

The Effect of Edge Stopping in Perona Malik Anisotropic Diffusion Model as Image Denoising on Digital Radiographic Image

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Image denoising is process of reducing image noise in order to improve the visual quality of the image. Many conventional filtering techniques are able to remove noise with the blurring effect that reduces the image quality. The introduction of nonlinear partial differential equations approach in image processing has grown significantly and gives varieties in filtering techniques. Perona Malik Anisotropic Diffusion (PMAD) is one of the nonlinear models that able to preserve the edge features well. But the sharp edge and fine details sometimes are not preserved well due to the used of inappropriate edge stopping function. Hence, the following objectives are been set up to propose a new edge stopping function as part of PMAD model and analyse the performance of the proposed function with the existing edge stopping function. In this paper, the PMAD model is solved using finite difference method. Then, different edge stopping functions are applied as part of the model. For experimental purposes, a set of digital radiographic image of welding defect is used. The performances of each function are evaluated using Structural Similarity Index Metric (SSIM) and Peak Signal to Noise Ratio (PSNR). The results show that different edge stopping function give different effect on the image quality after the denoising process. As the conclusion, utilizing edge stopping function preserves the sharp edge in the denoised image and hence the image quality is also improved.

Keywords: Perona Malik Anisotropic Diffusion, Structural Similarity Index Metric, Peak Signal to Noise Ratio, Edge Stopping Function, Radiographic Image, MATLAB

I. INTRODUCTION

In image processing, images undergo appropriate modifications in order to be improved. The images are enhanced with an improved visual quality that allows valuable information to be extracted which can be used for analysis purposes. High quality image gives better visualization and helps in producing more accurate evaluations.

Image denoising is one of the image processing processes. The aim of image denoising process is to remove

noise in the digital images in order to form a smooth clear image. However, the problem of conventional filtering techniques is often resulting in blurring effect and hence diminishes the object edges in the image. Since the edge information is important for image analysis and interpretation, the edge should be kept during the denoising process. Many methods have been proposed in order to remove noise and restore the quality of an image for better visualization.

Partial differential equations (PDEs) have recently dominated in the fields of computer vision and image

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processing due to its ability to extend its framework from two-dimensional to even higher dimensions. For image denoising case, it is possible to represent an image in a continuous domain in the form of PDEs.

Image denoising problem can be modelled as an extension of basic heat diffusion equation of PDE. The evolution of diffusion model from linear diffusion (isotropic diffusion) to nonlinear diffusion (anisotropic diffusion) has grown a lot of research in image processing area based on PDE. Generally the model is found to be able to remove noise from an image (Mhamed, Abid and Fnaiech, 2012; Perona and Malik, 1990).

In 1990, Perona and Malik first introduced a nonlinear two-dimensional diffusion model suitable for noise removal using a diffusion coefficient which involves an exponential function. Ever since then until now the model has been explored and modified without changing the exponential basis to produce better denoising tools. It is observed that the exponential feature has an asymptotic effect in the models. This led to the intuition of exploring the natural logarithmic function which also has an asymptotic feature in replacement of the exponential term in the diffusion coefficient in the nonlinear diffusion PDE model. Hence, in this study a denoising variant with this function is proposed. The objectives of this study are to propose a variant that governing the PDE with the diffusion equation, to apply finite difference method that produced finite difference scheme on PDE order two of diffusion equation and evaluate the performance proposed scheme on real data of digital radiographic image.

II. LITERATURE REVIEWS

There are many studies in image denoising methods using PDE-based concepts that have been proposed nowadays. However, this paper only focuses on the parabolic PDEs. A well-known parabolic PDE is the heat equation which is second order PDE that describes the heat flow and the diffusion processes (Cristobal, Schelkens and Thienpont, 2013). The advantages of using PDE for image processing are due to the well-grounded mathematical theories of PDEs and these theories offer more than classical techniques that led to discovery of new methods.

For the diffusion-based models, the anisotropic diffusion technique based on nonlinear PDE is involved in solving the initial value of input image using nonlinear heat diffusion equation (Lai, *et. al.*, 2011). Perona and Malik

(1990) had introduced a new definition of scale space technique and a class of algorithms using a diffusion process. In the diffusion equation framework, the diffusion coefficient is assumed to be a constant independent of the space location.

For PDE of order two based model, the diffusion coefficient function is required to control the diffusivity on the object's edge in the image. The diffusion coefficient is used in anisotropic diffusion that able to enhance the pixel levels on image. In addition, Behzad, Behrooz and Kian (2015) compared the performance of the three diffusion coefficients proposed by Perona Malik (1990), Buades, *et al.* (2008) and Green (1954), Kubo (1957). The PDE model on several real and synthetic 2D images of photography and 1D signal.

Yuan and Wang (2016) proposed a new diffusion coefficient on a second order derivative and local entropy information that able to eliminate the staircasing effect and enhance the texture information. Yahya, *et. al.*,(2017) demonstrates the new algorithm that blend total variation with anisotropic diffusion filter on standard test images.

III. METHODOLOGY

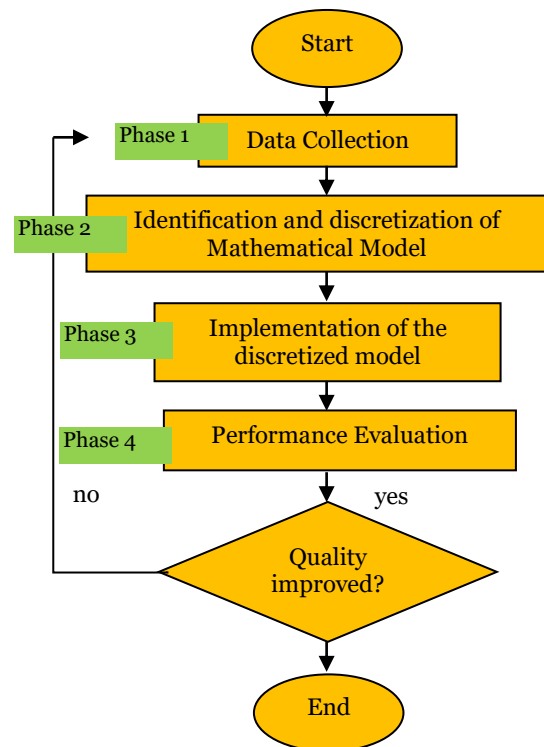


Figure 1. Phases in Research Methodology

This section discusses the methods involved in this study which involved four phases. The description of each phase of the research framework begins from data collection, identification and discretization of noise removal model, implementation of the proposed scheme on digital radiographic image and finally the performance evaluation as in Figure 1.

Phase 1 begins with acquisition of a flawed specimen plate which is termed as phantom in this study. Flawed specimen plate (U-C-17) is a specimen with specific flaws that are purposely induced for training and development purposes.

A radiographic machine is used to digitize the specimen in digital form. Figure 2 shows the captured digital image and the region of interest (ROI) that was selected manually by radiography inspectors in the area that contains weld flaw referred as the reference image.

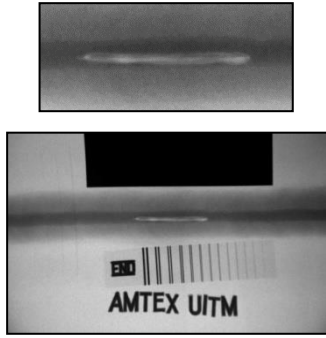


Figure 2. Captured Digital Image and Reference Image

In Phase 2, Mathematical model of PDE order two anisotropic diffusion is choose. The PDE is discretized using the forward, backward and central difference operators. Finite difference is applied because the pixel structure in digital image provides a natural discretization of a fixed rectangular grid (Weickert, 1998). Upon discretization the two-dimensional PDE equations of order two produced finite difference scheme which is explicit scheme. The parabolic PDE denoising model is as in the following equations.

$$I_t(x, y, t) = c(\|\nabla I(x, y, t)\|) (I_{xx}(x, y, t) + I_{yy}(x, y, t)), \quad (1)$$

$$(x, y) \in \Omega, t \in (0, T]$$

With different diffusion coefficient functions as in Table 1.

Table 1. Variation of Diffusion Coefficient Functions

Name	Diffusion Coefficient Function, $c(\ \nabla I(x, y, t)\)$
DC1	$e^{-\left(\frac{\ \nabla I(x, y, t)\ }{K}\right)^2}$
DC2	$\frac{1}{1 + \left(\frac{\ \nabla I(x, y, t)\ }{K}\right)^2}$
DC3	$\frac{1}{\sqrt{1 + \left(\frac{\ \nabla I(x, y, t)\ }{K}\right)^2}}$
DC4	$\frac{1}{\sqrt{1 + \left[\ln\left(\frac{\ \nabla I(x, y, t)\ }{K}\right)\right]^2}}, \frac{\ \nabla I(x, y, t)\ }{K} > 0$

DC1 and DC2 are well known functions proposed by Perona Malik (1990). The variant DC3 is the enhancement of DC2 that was proposed by Charbonnier, et al. (1994). In this study, DC4 was proposed as a new variant with similar characteristics as in DC1, DC2 and DC3. Note that all the coefficients require initial values of K to be simulated by users before the noise removal process was carried out.

The forward operator in time Δ_{+t} and central order in space δ_x^2, δ_y^2 are adopted on the parabolic PDE of order two as in equation (1) and the diffusion coefficient functions (Table 1).

This discretization produces a mesh of grid points. In these equations the term $I_{i,j}^n$ is the grid point in the mesh, Δt is the time step size, while Δx and Δy are the space step sizes such that $I(x_i, y_j, t_n) \approx I_{i,j}^n$ at the grid point (x_i, y_j) at time level $t = t_n$, where t_n is time at n^{th} level, $t_n = n\Delta t$. Then, $x_i = x_0 + i\Delta x$ and $y_j = y_0 + j\Delta y$.

The explicit scheme is produced as in equation (2).

$$I_{i,j}^{n+1} = \left(\mu_x c(\|\nabla I_{i,j}^n\|)\right) I_{i-1,j}^n + \left(\mu_y c(\|\nabla I_{i,j}^n\|)\right) I_{i,j-1}^n + \left(1 - 2(\mu_x + \mu_y) c(\|\nabla I_{i,j}^n\|)\right) I_{i,j}^n + \left(\mu_x c(\|\nabla I_{i,j}^n\|)\right) I_{i+1,j}^n + \left(\mu_y c(\|\nabla I_{i,j}^n\|)\right) I_{i,j+1}^n \quad (2)$$

In this study, the convergence and stability were investigated experimentally via graphs produced from the scheme (2).

In Phase 3, the discretized scheme is implemented on digital radiographic image contains weld defect. In order to evaluate the quality of the denoised image, image quality metric, *SSIM* (Dosselmann and Yang, 2008) and *PSNR* (Babawuro and Beiji, 2011; Kumar, Sinha and Thakur, 2011) were calculated in Phase 4.

IV. RESULTS AND DISCUSSIONS

The scheme (2) was applied on a set of 15 real data for noise removal. In order to determine how well the derived scheme performs in removing noise, a flawed specimen was used.

Table 2 illustrates the results obtained in terms of image quality with different values of *K* for the scheme (2) with $\Delta t = 0.05$. The maximum number of iterations is set to 500. The integer value of *n* in each table represents the number of iteration when the image quality based on *SSIM* is maximum.

Table 2. Image Quality Performance for the scheme (2) (initial *SSIM*=0.3535, *PSNR*=23.4668)

<i>K</i>	<i>DC1</i>			<i>DC2</i>		
	<i>n</i>	<i>SSIM</i>	<i>PSNR</i>	<i>n</i>	<i>SSIM</i>	<i>PSNR</i>
	50		23.744	50	0.4599	
0.4	0	0.35504	9	0	3	25.5235
	50		23.80	50	0.6616	
0.8	0	0.35851	27	0	5	28.7867
	50		23.851	50	0.7681	30.836
1	0	0.3614	3	0	9	6
			34.921		0.9195	
10	99	0.90418	8	44	0	35.9287
			35.987		0.9206	
50	35	0.92062	3	35	2	35.9875
10			35.98		0.920	
0	35	0.9206	9	35	6	35.989

<i>K</i>	<i>DC3</i>			<i>DC4</i>		
	<i>n</i>	<i>SSIM</i>	<i>PSNR</i>	<i>n</i>	<i>SSIM</i>	<i>PSNR</i>
	32	0.9159				35.999
0.4	8	2	35.7440	74	0.9202	5
		0.9167				
0.8	173	3	35.7968	56	0.9202	35.9901
					0.920	
1	143	0.9170	35.8209	53	2	35.9924
		0.9203			0.9190	35.806
10	40	1	35.976	62	0	0
					0.9193	
50	35	0.920	35.989	114	7	35.8424
10		0.9206				
0	34	4	35.9874	142	0.9196	35.8710

From Table 2, it can be observed that the performance of the scheme (2) on the noisy phantom in terms of number of iterations and in terms of image quality. In terms of number of iterations, DC1, DC2 and DC3 outperform DC4 as the *K*= 50 and 100 which the three coefficients required less number of iterations. But as *K*=0.4, 0.8 and 1, DC4 outperform DC1, DC2 and DC3 with less number of iterations that produced the highest image quality. In terms of image quality, DC4 is comparable with DC1, DC2 and DC3 as they are equal to the third decimal place.

It is observed that the image quality obtained for all schemes using DC4 with *K*=1 produces results that are close to second decimal place also. This observation leads to the idea that DC4 can be simplified by eliminating the parameter *K*. The proposition is that DC4 was modified as follows

$$MDC4 = \frac{1}{\sqrt{1 + [\ln(\|\nabla I(x,y)\|)]^2}}, \|\nabla I(x,y)\| > 0$$

The significance of this modification is that no simulation for *K* was required in the denoising process. The results of image quality with the processing time of 1 iteration using MDC4 for all the schemes are shown in Table 3 below

Table 3. Quality Performance of scheme using MDC4 for Phantom

Method	N	Image Quality Metric	
		SSIM	PSNR
Scheme (2)	53	0.920209	35.992446
Median	NA	0.7692	30.5889

Figure 3 below consists of intensity profiles of a selected segment extracted from the surface plot in the xz -plane resulting from the scheme (2) at $y = 95, \Delta x = 0.5$. The range time t selected was indicated by the values of n in the legend. The intensity profile from the reference image was drawn in thick black.

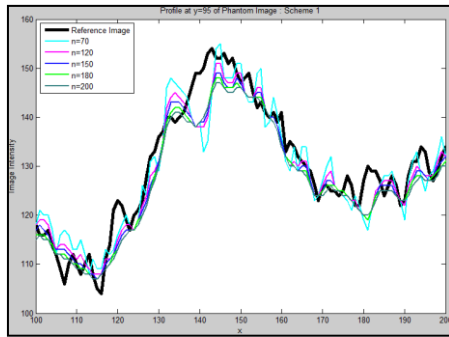


Figure 3. Intensity Profile of the scheme (2) ($\Delta t = 0.004, y = 95, \Delta x = 0.5$)

From Figure 3 it can be observed that as time t increases the intensity curves from the scheme (2) converge to the intensity curve (thick black) of the reference image.

Figure 4, Figure 5 and Figure 6 show the relative error (RE) profiles for the scheme (2) for different spatial step size ($\Delta x = \Delta y = 1.0; \Delta x = \Delta y = 0.5; \Delta x = \Delta y = 0.25$) which were plotted against number of iterations.

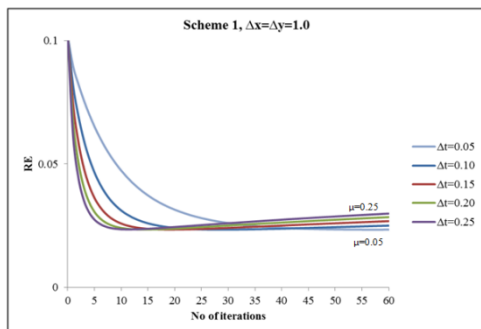


Figure 4. RE Profile for the scheme (2) with $\Delta x = \Delta y = 1.0$

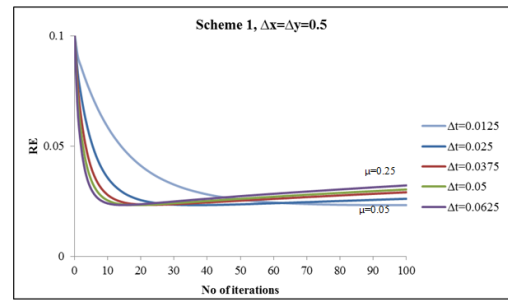


Figure 5. RE Profile for the scheme (2) with $\Delta x = \Delta y = 0.5$

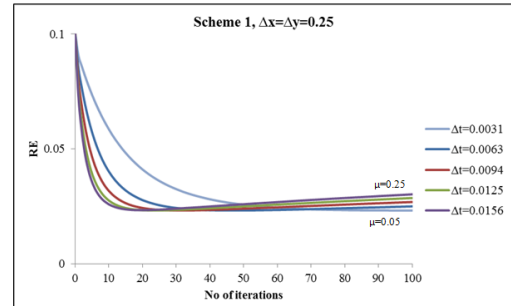


Figure 6. RE Profile for the scheme (2) with $\Delta x = \Delta y = 0.25$

This means each curve has different time span to complete a set of iterations. The RE curves in each figure were observed and the prospective curves for stability is taken into account.

Table 4 below illustrates the μ values for the different values of spatial step sizes chosen above.

Table 4. Stability Analysis for the scheme (2) with

$$\mu = \frac{\Delta t}{(\Delta x)^2}$$

$\Delta x=1.00$		$\Delta x=0.50$		$\Delta x=0.25$	
Δt	μ	Δt	μ	Δt	μ
0.05	0.05	0.0125	0.05	0.0031	0.05
0.10	0.10	0.0250	0.10	0.0063	0.10
0.15	0.15	0.0375	0.15	0.0094	0.15
0.20	0.20	0.0500	0.20	0.0125	0.20
0.25	0.25	0.0625	0.25	0.0156	0.25

The best RE curves identified in Figure 4, Figure 5 and Figure 6 above suggest that for the scheme (2), stability is established when $\mu \leq 0.05$.

In this study, a set of 15 real data was used. Only 15 samples of image data was used because it is difficult to produce a sample welding with defect for experimental

purposes.

Table 5 demonstrates the image quality measurement of the 15 samples real image data using the Scheme (2) with $\Delta t = 0.05$. The values in bold represent the image metric that reach the best quality of denoised image.

Table 5. Quality Measurement for Real Image Data

Image Name	Method	n	Image Quality Metrics	
			SSIM	PSNR
WD_1	Scheme (2)	65	0.9296	35.225
	Median	NA	0.7725	28.8993
WD_2	Scheme (2)	66	0.9362	36.4588
	Median	NA	0.7725	29.5251
WD_3	Scheme (2)	83	0.94795	37.4233
	Median	NA	0.7829	30.0437
WD_4	Scheme (2)	67	0.8524	31.3793
	Median	NA	0.7293	26.4581
WD_5	Scheme (2)	49	0.9144	34.1645
	Median	NA	0.7650	28.8002
WD_6	Scheme (2)	28	0.7536	28.5439
	Median	NA	0.6421	25.6452
WD_7	Scheme (2)	50	0.9201	36.3402
	Median	NA	0.7725	31.2253
WD_8	Scheme (2)	46	0.8572	33.204
	Median	NA	0.8228	28.0066
WD_9	Scheme (2)	48	0.8211	32.9448
	Median	NA	0.7944	27.6548
WD_10	Scheme (2)	56	0.8783	34.6954
	Median	NA	0.7896	30.6095
WD_11	Scheme (2)	64	0.8551	33.4007
	Median	NA	0.8008	27.4507
WD_12	Scheme (2)	53	0.7830	33.9018
	Median	NA	0.7380	29.9954

WD_13	Scheme (2)	73	0.957558	36.206835
	Median	NA	0.7755	28.5851
WD_14	Scheme (2)	54	0.9580788	36.6500823
	Median	NA	0.8500	29.8806
WD_15	Scheme (2)	62	0.9632	34.0089
	Median	NA	0.8121	26.0336

It can be observed that for the 15 images, scheme (2) produced ad good quality of image as compared with median filter.

V. CONCLUSION

Numerical results obtained via experiments indicate that the scheme (2) with the introduction of the natural logarithmic function in the nonlinear diffusion equation shows converging and stability feature. The scheme also successfully removes noise in the images that implies the variant, and the scheme is prospective denoising agent for digital images.

It is recommended that for nonlinear two-dimensional PDE of order two only where a new variant diffusion coefficient is introduced. This opens room for further research whereby the function can be modified to improve the denoising ability of the PDE. Besides that, it also opens rooms for other functions with similar feature to be explored to produce other denoising variants.

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