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Generalizations of Burr Type X Distribution with Applications

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We propose the generalizations of Burr Type X distribution with two parameters by using the methods of Beta-G, Gamma-G and Weibull-G families of distributions. We discuss maximum likelihood estimation of the model's parameters. The performances of the parameter's estimates are assessed via simulation studies under different sets of conditions. In the applications to real data sets, three sets of data are used whereby from the results we can deduce that these models can be used quite effectively in analysing lifetime data.

Keywords: cumulative density function; lifetime data; maximum likelihood estimation

I. INTRODUCTION

Statistical distributions are constantly and extensively used to describe and predict real world phenomena in broad spectrum of areas such as engineering, medicine, biology, demography, environment, economics and many others. However, there is a need for an extended form of these distributions such as in survival analysis whereby the hazard function might be of various forms. Due to this over the past decades many studies have been put forward to develop such distributions that are more flexible and useful. Attempts have also been made in defining new families of probability distributions by extending well-known families of distributions such as the generation of the broad family of univariate distributions from the Weibull distribution (Gurvich *et. al.*, 1997).

Eugene et. al. (2002) proposed and study a general class of distributions based on the logic of a beta random variable named Beta-Generator (Beta-G) family distribution. The Gamma-Generator (Gamma-G) was developed by (Ristic & Balakrishnan, 2012). More recently, Bourguignon et al. (2014) proposed and studied the generality family of a univariate distribution with two additional parameters using the Weibull-Generator (Weibull-G) applied to the odds ratio $K(t)/1-K(t)$.A familiar trait of these generalized distributions is that they contain more parameters. According to Johnson & Kotz (1994), four-parameter distributions should be reasonably useful for most practical applications.

Burr (1942) introduced twelve different forms of cumulative functions for modelling data. These distributions have been widely used in the analysis of real-life data in the areas of health, agriculture, reliability and others. Several have received great attention including Burr Type X with two parameters (BX) that has a nonmonotone hazard function, unlike the Weibull, Generalized Exponential and Gamma distributions. BX is thus attractive in term of flexibility in dealing with hazard function variants.

This paper presents the extension of Burr Type X with two parameters (BX) through Beta-G, Gamma-G and Weibull-G. We explore some properties of the estimators through simulation under different settings. The estimation of the parameters is via maximum likelihood estimation (MLE) method. The applicability of these new models is illustrated by utilizing several real data sets. We use several criteria to compare the fit of these three models with the baseline model that is the BX.

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II. MATERIALS AND METHODS

The probability density function (pdf) of Burr Type X with one parameter (BXI) is

$$\boldsymbol{f(y,\vartheta) = 2\vartheta y e^{-y^2}\left(1-e^{-y^2}\right)^{\vartheta-1}}, \quad (1)$$

and the cumulative density function (cdf) is

$$\boldsymbol{F\,(y,\vartheta) = \left(1-e^{-y^2}\right)^{\vartheta}}, \quad (2)$$

whereϑ is the shape parameter.

Surles & Padgett (2001) modified the BXI distribution following the method of Mudholkar & Srivastava (1993) for the exponentiated Weibull and named it Burr Type X with two parameters (BX) distribution.

The pdf and cdf respectively of BX are

$$\boldsymbol{f(y,\eta,\vartheta) = 2\vartheta\eta^2 y e^{-(y\eta)^2}\left[1-e^{(y\eta)^2}\right]^{\vartheta-1}}, \quad (3)$$

and

$$\boldsymbol{F(y,\eta,\vartheta) = \left[1-e^{-(y\eta)^2}\right]^{\vartheta}}, \quad (4)$$

whereϑ is the shape parameter and η is the scale parameter.

A. Beta Burr Type X

If $F(y)$ is the cumulative function of any random variable Y, the cdf of Beta-G family of distribution with $\upsilon > 0$ and $\omega > 0$ is defined by Eugene et al. (2002),

$$\Phi(y,\upsilon,\omega) = \frac{1}{B(\upsilon,\omega)}\int_0^{F(y)} t^{\nu-1}(1-t)^{\omega-1}dt, \quad (5)$$

where ν and ω are the additional shape parameters. The pdf corresponding to (5) is

$$\phi(y,\nu,\omega) = \frac{1}{B(\nu,\omega)}[F(y)]^{\nu-1}[1-F(y)]^{\omega-1}f(y)$$

$$(6) B(\nu,\omega) = \frac{\Gamma(\nu)\Gamma(\omega)}{\Gamma(\upsilon+\omega)}$$

The pdf of Beta Burr Type X (BBX) from (3), (4) and (6) is

$$\phi(y,\nu,\omega,\eta,\vartheta) = \frac{2\vartheta\eta^2 y}{B(\nu,\omega)}\left[\left[1-e^{-\eta y^2}\right]^{\vartheta}\right]^{\nu-1}\left[1 - \left[1-e^{-\eta y^2}\right]^{\vartheta}\right]^{\omega-1} \times e^{-(\eta y)^2}\left[1-e^{-(\eta y)^2}\right]^{\vartheta-1}, \quad (7)$$

where $y > 0$, the shape parameters $\nu,\omega,\vartheta > 0$ and the scale parameter $\eta > 0$.The cdf of BBX is

$$\Phi(y,\nu,\omega,\eta,\vartheta) = \frac{1}{B(\nu,\omega)}\int_0^{\left[1-e^{-(\eta y)^2}\right]^{\vartheta}} t^{\nu-1}(1-t)^{\omega-1}dt \quad (8)$$

The hazard function is

$$h(y,\nu,\omega,\eta,\vartheta) = \frac{\phi(y,\nu,\omega,\eta,\vartheta)}{1-\Phi(y,\nu,\omega,\eta,\vartheta)}$$

B. Gamma Burr Type X

From Ristic & Balakrishnan (2012), for any baseline cdf$F(y)$ of a random variable $y \in R$ and $f(y)$ is the pdf, they define the generalized class of Gamma distribution with cdf

$$\Phi(y,\nu) = 1 - \frac{1}{\Gamma(\nu)}\int_0^{-log[F(y)]} t^{\nu-1}e^{-t}\,dt \quad y \in R \quad \nu > 0 \quad (9)$$

The pdf corresponding to (9) is

$$\phi(y,\nu) = \frac{f(y)[-log[F(y)]]^{\nu-1}}{\Gamma(\nu)} \quad y \in R \quad \nu > 0 \quad (10)$$

By substituting the pdf (3) and cdf (4) of BX distribution into (10), we obtain the pdf of Gamma Burr Type X (GBX) for $y > 0$ as

$$\phi(y,\nu,\vartheta,\eta) = \frac{2\vartheta\eta^2 y e^{-(\eta y)^2}}{\Gamma(\nu)}\left[1 - e^{-(\eta y)^2}\right]^{\vartheta-1}\left[-\vartheta log\left[1-e^{-(\eta y)^2}\right]\right]^{\nu-1} \quad (11)$$

The cdf of (11) is

$$\Phi(y,\nu,\vartheta,\eta) = 1 - \frac{\gamma\left[\nu,-\vartheta log\left[1-e^{-(\eta y)^2}\right]\right]}{\Gamma(\nu)}, \quad (12)$$

where $[\nu,y] = \int_0^y t^{\nu-1}e^{-t}\,dt$, is the incomplete gamma function.

The two shape parameters of GBX make the new distribution more suitable for analysing skewed positive real data.

C. Weibull Burr Type X

Let $F(y,\xi)$ and $f(y,\xi)$ respectively be a cumulative and density functions of the baseline model with parameter vector ξ. The Weibull cdf is $\Phi(y,\nu,\omega) = 1-\left(e^{-\nu y^{\omega}}\right)$ for $y > 0$ with positive parameters ν and ω. Based on this density, by replacing y with $F(y,\xi)/1-F(y,\xi)$, the cdf of the Weibull-G distribution with two extra parameters ν and ω as defined by Bourguignon et al. (2014) is

$$\Phi(y,\nu,\omega,\xi) = \int_0^{F(y,\xi)/1-F(y,\xi)} \nu\omega y^{\omega-1}e^{-\nu y^{\omega}}$$

$$=1-exp\left(-\nu\left[\frac{F(y,\xi)}{1-F(y,\xi)}\right]^{\omega}\right) \quad (13)$$

The pdf corresponding to (13) is

$$\phi(y,\nu,\omega,\xi) = \nu\omega f(y,\xi)\frac{F(y,\xi)^{\omega-1}}{[1-F(y,\xi)]^{\omega+1}}exp\left(-\nu\left[\frac{F(y,\xi)}{1-F(y,\xi)}\right]^{\omega}\right)$$

By substituting the cdf (4) into (13), the cdf of Weibull Burr Type X (WBX) distribution is

$$\Phi(y,\nu,\omega,\eta,\vartheta) = 1-e\left(-\nu\frac{\left[1-e^{-(\eta y)^2}\right]^{\vartheta\omega}}{[1-[1-e^{-(\eta y)^2}]^{\vartheta}]^{\omega}}\right) \quad (14)$$

The pdf corresponding to (14) is

$$\phi(y,\nu,\omega,\eta,\vartheta) = 2\nu\omega\eta^2\vartheta y e^{-(\eta y)^2}\frac{\left[1-e^{-(\eta y)^2}\right]^{\vartheta\omega-1}}{[1-[1-e^{-(\eta y)^2}]^{\vartheta}]^{\omega+1}}$$

$$\times exp\left(-\nu\frac{\left[1-e^{-(\eta y)^2}\right]^{\vartheta\omega}}{\left[1-\left[1-e^{-(\eta y)^2}\right]^{\vartheta}\right]^{\omega}}\right) \quad (15)$$

where $\nu>0,\omega>0,\eta>0$ and $\vartheta>0$, with ν and ω the two additional parameters.

D. Parameter Estimation

Let $Y_1,Y_2,\dots,Y_n$ be an i.i.d random variable with common BBX $(\nu,\omega,\eta,\vartheta)$ distribution, WBX distribution with the unknown parameters ν,ω,η and ϑ and GBX distribution with the unknown parameters ν,ϑ and η.

The log- likelihood function for the vector of parameters $\Theta = (\nu,\omega,\eta,\vartheta)^T$ can be expressed as

$$l = log[L(\Theta)] = n[log2+log\vartheta+2log\eta+logy+log\Gamma(\nu+\omega)-log\Gamma(\nu)-log\Gamma(\omega)]-\sum_{i=1}^{n}(\eta y_i)^2+(\nu\vartheta-1)\sum_{i=1}^{n}log\left[1-e^{-(n y_i)^2}\right]+(\omega-1)\sum_{i=1}^{n}log\left[1-\left[1-e^{-(\eta y_i)^2}\right]^{\vartheta}\right] \quad (16)$$

The logarithm of likelihood function for the vector of parameters $\Theta=(\nu,\eta,\vartheta)^T$ of GBX is

$$l = log[L(\Theta)] = n[log2+\nu log\vartheta+2log\eta-log\Gamma(\nu)] + log\left(\sum_{i=1}^{n}y_i\right) + (\vartheta-1)\sum_{i=1}^{n}log\left[1-e^{-(\eta y_i)^2}\right] - \sum_{i=1}^{n}(\eta y_i)^2 - (\nu-1)\sum_{i=1}^{n}log\left[log\left[1-e^{-(\eta y_i)^2}\right]\right] \quad (17)$$

The logarithm of likelihood function for the vector of parameters $\xi=(\nu,\omega,\eta,\vartheta)^T$ of WBX is

$$l = log(L) = n\log 2 + n\,log(\nu\omega\eta^2\vartheta) - \sum_{i=1}^{n}(\eta y_i)^2 + \sum_{i=1}^{n}logy_i + (\vartheta\omega-1)\sum_{i=1}^{n}log\left[1-e^{-(\eta y_i)^2}\right] - (\omega-1)\sum_{i=1}^{n}log\left[1-\left[1-e^{-(\eta y_i)^2}\right]^{\vartheta}\right] - \nu log\left(\sum_{i=1}^{n}\frac{\left[1-e^{-(\eta y)^2}\right]^{\vartheta\omega}}{\left[1-\left[1-e^{-(\eta y)^2}\right]^{\vartheta}\right]^{\omega}}\right) \quad (18)$$

Take the partial derivatives of log-likelihoods in (16), (17) and (18) with respect to their parameters and equate them to zero (Merovci *et. al.,* 2016; Khaleel *et. al.,* 2016; Ibrahim *et al.,* 2017). Since the models do not have close form solutions, solve the equations numerically using iterative methods to obtain the maximum likelihood estimation (MLE) for the parameters.

E. Simulation Study

We conduct simulation studies to examine the behaviour of the maximum likelihood estimators of the parameters of BBX, GBX and WBX distributions for different sample size and initial values of the parameters. Random sample can be generated from the inverse function of (8), (12) and (14) respectively. The sample size considered are n=50, 100 and 150 with the number of repetitions to be 1000 for each. The initial parameters values for BBX$(\nu,\omega,\eta,\vartheta)$ are arbitrarily chosen. Three sets were considered but only Set 1 for each is illustrated. They are Set1=(2,2,2,2) for BBX; for GBX(ν,η,ϑ), Set1=(2,4,3), for WBX$(\nu,\omega,\eta,\vartheta)$, Set1=(3,3,3,3). The mean (AvE), bias and root mean square error (RMSE) of the maximum likelihood estimators are calculated as measures to assess the performance of the estimators.

$\mathrm{AvE}(\widehat{\Theta}) = \frac{\sum_{i=1}^{1000} \widehat{\Theta}}{1000}$ $\mathrm{Bias}(\widehat{\Theta}) = \mathrm{AvE}(\widehat{\Theta}) - \Theta$,

$\mathrm{RMSE} = \sqrt{\sum_{i=1}^{1000} \frac{(\widehat{\Theta}-\Theta)^2}{1000}}$

where $\widehat{\Theta}$ is the MLE for Θ for the respective distributions.

F. Application of Real Data

We fit three data sets by using Beta Burr Type X (BBX), Gamma Burr Type X (GBX) and Weibull Burr Type X (WBX) to illustrate the potentiality and capability of these distributions. The real data sets used are either left-skewed or right-skewed or approximately symmetrical. The criteria used to compare the three models with the baseline are log likelihood and the Akaike information criterion (AIC). The likelihood ratio (LR) test can be used for testing the null hypothesis of the three distributions against the baseline distribution. The BBX, GBX and WBX distributions are reduced to BX distribution when $\nu = \omega = 1, \nu = 1$ and $\nu = \omega = 1$ respectively.

1. Time-to-Failure of the Turbochargers Diesel Engines Data

This data set is time in hours of the failure of 40 turbochargers diesel engines. These data were also used by Xu *et. al.,* (2003) for reliability in forecasting engine. Table 1 provides descriptive statistics of the data such as item number (N), mean, standard deviation (SD), median, skewness (Skew), kurtosis and standard error (SE).

Table 1. Descriptive statistics of turbochargers failure data set

N	Mean	SD	Median	Skew	Kurtosis	SE
40	6.25	1.96	6.5	-0.64	-0.49	0.31

2. Waiting Times (minutes) of Bank Customers Data

This data set is from Ghitany *et. al.,* (2008) which represents the waiting time (minutes) before service of 100 bank customers. Table 2 provides the same descriptive statistics as in Table 1.

Table 2. Descriptive statistics of waiting times (minutes) of 100 bank customers data set

N	Mean	SD	Median	Skew	Kurtosis	SE
100	9.88	7.24	8.1	1.45	2.43	0.72

3. The Strengths of 1.5 cm Glass Fibres Data Set

This data set consists of 63 observations of the breaking strengths of 1.5 cm glass fibres and have been analysed by Smith & Naylor (1987).The descriptive statistics is presented in Table 3.

Table 3. Descriptive statistics of the strengths of 1.5 cm glass fibres data set

N	Mean	SD	Median	Skew	Kurtosis	SE
63	1.51	0.32	1.59	-0.88	0.80	0.04

III. RESULTS AND DISCUSSION

This section provides the results and discussion of the performance of the estimators of the three models: BBX, GBX and WBX based on the results of the simulation studies. The fit of the three models is assessed via real data application.

Tables 4-6 present the AvE, bias and RMSE of the estimators for different initial values and *n* for BBX, GBX and WBX respectively. In general, the AvE of all estimators are closer to the selected initial values as sample size increases. The bias of each tend to decrease with the increase of sample size.

The RMSE of the estimators which measure the average error behave accordingly that is the values decrease as sample size increases. The simulation results of the other sets depicted similar trend. The simulation results indicate that the estimators perform well.

Table 4. The AEs, biases and RMSEs based on 1,000 simulations of the BBX distribution and n=50, 100 and 150

Set 1: v=2; $\omega = 2$; $\eta = 2$; and $\vartheta = 2$				
n	**Φ**	AEs	Bais	RMSE
50	v	2.0688	0.0688	0.4019
	ω	2.0580	0.0580	0.5167
	η	2.0472	0.0472	0.2361
	ϑ	2.1212	0.1212	0.4741
100	v	2.0477	0 .0477	0.3038
	ω	2.0198	0.0198	0.3971
	η	2.0358	0.0358	0.1741
	ϑ	2.0641	0.0641	0.3652
150	v	2.0241	0.0241	0.2450
	ω	2.0037	0.0037	0.3233
	η	2.0138	0.0138	0.1293
	ϑ	2.0327	0.0327	0.2567

Table 5. The AEs, biases and RMSEs based on 1,000 simulations of the GBX distribution and n=50, 150 and 300

Set 1: η=2; $\vartheta = 4$ and $v = 3$				
n	**Φ**	AEs	Bais	RMSE
50	η	2.0733	0.0733	0.6021
	ϑ	4.1382	0.1382	0.8219
	v	3.2646	0.2646	0.7959
100	η	2.0182	0.0182	0.4233
	ϑ	4.0842	0.0842	0.5385
	v	3.1224	0.1224	0.4518
150	η	1.9838	-0.0162	0.2708
	ϑ	4.0418	0.0418	0.2792
	v	3.0197	0.0197	0.2435

Table 6. The AEs, biases and RMSEs based on 1,000 simulations of the WBX distribution and n=50, 100 and 150

Set 1: v=3; $\omega = 3$; $\eta = 3$; and $\vartheta = 3$				
n	**Φ**	AEs	Bais	RMSE
50	v	3.1898	0.1898	0.2433
	ω	3.0642	0.0642	0.3759
	η	3.0134	0.0134	0.0401
	ϑ	3.0482	0.0482	0.1301
100	v	3.1555	0.1555	0.1942
	ω	3.0096	0.0096	0.2065
	η	3.0054	0.0054	0.0216
	ϑ	3.0395	0.0395	0.0718
150	v	3.1058	0.1058	0.1731
	ω	2.9969	-0.0031	0.1404
	η	3.0022	0.0022	0.0148
	ϑ	3.0318	0.0318	0.0571

Tables 7-9 present the MLEs of the parameters, the –log likelihood (-l) and AIC. From Table 7 the values of AIC of WBX is the smallest. These data are left-skewed depicted by Figure 1.From Figure 2 we can see that the cdf of WBX distribution is very close to the empirical cdf. The results suggest that WBX is a good fit for for left-skewed data.

Table 7. The ML estimates, $-l$ and $AIC$of turbocharges failure data set

Model	ML Estim.	$-l$	AIC
BBX	$\hat{v} = 0.119$ $\hat{\omega} = 164.8$ $\hat{\eta} = 0.121$ $\hat{\vartheta} = 14.056$	79.4	167
GBX	$\hat{v} = 2.145$ $\hat{\eta} = 0.142$ $\hat{\vartheta} = 2.811$	85.5	177
WBX	$\hat{v} = 0..0532$ $\hat{\omega} = 1.731$ $\hat{\eta} = 0.343$ $\hat{\vartheta} = 0.136$	79.6	165
BX	$\hat{\eta} = 0.343$ $\hat{\vartheta} = 2.386$	85.8	176

Table 8. indicates that the GBX is a strong competitor to BBX, WBX and BX for fitting the waiting times of 100 bank customers data set.

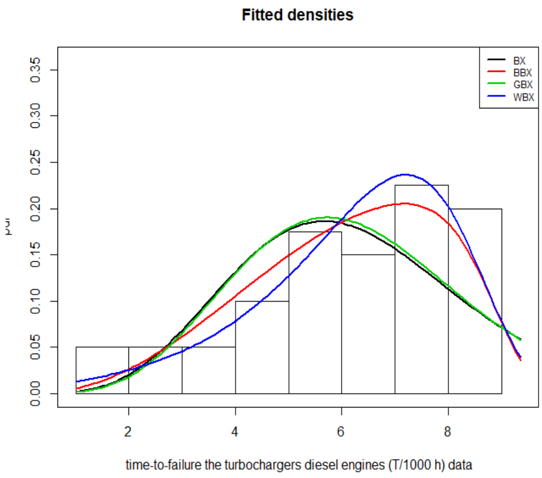

Figure 1. Histogram and plots of fitted densities time-to-failure of turbochargers diesel engine data

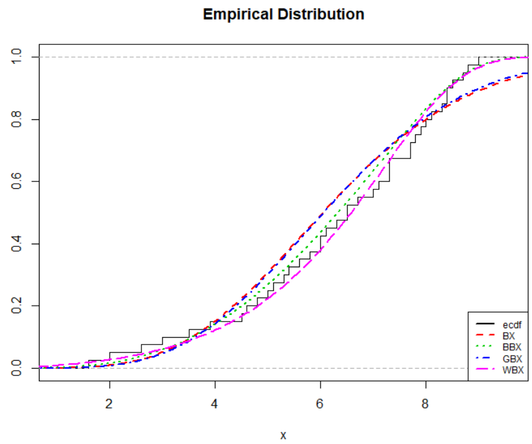

Figure 2. Empirical cdf and cdfs for time-to-failure of turbochargers diesel engine data

GBX is the better fit followed by WBX. This data set is right-skewed (Figure 3). This application suggests that GBX fits very well the right-skewed data. Figures 3 and 4 clearly indicate that GBX is the best fit.

For the breaking strengths data set it is clearly shown in Table 9 that WBX is the best fit for these data based on the largest log likelihood and smallest AIC. Figures 5 and 6 emphasize the strength of WBX. Figure 5 shows that this data set is approximately a symmetric data that is slightly skewed to the left. This illustration also indicates that WBX distribution is also suitable to fit approximately symmetric data.

Table 8. The ML estimates, $-l$ and AIC of waiting times (minutes) of 100 banks customers data set

Model	ML Estim.	$-l$	AIC
BBX	$\hat{\nu} = 2.6858$ $\hat{\omega} = 6.968$ $\hat{\eta} = 0.0183$ $\hat{\vartheta} = 0.3565$	318	644
GBX	$\hat{\nu} = 5.112$ $\hat{\eta} = 0.022$ $\hat{\vartheta} = 1.409$	318	641
WBX	$\hat{\nu} = 13.968$ $\hat{\omega} = 0.247$ $\hat{\eta} = 3.037$ $\hat{\vartheta} = 0.016$	318	643
BX	$\hat{\eta} = 0.0694$ $\hat{\vartheta} = 0.6289$	322	647

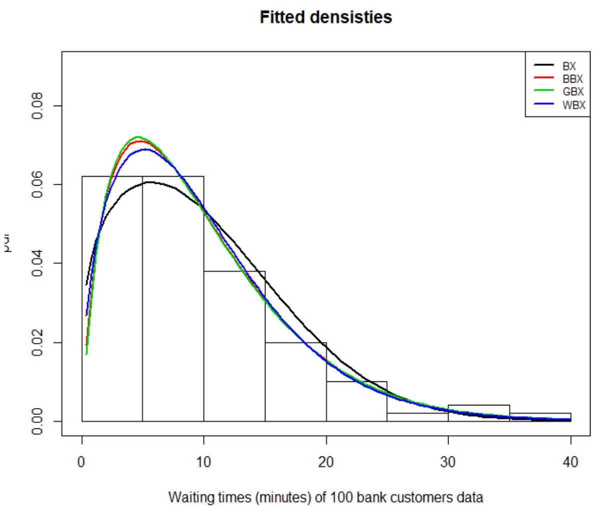

Figure 3. Histogram and plots of fitted densities for waiting times (minutes) of 100 bank customers data

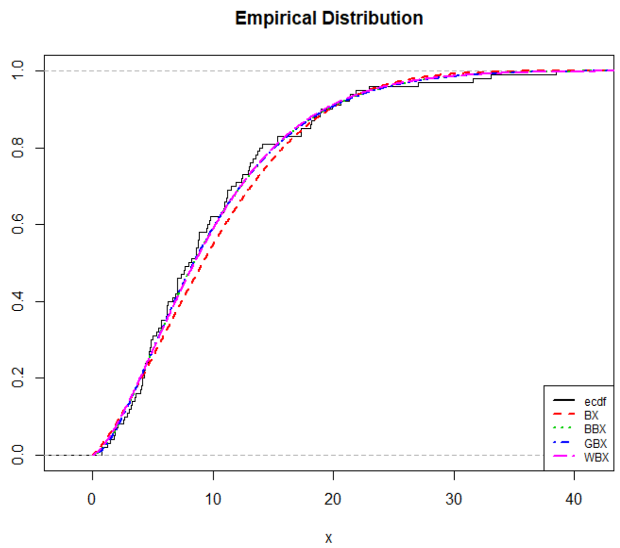

Figure 4. Empirical cdf and cdfs for waiting times (minutes) of 100 bank customers data

Table 9. The ML estimates, $-l$ and AIC of breaking strengths data set

Model	ML Estim.	$-l$	AIC
BBX	$\hat{\nu} = 0.354$ $\hat{\varpi} = 37.42$ $\hat{\eta} = 0.55$ $\hat{\vartheta} = 8.539$	14.9	37.7
GBX	$\hat{\nu} = 4.88$ $\hat{\eta} = 0.561$ $\hat{\vartheta} = 6.575$	21.6	49.3
WBX	$\hat{\nu} = 11.142$ $\hat{\varpi} = 6.717$ $\hat{\eta} = 0.262$ $\hat{\vartheta} = 0.013$	14.6	35.1
BX	$\hat{\eta} = 0.987$ $\hat{\vartheta} = 5.486$	23.9	51.9

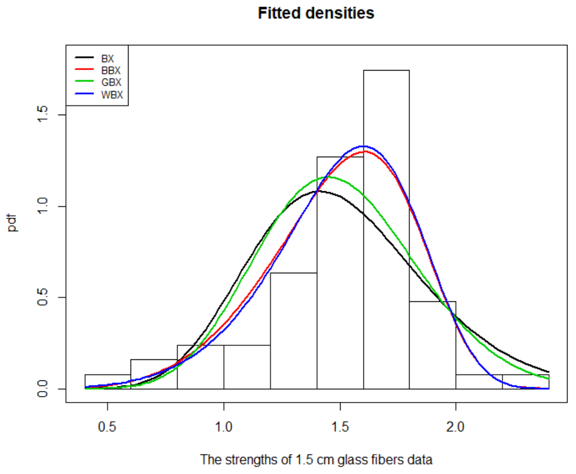

Figure 5. Histogram and plots of fitted densities for glass fibres data.

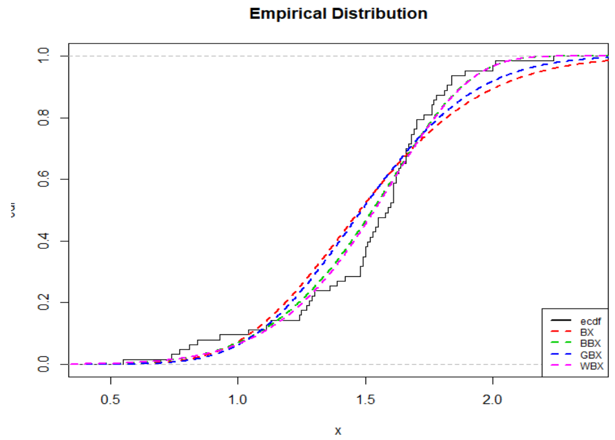

Figure 6. Empirical cdf and cdfs for glass fibres data.

IV. SUMMARY

The main ingredient of this work is in the generalization of Burr Type X distribution with two parameters (BX) which is the baseline distribution used to fit normally a right-skewed data sets. We extend this distribution by using three different families namely Beta-G, Gamma-G and Weibull-G. The parameters of the new distributions perform accordingly based on the simulation results. The real data applications illustrate the potentiality and flexibility of the new distributions and are able to model the data well. applications in all related areas.

V. ACKNOWLEDGEMENT

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