equalization (SC-FDE) transmission method is employed to solve the PAPR problem of OFDM (Falconer *et al.*, 2002). SC-FDE, which is a system using FDE, is similar to OFDM. Both methods are performed in block units in digital transmission and rely on FFT and IFFT operations. In addition, frequency domain equalization is used. SC-FDE has similar complexity to OFDM without the need for high-precision frequency synchronization and linear power amplification (Pancaldi *et al.*, 2008). Therefore, SC-FDE method has relatively low PAPR, enabling the use of SC-FDE as an alternative to the OFDM method.

However, in order to perform channel estimation, the conventional SC-FDE transmission structure transmits a pilot in every data frame, resulting in a huge pilot overhead. In this paper, we modify the pilot transmission method and propose a new channel estimation/equalization technique. Specifically, instead of transmitting the pilot in every data frame, the proposed structure transmits the pilot once every several frames. Thus, the proposed structure has the advantage of low pilot overhead. To estimate the channel using reduced pilots, linear interpolation is introduced. In detail, during the pilot period, the receiver performs channel estimation, and the channels between the pilots are obtained via linear interpolation of the channel estimates in the pilot periods. By doing this, channel estimation under time-varying mobile environments can be possible. The channel estimation and equalization performances are verified through computer simulation. The simulation results confirm that the proposed method can reduce the pilot overhead to

1/10 of the conventional techniques while provide negligible performance loss in channel estimation and equalization.

The structure of this paper is as follows. The structure of the conventional SC-FDE, channel estimation, equalization, and the problem of overhead caused by pilots are explained in Section II. Section III proposes a new SC-FDE structure that can solve the aforementioned problem. Section IV presents the environments of simulation experiments and the performance of the proposed SC-FDE. Section V concludes this paper.

## II. CONVENTIONAL TRANSMISSION STRUCTURE OF SC-FDE

Figure 1 shows the conventional SC-FDE transmission structure. The SC-FDE system performs frequency-domain equalization by a method similar to OFDM, but it differs from OFDM in that there is no IFFT block at the transmitter. The transmitter structure is more similar to the conventional single carrier transmitter than to the OFDM transmitter. In Figure 1, the length of the transmission data is  $L_D$ ; the length of the pilot symbol for channel estimation and equalization is  $L_P$ ; and the length of the cyclic prefix (CP) is  $L_{CP}$ . The total length of  $L_D$  data and  $L_P$  pilot is  $N$ , i.e.,  $N = L_D + L_P$ , where  $N$  is the FFT size for the frequency domain equalization. IFFT is not required at the transmitter, but the block size  $N$  should be chosen carefully because channel distortion can be compensated only when the channel does not change in  $N$

time period. The CP is the copy of the end of the data block, and it converts the linear convolution between the Tx signal and the channel into circular convolution, enabling frequency-domain equalization at the receiver. The CP should be longer than the channel impulse response. Therefore, it is necessary to appropriately set the lengths of  $L_{CP}$ ,  $N_F$  and  $L_p$  in order to perform normal channel estimation, equalization, and data recovery with affordable pilot and CP overheads.

The equalization process at the receiver is shown in Figure 2. First, it removes the CP and estimates the time-domain channel. Let  $d(k), k = 0, \dots, N-1$  be the transmitted signal, and  $s(k), k = 0, \dots, N-1$  be the received signal after CP removal. The length of the channel is  $L_h$ , and the channel impulse response is denoted by  $h(k), k = 0, \dots, L_h-1$ . Then, the received signal can be written as follows.

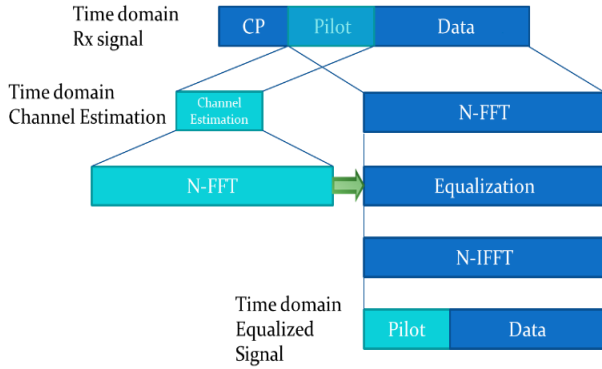


Figure 2. Receiver structure for the conventional SC-FDE transmission structure

$$\begin{aligned} s(k) &= \sum_{q=0}^{L_h-1} h(q)d(k-1) + n(k) \\ &= \mathbf{d}^T(k)\mathbf{h} + n(k) \end{aligned} \quad (1)$$

Where  $\mathbf{d}(k) = [d(k), d(k-1), \dots, d(k-L_h+1)]^T$  and  $\mathbf{h} = [h(k), h(k-1), \dots, h(L_h+1)]^T$ . In (1),  $n(k)$  represents Gaussian noise. In the conventional SC-FDE systems, channel estimation can be performed as follows: Unlike OFDM, the conventional SC-FDE uses a fixed time domain pilots, not frequency domain pilots. To obtain frequency domain channel using the time domain pilots, time domain channel estimation is performed first, and then convert it to the frequency-domain channel by FFT. Suppose that the pilot symbol in the receive block is  $s(k), 0 \leq k \leq L_p$ , and the pilot symbol in the transmit block is  $d(k), 0 \leq k \leq L_p$ . The received pilot can be expressed as follow.

$$s(k) = \mathbf{d}^T(k)\mathbf{h} + n(k), k = 0, \dots, L_p-1 \quad (2)$$

The channel impulse response vector,  $\mathbf{h}$ , can be found using any estimation method, and this paper employs the least squares. The least squares estimation of  $\mathbf{h}$  is obtained as follows.

$$\begin{aligned} \hat{\mathbf{h}} &= (\mathbf{D}_p^H \mathbf{D}_p)^{-1} \mathbf{D}_p^H \mathbf{s}_p \\ \mathbf{D}_p &= [\mathbf{d}(0), \mathbf{d}(1), \dots, \mathbf{d}(L_p-1)]^T \\ \mathbf{s}_p &= [s(0), s(1), \dots, s(L_p-1)]^T \end{aligned} \quad (3)$$

Thereafter, FFT of the received signal is performed for channel equalization. Channel equalization employs MMSE equalization to reduce the amplification of noise without incurring large loss in the low signal-to-noise ratio (SNR) region. The MMSE equalization performs as

$$\hat{D}(k) = \frac{S(k)\hat{H}^*(k)}{|\hat{H}(k)|^2 + \sigma_z^2} \quad (4)$$

Where  $\sigma_z^2$  indicates the variance of noise. We assume that the value of the variance is known. After frequency domain equalization, time domain equalized signals can be obtained via IFFT of  $\hat{D}(k)$ .

In the conventional SC-FDE transmission structure, it is necessary to insert a CP for each SC-FDE symbol and a pilot to perform channel estimation and equalization. Since CP and pilot are needed for each transmitted SC-FDE symbol, the overhead due to the pilot and CP is considerably large. In general, CP length is determined by the length of the channel impulse response which makes reduction of CP overhead difficult. This paper focuses on pilot overhead reduction. In order to decrease the pilot overhead, we propose a new SC-FDE structure. The pilot overhead of the conventional SC-FDE transmission structure can be expressed as

$$\text{pilot overhead} = \frac{L_p}{L_p + L_D} \quad (5)$$

The pilot overhead is defined by ratio of pilot length to the total transmission signal length. If a longer pilot is used, channel estimation performance improves, but the pilot overhead also increases. In the conventional structure, pilots are transmitted for each SC-FDE frame, which imposes a large pilot overhead.

To overcome this problem, the overhead of the pilot is reduced by sending multiple data between two neighbouring pilots. This structure will be explained in the next section.

pilots at both ends is the same as that in the conventional structure. We use RLS algorithm for time domain channel estimation (Toumazou *et al.*, 1996).

The operation of the receiver will be described in detail. First, channel estimation is performed by using two pilots in the time domain. Next, FFT is performed for each of the two channel coefficients found in the channel estimation, and the coefficients are converted into the frequency domain. The index of the first pilot frame is denoted as 0, that of the second pilot frame as  $M + 1$ , and those of the data frames as  $1, \dots, M$ . We assume that the

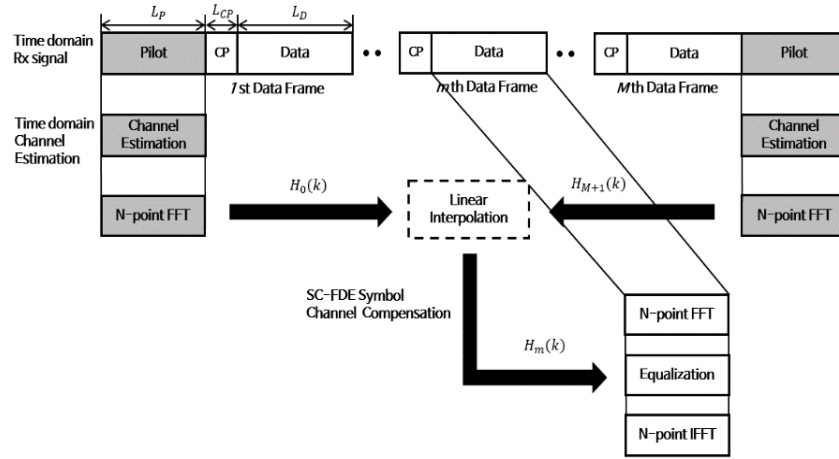


Figure 3. Structure of the proposed SC-FDE transmission structure

### III. PROPOSED SC-FDE TRANSMISSION STRUCTURE

The SC-FDE transmission structure proposed in this paper is shown in Figure 3. The proposed structure, unlike the conventional structure, transmits multiple SC-FDE symbols between two pilots (Xiaodai *et al.*, 2017). Therefore, it differs from the conventional structure that transmits pilot at each SC-FDE symbol. Due to the new structure, the pilot overhead can be significantly lowered compared to that in the conventional structure. The operation of the proposed structure is as follows. First, the channel estimation is performed by using the two pilots at both ends from the received signal. If the channel estimated at a pilot is used for equalization until the next pilot, the channel compensation may not be achieved well because the channel is time-varying in mobile environments. To cope with the time-varying environments, linear interpolation is performed to find channel coefficients for SC-FDE data frames between two pilots. The method of performing channel estimation using

channel estimates at  $k$ -th subcarrier of the first and second pilot frames are denoted by  $H_0(k)$  and  $H_{M+1}(k)$ , respectively. Using linear interpolation, we determine the frequency-domain channel coefficient of the  $M$  data frames. Specifically, the channel coefficient of the  $m$ -th data frame is obtained as follows.

$$\hat{H}_m(k) = H_0(k) + \frac{m(H_{M+1}(k) - H_0(k))}{M+1}, 0 \leq k \leq N-1 \quad (6)$$

Using the channel coefficients of each data item found via linear interpolation, channel equalization applies MMSE for channel equalization. A transmission signal that has undergone channel equalization in the frequency domain is presented as  $\hat{X}_m(k)$ .

$$\hat{X}_m(k) = \frac{Y_m(k)\hat{H}_m(k)}{|\hat{H}_m(k)|^2 + \sigma_z^2} \quad (7)$$

$Y_m(k)$  is a frequency-domain received signal at the  $k$ -th subcarrier;  $\hat{H}_m(k)$  is the channel coefficient at the  $k$ -th subcarrier; and  $\sigma_z^2$  noise variance.  $\hat{X}_m(k)$  for  $k = 0, \dots, N -$

1 are converted into the time-domain equalized signal via the  $N$ -point IFFT. Specifically, the equalized signal of the  $m$ -th data frame in the time domain is as follows.

$$\hat{x}_m(k) = \sum_{t=0}^{N_F} \hat{X}_m(t) \exp\left(j2\pi \frac{kt}{N_F}\right), k = 0, \dots, N_F - 1 \quad (8)$$

The overhead by the pilot of the proposed structure can be expressed as follows.

$$\text{pilot overhead} = \frac{L_p}{L_p + ML_D} \quad (9)$$

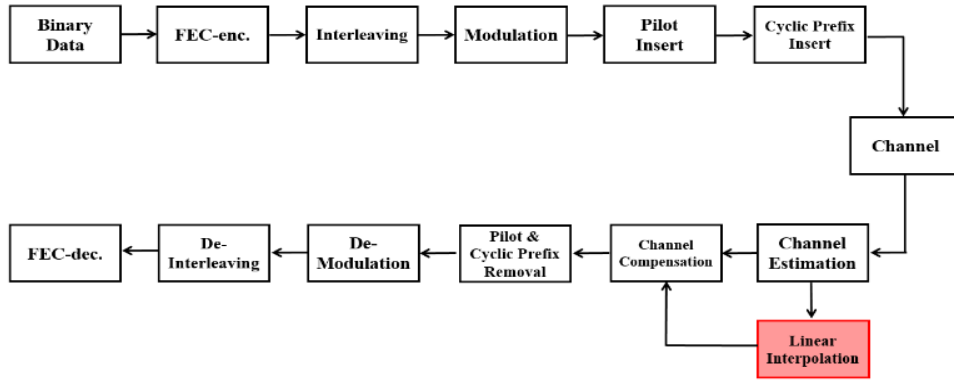


Figure 4. Proposed SC-FDE simulation block diagram

If  $L_p \gg L_D$  in the above equation, the proposed structure can reduce the pilot overhead by  $1/M$  compared to that of the conventional structure in (5). Therefore, fewer pilots than that in the conventional structure can be transmitted, and reliable channel estimation and equalization can be performed using linear interpolation in time-varying environments. In addition, since many data frames are inserted and transmitted between pilots, a large amount of data can be transferred.

However, the number of data frames that can be sent between two pilots should be determined according to the temporal channel change caused by the moving speed (or The Doppler effect). If the speed is low, many data frames can be sent between the pilots, whereas if the speed is high, only a few data frames can be sent. The number of transmittable data frames depending on the moving speed was determined via computer simulation. Therefore, the pilot overhead of the proposed scheme can be reduced if the speed is low, while it cannot be reduced much if the speed is high. In this research, the proposed SC-FDE method was simulated using MATLAB (Lee, 2017).

#### IV. SIMULATION RESULTS

Figure 4 is a simulation block diagram of the proposed SC-FDE. First, binary data is generated, and FEC encoding, interleaving, and modulation are performed. Next, insert a pilot on both sides of the data frame, and send it with CP inserted. The receiver executes channel estimation on a transmission frame passing through the channel, and channel equalization using a linear interpolation method. Finally, we verify the performance of the proposed SC-FDE after inverse processing of the transmitter structure.

In order to compare the performance of the SC-FDE transmission structure proposed in this paper, computer simulation was performed to verify the performance. In the first simulation, channel estimation is performed using only the first pilot and channel equalization of all the data frames is based on the estimated channel. In the second simulation, the channel coefficients for the data frames between two pilots are obtained through the linear interpolation using two pilots, and verify the equalization performance. The performance of the above two is confirmed by simulation. Table 1 shows simulation environments.

Table 1. Simulation environments

Parameters	Value
Channel model	Rician model with 7taps, K-factor = 14 [dB]
Modem type	QPSK
Channel code	LDPC, $r = 1/2$
Pilot length	128 [symbols]
CP length	24
DATA length	1024 [bits]
FFT size	512

Velocity[km/h]	100, 200, 300
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The channel estimation and bit error ratio (BER) performances were verified via computer simulation. Table 1 lists the simulation environments. For the channel model, we used the Rician model of 7 taps and the value of K-factor at 14 dB. We used the quadrature phase shift keying (QPSK) as a modulation scheme and  $R = 1/2$  LDPC (Low-Density Parity-Check) code for the channel code. The LDPC code has an advantage that it can perform high-speed processing and has lower decoding complexity than the turbo code (Gallager, 1962; MacKay & Neal, 1996). The lengths of the pilot, CP, and data were 128 symbols, 24 bits, and 1024 bits, respectively. The FFT size was 512.

#### A. Without Channel Interpolation

In this section, performance of the channel equalization using only the first pilot is shown with various moving speeds.

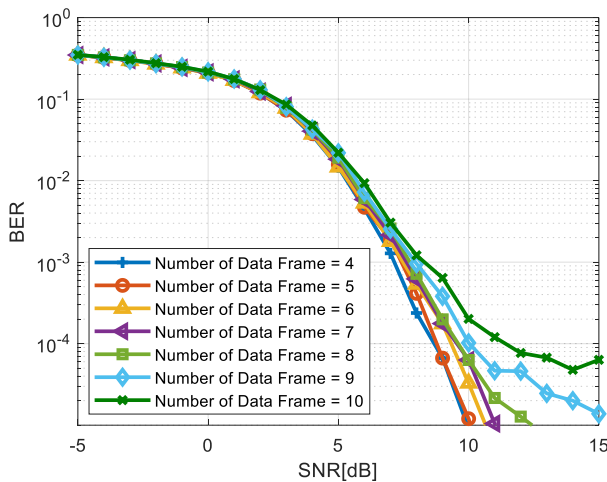


Figure 5. BER performance without channel interpolation when the velocity is 100 km/h

Figure 5 shows the BER curve corresponding to the number of data frames between two pilots without the linear interpolation for channel estimation. The moving speed is 100 km/h. The BER curves confirm that the BER decreases when SNR increases when transmitting 7 or fewer data frames. However, the performance degradation is observed for 8 or more data frames. This results indicate that linear interpolation can support for 7 data frames at 100 km/h.

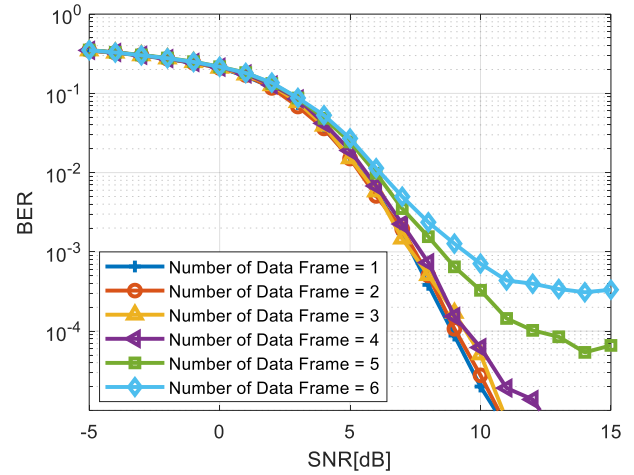


Figure 6. BER performance without channel interpolation when the velocity is 200 km/h

Figure 6 shows the BER curve corresponding to the number of data frames between two pilots without using the linear interpolation of channel estimation when the moving speed is 200 km/h. The BER curves confirms that the BER decreases when SNR increases when transmitting 4 or fewer data frames. Performance degradation occurs for 5 or more data frames. Due to the faster change of the channel, the linear interpolation can work for 5 data frames.

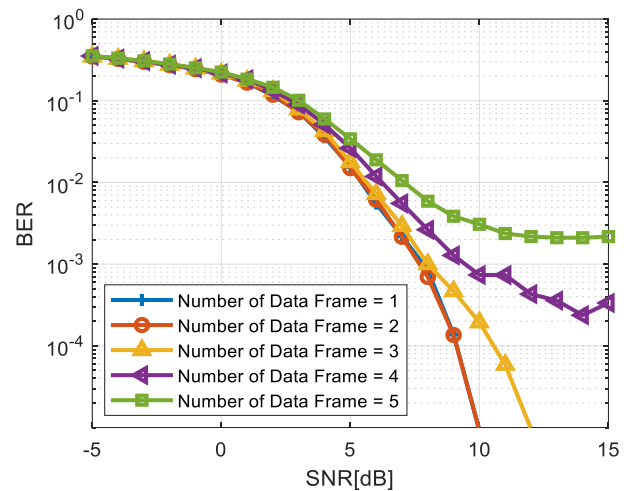


Figure 7. BER performance by the number of transmitted data frames when the velocity is 300 km/h

Figure 7 shows the BER curve corresponding to the number of data frames between two pilots without using the linear interpolation of channel estimation when the moving speed is 300 km/h. The BER curves shows that the BER decreases when SNR increases when transmitting 3 or fewer data frames. Performance



degradation happens for 4 or more data frames. From those results, it can be seen that the pilot overhead does not decrease a lot without channel interpolation.

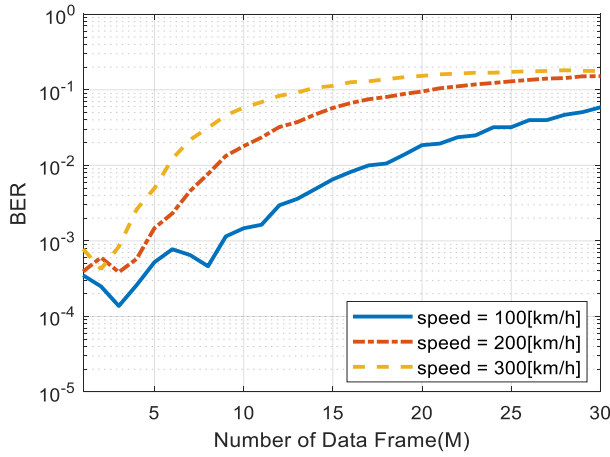


Figure 8. BER performance by the number of data frames when the SNR is 8dB

Figure 8 shows the BER performance for three moving speeds by the number of data frames transmitted between pilots for a fixed SNR of 8 dB. The BER performances show that when the moving speed increases, the performance degradation is remarkable when the number of data frames is large. This indicates that there is a limit on the number of data frames that allow reliable channel estimation and equalization processing for high moving speeds. Also, when the channel equalization is performed without using the linear interpolation of channel estimation, it can be seen that the number of data frames is low that can be transmitted without degrading performance.

### B. With Channel Interpolation

In this section, performance of channel equalization through linear interpolation channel estimation using two pilots is shown according to each speed.

Figure 9 shows the channel coefficients of the data frame transmitted between two pilots using linear interpolation after estimating the channel using two pilots. Among 512 FFT indices, only 256 indices are shown for convenience. In the simulation, the number of data frames is 5 ( $M = 5$ ). It can be confirmed that the channel coefficients are equally spaced at data frames by the linear interpolation of two channel estimates in pilots.

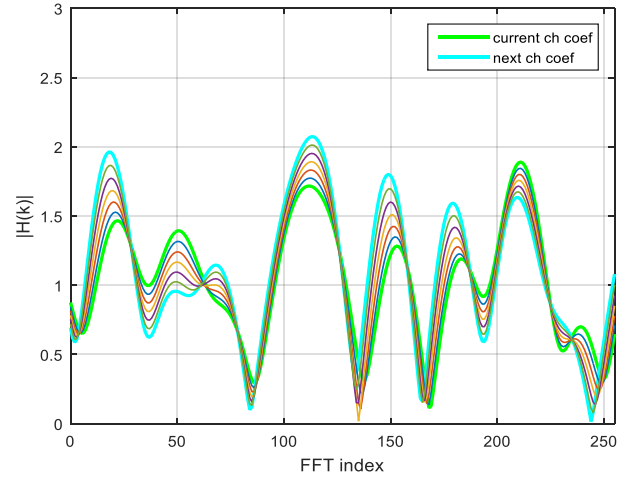


Figure 9. Channel coefficient

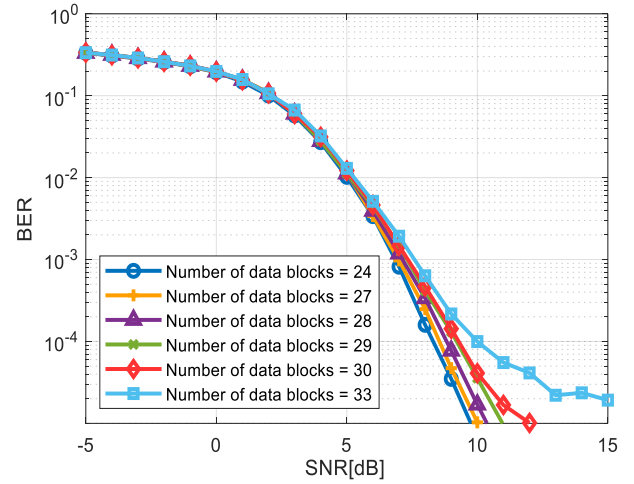


Figure 10. BER performance by the number of transmitted data frames when the velocity is 100 km/h

Figure 10 shows the BER curve corresponding to the number of data frames between two pilots using the linear interpolation of channel estimation when the moving speed is 100 km/h. Comparison of BER curves shows that the BER curves coincide when SNR increases when transmitting 28 or fewer data frames. Further, the performance degradation occurs for 29 or more data frames.

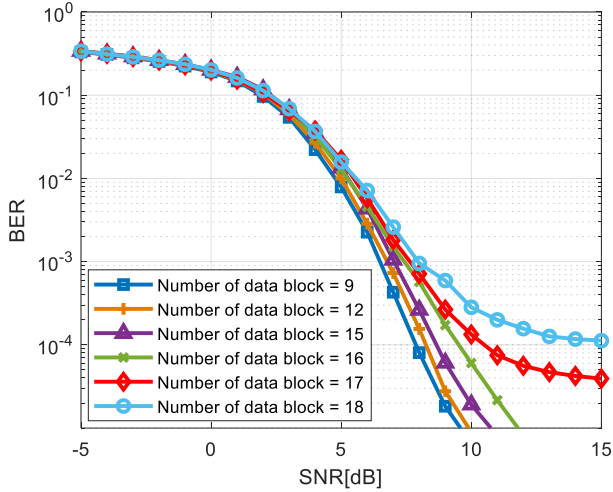


Figure 11. BER performance by the number of transmitted data frames when the velocity is 200 km/h

Figure 11 shows the BER curve corresponding to the number of data frames between two pilots using the linear interpolation of channel estimation when the moving speed is 200 km/h. Comparison of the BER curves confirms that the BER performances coincide when SNR increases when transmitting 15 or fewer data frames. Performance degradation occurs for 16 or more data frames.

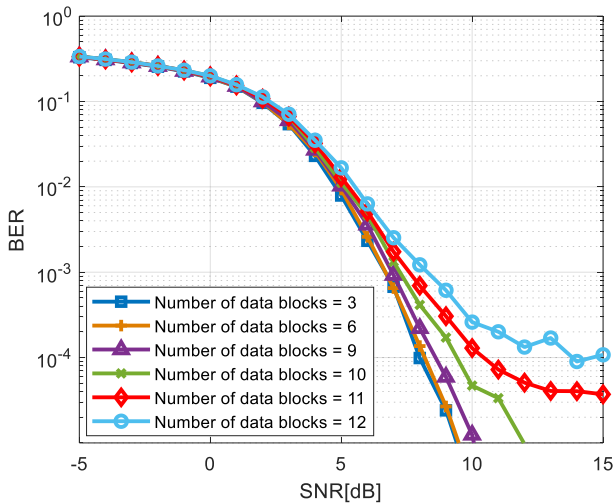


Figure 12. BER performance by the number of transmitted data frames when the velocity is 300 km/h

Figure 12 shows the BER curve corresponding to the number of data frames between two pilots with the linear interpolation of channel estimation when the moving speed is 300 km/h. Comparison of the BER curves confirms that BER performances are the best when transmitting 9 or fewer data

frames. Performance degradation occurs when 10 or more data frames are transmitted.

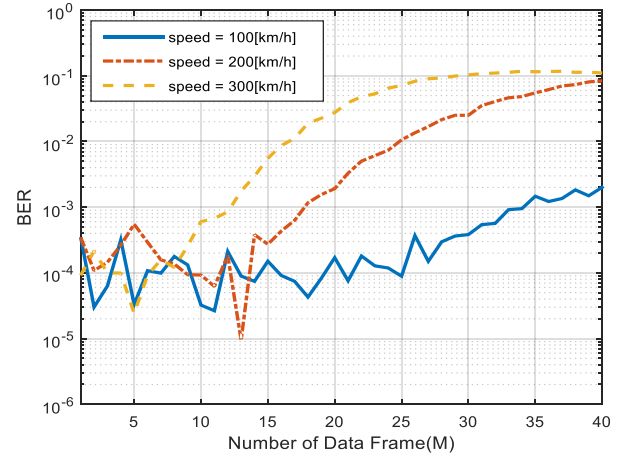


Figure 13. BER performance by the number of data frames when the SNR is 8dB

Figure 13 shows the BER performance for each moving speed according to the number of data frames transmitted for a fixed SNR of 8 dB. Comparison of the BER performances shows that when the moving speed increases, the performance degradation is remarkable when the number of data frames is large. This indicates that there is a limit on the number of data frames that allow reliable channel estimation and equalization processing for high moving speeds. We confirmed the performance of the SC-FDE through simulation. If we determine the appropriate number of data frames according to the moving speed, we can ensure that the BER performance is not degraded and that the pilot overhead is reduced.

## V. RESULT AND DISCUSSION

In this paper, we proposed a new SC-FDE structure to reduce the pilot overhead of the conventional SC-FDE. The proposed structure transmits one or more data frames between pilots instead of transmitting pilots for each SC-FDE data frame. Through simulation, we confirmed that channel estimation and equalization were performed well while reducing pilot overheads. However, it is difficult to restore the received data until the next pilot is received and in an environment where the channel suddenly changes. Further, there are limitations to the number of



data frames that can be estimated and equalized; these limitations are determined by the moving speed. Hence, it is necessary to set the number of data frames to be transferred taking the speed into consideration.

## VI. REFERENCES

- 3GPP TS 36.201 2007, 'E-UTRA; LTE Physical Layer-General Description', v.8.0.0.
- Abdelrahman, RBM, Mustafa, A & Osman, A 2015, 'A comparison between IEEE 802.11a, b, g, n and ac standards', *IOSR Journal of Computer Engineering*, vol. 17, no. 10, pp. 26-29.
- Abed, GA, Ismail, M & Jumari, K 2012, 'The evolution to 4G cellular systems: architecture and key features of LTE-advanced networks', *International Journal of Computer Networks and Wireless Communications*, vol. 2, no. 1, pp. 21-26.
- Falconer, D, Ariyavisitakul, SL, Benyamin-Seeyar, A & Eidson, B 2002, 'Frequency Domain Equalization for Single-Carrier Broadband Wireless Systems', *IEEE Communications Magazine*, vol. 40, no. 4, pp. 58-66.
- Gallager, RG 1962, 'Low-Density Parity-Check Codes', *IRE Transactions on Information Theory*, vol. 8, no. 1, pp. 21-28.
- Karam, G and Sari, H 1989, 'Analysis of predistortion, equalization, and ISI cancellation techniques in digital radio systems with nonlinear transmit amplifiers', *IEEE Transactions on Communications*, vol. 37, no. 12, pp.1245-1253.
- Lee, SR 2017, 'Dispersion-Managed Links for WDM Transmission Arranged by Linearly or Nonlinearly Incremented Residual Dispersion per Span', *Journal of Information and Communication Convergence Engineering*, vol. 15, no. 4, pp. 205-211.
- MacKay, DJC & Neal RM 1996, 'Near Shannon Limit Performance of Low Density Parity Check Codes', *Electronics Letters*, vol. 32, no. 18, pp.1645-1646.
- Moose, PH 1994, 'A technique for orthogonal frequency division multiplexing frequency Offset Correction', *IEEE Transactions on Communications*, vol. 42, no.10, pp. 2908-2914.
- Nee, R & Prasad, R 2000, *OFDM for Wireless Multimedia communications*, 1 edn, Artech House Publishers, MA, USA.
- O'Neill, R & Lopes, LB 1995, 'Envelope Variations and Spectral Splatter in Clipped Multicarrier Signals', in *Proceedings of 6th International Symposium on Personal, Indoor and Mobile Radio Communications*, 27-29 Sept. 1995, Toronto, Ontario, Canada.
- Pancaldi, F, Vitetta, G, Kalbasi, R, Al Dhahir, N, Uysal, M & Mheidat H 2008, 'Single-carrier frequency domain equalization', *IEEE Signal Processing Magazine*, vol. 25, no. 5, pp. 37-56.
- Tellado, J 2000, *Multicarrier Modulation with Low PAR: Applications to DSL and Wireless*, Kluwer Academic Publisher, MA, USA.
- Toumazou, C, Battersby, NC & Porta S 1996, *Circuits and Systems Tutorials*, IEEE Press Published, NewYork, USA.
- Wang, Z & Giannakis, GB 2000, 'Wireless Multicarrier Communications', *IEEE Signal Processing Magazine*, vol. 17, no.3, pp.29-48.
- Xiaodai, D, Wu-Sheng, L & Soong ACK 2017, 'Linear Interpolation in Pilot Symbol Assisted Channel Estimation for OFDM', *IEEE Transactions on Wireless Communications*, vol. 6, no. 5, pp.1910-1920.