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An Integrated Fuzzy Approach for Evaluating Mathematics Mobile Application

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Various studies have been carried out to evaluate the effectiveness of mathematics mobile application. However, there are limited studies that consider the uncertainty of human judgement. Due to human perception which is vague and uncertain, with fuzziness condition during the assessment process, this paper proposes an integrated fuzzy approach for evaluating the effectiveness of mathematics mobile application. Firstly, the students' answer scripts in pre-test and post-test were evaluated using triangular fuzzy numbers associated with the degree of confidence concept. Then, an enhanced fuzzy conjoint model (FCM) based on triangular fuzzy numbers is presented for analysing students' opinion on the mobile application. The integrated approach is applied in the evaluation of a mathematics mobile application, namely as Anti-Derivatives Mobile Learning Tool based on Types of Integrand (ADMLTI). The findings show that there is a significant increase in performance from the pre-test to post-test. Students agreed at more than 0.9 degrees of similarity that ADMLTI gives positive impact on learning Mathematics. The sensitivity analysis based on the degree of fuzziness δ shows that the method produces an almost similar ranking for different values of $\delta \in [0,2]$. The results demonstrate that ADMLTI can assist students in improving their Mathematics learning productivity.

Keywords: ADMLTI; degree of confidence; fuzzy conjoint model; learning mathematics; mathematics mobile application

I. INTRODUCTION

The increasing number of mobile devices' user and rapid advancement in wireless technology brings a new approach to teaching and learning. The ever increasing speed of mobile apps development is also a factor that contributes to the surge of the use of mobile devices as an educational tool. People can conveniently access information resources round the clock without time-bound and place restrictions by using mobile devices. A study by Huet and Tcheng (2010) found that mobile apps give a positive influence on students' attitude in learning activities. Furthermore, Kay and Lauricella (2011) also found that students perceived mobile apps as helping them to stay focused, be more organised and efficient in their learning activities. The use of mobile apps can also reduce confusion and is easier to transfer the conceptual information to students, especially in some subjects such as Mathematics (Skiada *et al*., 2014). Various studies such as Supandi *et al.* (2018), Fabian *et al.* (2016), and Taleb *et al.* (2015) have evaluated the usefulness of mobile apps in mathematics learning. Instead, the assessment of the effectiveness of mobile apps in learning mathematics that considers the uncertainty of human judgement is largely understudied. According to Biswas (1995), it is critical that educational institutions provide students with an evaluation report as adequately as possible with the smallest possible inevitable error.

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The fuzzy approach with grade membership functions provides a useful way of evaluating students' assessment in a fair and intelligent manner. Many studies have been carried out to evaluate students' answer script using a fuzzy approach such as matching function by Biswas (1995), satisfaction levels and degree of satisfaction by Chen and Lee (1999), type-2 fuzzy set (Wang & Chen, 2006), vague sets (Wang & Chen, 2008a), and triangular fuzzy numbers (TFNs) with a degree of confidences (DoC) by Wang and Chen (2008b). The Wang and Chen's (2008b) method is more flexible and comprehensive compared to Biswas's (1995), and Chen and Lee's (1999) approaches as it can deal with the assessment values in fuzzy numbers form and take into account the evaluator's degree of optimism.

Perception, opinion or satisfaction level towards learning attributes are subjective in nature and depends on human interpretation. The fuzzy conjoint model (FCM) (Turksen & Willson, 1994) has been widely used for measuring preference under fuzzy environment. The FCM has been applied in different areas, for example in evaluating students' perception on computer algebra system (Abdullah *et al.*, 2011), students' expectation on mathematics learning (Sarala & Kavitha, 2017), teachers' belief on mathematics (Lazim & Osman, 2009), employers' satisfaction level for graduates' performance (Yusoff *et al.*, 2013), customers' opinion on credit card services (Baheri *et al.*, 2011) and consumers' opinion on cycling transport (Yaakub *et al.*, 2018). Nevertheless, the above-mentioned FCM methods used the fuzzy sets to describe the membership function of the linguistic value, in which the α-cut cannot be obtained for different values of $\alpha \in [0,1]$.

Thus, in this paper, an improvised FCM method based on TFNs is presented. This paper integrates Wang and Chen's (2008b) method and the improvised FCM based on TFNs in the evaluation of a mathematics mobile application, namely as Anti-Derivatives Mobile Learning Tool based on Types of Integrand (ADMLTI). Firstly, the students' answer scripts in pre and post-test were evaluated using TFNs associated with the DoC concept. Then, the FCM based on TFNs is applied in analysing students' opinion on the mobile application.

The paper is arranged accordingly: Section 2 briefly reviews some basic concept of TFNs, Xu *et al.*'s (2010) similarity measure, and Wang and Chen's (2008b) method. Section 3 presents the proposed integrated fuzzy approach for evaluating the effectiveness of mathematics mobile application. Section 4 presents an illustrative example and results of evaluation of ADMLTI, and the paper is concluded in Section 5.

II. PRELIMINARIES

This section takes a look at some core definitions of TFNs, the α-cut of the fuzzy numbers, the similarity measure of TFNs from Xu *et al.* (2010) and the fuzzy assessment from Wang and Chen (2008b).

A. Triangular Fuzzy Number and α -cut

A triangular fuzzy number (TFN) as given in Figure 1, denotes as $\tilde{M} = (a, b, c)$ has a membership function defined as:

$$\mu_{\tilde{M}}(x) = \begin{cases} \dfrac{x-a}{b-a}, & a \le x \le b \\ \dfrac{c-x}{c-b}, & b \le x \le c \\ 0, & \text{otherwise} \end{cases}$$

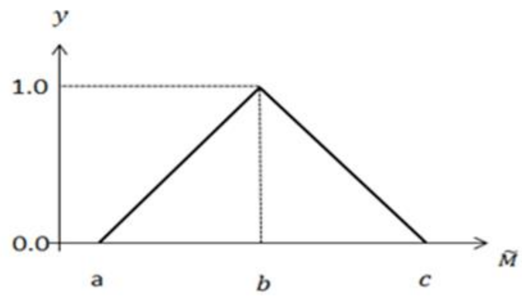

Figure 1. Triangular fuzzy number, $\tilde{M}$

The α -cut of a fuzzy number $\tilde{M}$ in the universe of discourse X and denoted as $\tilde{M}_\alpha$ is defined as:

$$\tilde{M}_\alpha = \{x_i \in X: \mu_M(x_i) \ge \alpha\} = [\alpha_1{}^{(\alpha)}, \alpha_2{}^{(\alpha)}]$$

whereby $\alpha \in [0,1]$.

The α-cut of TFN $\tilde{M} = (a, b, c)$ is given as $\widetilde{M_\alpha} = [(b-a)\alpha + a,\ c - (c-b)\alpha]$.

B. Similarity Measure of TFNs (Xu et al., 2010)

The degree of similarity measure between two TFNs $A =$

(a_1, a_2, a_3) and $B = (b_1, b_2, b_3)$ is defined as:

$$S(A,B) = 1 - \frac{1}{8}\{|a_1 - b_1| + 2|a_2 - b_2| + |a_3 - b_3|\} - \frac{d(A,B)}{2} \qquad (1)$$

whereby $d(A,B) = \frac{\sqrt{(x_A - x_B)^2 + (y_A - y_B)^2}}{\sqrt{1.25}}$,

$y_A = \begin{cases} \frac{1}{3} & , a_1 \neq a_3 \\ \frac{1}{2} & , a_1 = a_3 \end{cases}$ and $x_A = 2y_A a_2 + (a_3 + a_1)(1 - y_A)$.

C. Fuzzy Assessment with Degree of Confidence (DoC) for Evaluating Students' Answer Script (Wang & Chen, 2008b)

The fuzzy assessment with DoC for evaluating the students' answer script from Wang and Chen (2008b) is presented. First, we consider the situation in which there are n questions with the total marks as M. The distribution of marks for each question is given as Question 1 has m_1 marks, Question 2 has m_2 marks, Question 3 has m_3 marks, ..., Question n has m_n marks whereby $\sum_{i=1}^{n} m_i = M$ and $1 \leq i \leq n$. The assessor is assumed to have a level of optimism θ whereby $\theta \in [0,1]$. The assessor is a pessimistic assessor for $\theta \in [0,0.5)$, $\theta = 0.5$ the assessor is a normal assessor and $\theta \in (0.5,1]$ the assessor is an optimistic assessor. The fuzzy assessment consists of the following four steps:

Step 1 : The assessor evaluates the student's answer script by giving the satisfaction level for each question $1 \leq i \leq n$ in linguistic values such as V_1, V_2, V_3, ..., V_n whereby V_i is represented with triangular fuzzy numbers. Furthermore, the evaluator gives the DoC of each satisfaction level V_i awarded to Question i. Assume $\alpha, \beta, \gamma, \ldots, \delta$ represent the DoC of the satisfaction level V_1, V_2, V_3, ..., V_n respectively which is awarded to Question 1, 2, 3, ..n respectively whereby α, $\beta, \gamma, \ldots, \delta \in [0,1]$.

Step 2: Calculate the α- cut of the TrFN V_1 $(V_1)_\alpha$, the β- cut of the TrFN V_2 $(V_2)_\beta$, the γ- cut of the TrFN V_3 $(V_3)_\gamma$, ..., and the δ- cut of the TrFN V_n $(V_n)_\delta$. Assume $(V_1)_\alpha = [a_1, a_2]$, $(V_2)_\beta = [b_1, b_2]$, $(V_3)_\gamma = [c_1, c_2]$, ..., $(V_n)_\delta = [z_1, z_{n2}]$.

Step 3: Calculate the total mark in interval form $[t_1, t_2]$ whereby

$$t_1 = \frac{m_1}{M} \times (V_1)_{\alpha_L} + \frac{m_2}{M} \times (V_2)_{\beta_L} + \frac{m_3}{M} \times (V_3)_{\gamma_L} + \ldots + \frac{m_n}{M} \times (V_n)_{\delta_L}$$
$$= \frac{m_1}{M} \times a_1 + \frac{m_2}{M} \times b_1 + \frac{m_3}{M} \times c_1 + \ldots + \frac{m_n}{M} \times z_1$$
$$t_2 = \frac{m_1}{M} \times (V_1)_{\alpha_U} + \frac{m_2}{M} \times (V_2)_{\beta_U} + \frac{m_3}{M} \times (V_3)_{\gamma_U} + \ldots + \frac{m_n}{M} \times (V_n)_{\delta_U}$$
$$= \frac{m_1}{M} \times a_2 + \frac{m_2}{M} \times b_2 + \frac{m_3}{M} \times c_2 + \ldots + \frac{m_n}{M} \times z_2.$$

Step 4: Calculate the total mark defined as $(1 - \theta) \times t_1 + \theta t_2$ where θ denotes the index of optimism. The DoC of the total mark awarded is defined as $min(\alpha, \beta, \gamma, \ldots, \delta)$.

III. THE PROPOSED INTEGRATED FUZZY METHOD FOR MEASURING THE EFFECTIVENESS OF MATHEMATICS MOBILE APPLICATION

The proposed integrated fuzzy method for measuring the effectiveness of mathematics application consists of two different kinds of fuzzy multi-criteria decision making approaches. First, the fuzzy evaluation from Wang and Chen's (2008b) is used to evaluate students' answer script for the pre and post-test assessment.

Then, the improvised FCM based on TFNs is applied in analysing students' opinion on the mobile application. The improvised FCM is presented below.

First, consider the situation where M attributes for the questionnaires with j-th linguistic values as L_j (j = 1,2,3 ..., p) and p is the number of linguistic values used.

The FCM based on TFNs consists of the following five steps:

Step 1: Collect the respondents' opinion based on p linguistic values L_j (j = 1, 2, ..., p).

Step 2: Determine the number of opinion of the respondent n_{ij} for attribute i, and linguistic value j.

Step 3: Compute the weight of attribute i with linguistic value j as:

$$w_{ij} = \frac{n_{ij}}{\sum_{j=1}^{p} n_{ij}} . \qquad (2)$$

Step 4: Determine the overall membership function of attribute i as:

$$R_i = \sum_{j}^{p} w_{ij} L_j \quad \text{for } i = 1, 2, 3, \ldots, M. \qquad (3)$$

Step 5: Compute the similarity degree between TFNs R_i and L_j using similarity measure from Xu *et al.* (2010) as in Equation (1). Then, choose the maximum similarity of each R_i.

Figures 2 and 3 summarise the procedure for evaluating the students' answer script from Wang and Chen (2008b) and the improvised FCM based on TFNs, respectively.

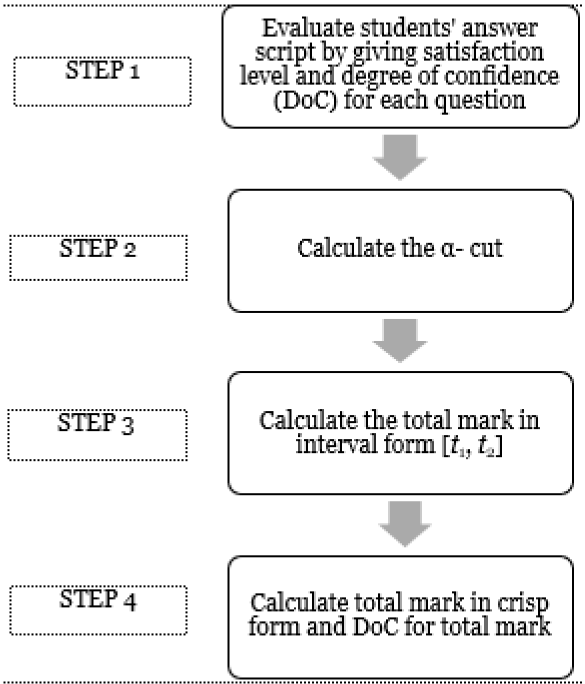

Figure 2. Procedure for evaluating students' answer script (Wang & Chen, 2008b)

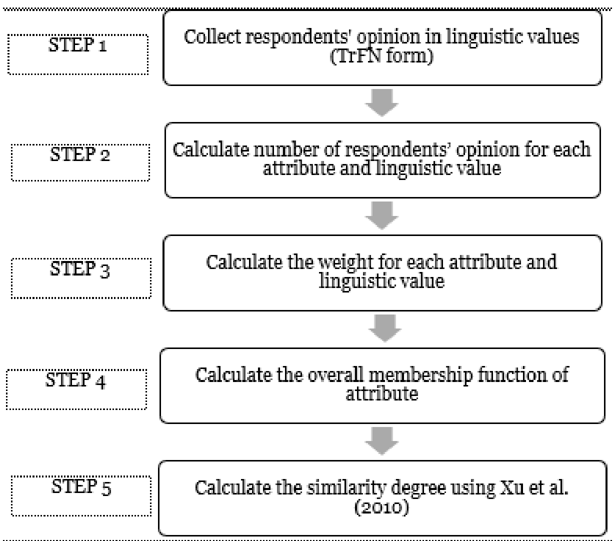

Figure 3. Procedure for FCM based on TFNs

IV. ILLUSTRATIVE EXAMPLE

The integrated fuzzy approach for evaluating the effectiveness of mathematics mobile application is applied in the evaluation of Anti-Derivatives Mobile Learning Tool based on Types of Integrand (ADMLTI). Thirty-six students in the science and technology degree program at one higher institution in the East Coast of Malaysia were involved in the ADMLTI evaluation. The students started using the ADMLTI in the third week of the semester, the pre-test was carried out in the earlier part of week 3, and the post-test and evaluation on satisfaction were carried out on week 12 of the semester.

A. Evaluating Students' Answer Script based on Triangular Fuzzy Numbers with DoC (Wang & Chen, 2008b)

First, the students answer a mathematics quiz consisting of 20 pre and post-test questions. Both tests have a total mark of 40 with different questions for each test but within the same cognitive level. The distribution of full marks for each question is given in Table 1. The assessor's index of optimism θ is 0.6.

Table 1. Distribution of full marks for each question

Question	Full Mark(s)	Question	Full Mark(s)
1	1	11	2
2	1	12	3
3	2	13	1
4	2	14	2
5	2	15	3
6	2	16	1
7	3	17	2
8	2	18	3
9	1	19	2
10	2	20	3

Table 2. Linguistic values for satisfaction level

Linguistic Values	Triangular Fuzzy Number
Extremely Good (EG)	(100, 100, 100)
Very Good (VG)	(90, 100, 100)
Good (G)	(70, 90, 100)
More or Less Good (MLG)	(50, 70, 90)
Fair	(30, 50, 70)
More or Less Bad (MLB)	(10, 30, 50)
Bad (B)	(0, 10, 30)
Very Bad (VB)	(0, 0, 10)
Extremely Bad (EB)	(0, 0, 0)

The fuzzy assessment for evaluating students' answer script in pre and post-test is given as follows:

Step 1: An assessor evaluates the answer script in terms of satisfaction level and DoC of the satisfaction level for each question as shown in Tables 3(a) and 3(b).

Step 2: The satisfaction level involved are VG, G, MLG, VB, B, MLB, and EB. Based on the TFNs in Table 2, the α- cut of each satisfaction level is given as in Tables 3(a) and 3(b).

Table 3(a). Distribution of full marks for questions 1 to 10

Question No.	Satisfaction Level	DoC of Satisfaction Level	α-cut
1	VG	0.9	[99, 100]
2	VG	0.9	[99, 100]
3	EB	0.8	[0, 0]
4	B	0.9	[9, 12]
5	VG	0.9	[99, 100]
6	B	0.8	[8, 14]
7	VG	0.9	[99, 100]
8	G	0.8	[86, 92]
9	MLG	0.7	[64, 76]
10	VG	0.9	[99, 100]

Table 3(b). Distribution of full marks for questions 11 to 20

Question No.	Satisfaction Level	DoC of Satisfaction Level	α-cut
11	VG	0.9	[99, 100]
12	VG	0.9	[99, 100]
13	VB	0.8	[0, 2]
14	VG	0.9	[99, 100]
15	VG	0.9	[99, 100]
16	VB	0.8	[0, 2]
17	MLB	0.7	[24, 36]
18	VG	0.9	[99, 100]
19	VG	0.9	[99, 100]
20	G	0.8	[86, 92]

Step 3: The calculation of the total mark in interval form [t_1, t_2] is given as follows:

$$t_1 = \frac{1}{40} \times 99 + \frac{1}{40} \times 99 + \frac{2}{40} \times 0 + \frac{2}{40} \times 9 + \frac{2}{40} \times 99 + \frac{2}{40} \times 8 + \frac{3}{40} \times 99 + \frac{2}{40} \times 86 + \frac{1}{40} \times 64 + \frac{2}{40} \times 99 + \frac{2}{40} \times 99 + \frac{3}{40} \times 99 + \frac{1}{40} \times 0 + \frac{2}{40} \times 99 + \frac{3}{40} \times 99 + \frac{1}{40} \times 0 + \frac{2}{40} \times 24 + \frac{3}{40} \times 99 + \frac{2}{40} \times 99 + \frac{3}{40} \times 86 = 73.8$$

$$t_2 = \frac{1}{40} \times 100 + \frac{1}{40} \times 100 + \frac{2}{40} \times 0 + \frac{2}{40} \times 12 + \frac{2}{40} \times 100 + \frac{2}{40} \times 14 + \frac{3}{40} \times 100 + \frac{2}{40} \times 92 + \frac{1}{40} \times 76 + \frac{2}{40} \times 100 + \frac{2}{40} \times 100 + \frac{3}{40} \times 100 + \frac{1}{40} \times 2 + \frac{2}{40} \times 100 + \frac{3}{40} \times 100 + \frac{1}{40} \times 2 + \frac{2}{40} \times 36 + \frac{3}{40} \times 100 + \frac{2}{40} \times 100 + \frac{3}{40} \times 92 = 76.6$$

Thus, the interval of the total marks for satisfaction level and DoC given in Table 3 is [73.8, 76.6].

Step 4: The assessor's index of optimism is 0.6 and thus, we calculate the total mark as follows:

$$(1 - 0.6) \times 73.8 + 0.6 \times 76.6 = 75.48 \cong 75 .$$

The DoC of the total mark is $min(0.9,\ 0.8, 0.7) = 0.7$. This suggests that the assessor's level of certainty for the post-test sample in Table 3 is 0.7 with total mark 75.

Using similar procedure as above, the pre and post-test marks for all students are obtained as shown in Figure 4. We found that the degree of confidence for pre and post-test is equal to 0.7. Figure 4 shows a comparison of the marks in pre and post-test. There is an obvious increment in terms of marks for several students. More than 55% of students show better performance in the pre-test after the implementation of ADMLTI as a tool in learning Mathematics.

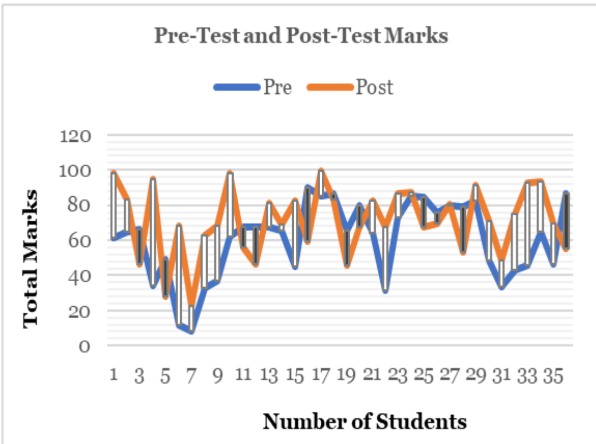

Figure 4. Pre and post-test results

Furthermore, to check whether there is a significant difference before and after the ADMLTI is introduced, the marks awarded were analysed using the t-test. However, before the paired sample t-test can be calculated, all the assumptions were checked using R. The marks obtained are continuous interval data for the related samples. As for this analysis, only 36 data were obtained, the normality test of the Shapiro-Wilk was used to verify the normality of the data. Based on the Shapiro-Wilk normality test, we get diff, W = 0.9666, p-value = 0.34.

Based on the output obtained in R, the p-value > 0.05 implies the distribution of the data is not significantly different from the normal distribution. Thus, it can be assumed that the data is normally distributed. Furthermore, based on boxplots in Figure 5, there are no outliers in the data. The comparison for the test marks obtained prior and after the use of ADMLTI can be easily seen from the boxplots. The lowest mark and the highest mark for the post-test are [23, 100], meanwhile, the lowest mark and the highest mark for pre-test are [8, 90].

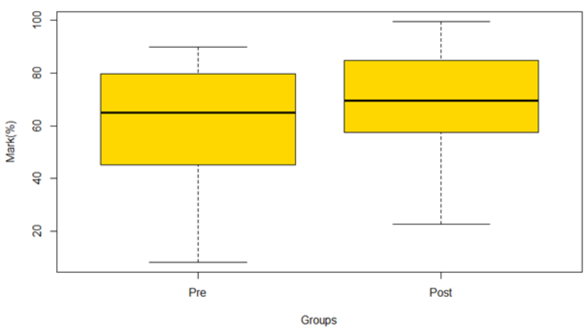

Figure 5. Boxplots for pre and post-test marks using fuzzy assessment

The hypothesis H_0: $\mu_D = 0$, thus, based on the $|t| > t_{\alpha/2}$ with 35 degrees of freedom, the value of calculated t = 2.537 is greater than the tabulated t which means H_0 is rejected. It can therefore be concluded that there is a significant difference between the mean of pre-test marks and mean of post-test marks after learning Mathematics through ADMLTI. On average, the scores increase by 10% with a confidence interval ($2.1303 < \mu_D < 19.1978$) after learning Mathematics using ADMLTI.

B. Measuring Students' Perception based on Improvised Triangular Fuzzy Number Conjoint Model

All the thirty-six students involved in answering the questionnaire on students' perception of ADMLTI were asked to answer the questionnaire which consists of six attributes (A_1 to A_6) which are as follows:

A_1: ADMLTI is positively beneficial for learning Mathematics

A_2: ADMLTI makes the class more enjoyable

A_3: Regular usage of ADMLTI in class gives positive effect in education

A_4: ADMLTI is useful in students' learning process

A_5: ADMLTI enables to accomplish learning activities more quickly.

A_6: ADMLTI increases learning productivity.

The FCM based on triangular fuzzy number consists of five steps which are as follows:

Step 1: Collect students' opinion on each attribute based on five linguistic values L_1 to L_5, whereby L_1, L_2, L_3, L_4 and L_5 represent strongly disagree, disagree, indifferent, agree, and strongly agree, respectively. The linguistic values L_1 to L_5 in the form of TFNs are shown in Table 4.

Table 4. Linguistic values L_1 to L_5 in TFNs form

Linguistic Values	**Triangular fuzzy number**
Strongly Disagree (L_1)	(0,0,2)
Disagree (L_2)	(0,2,4)
Neutral (L_3)	(3,5,7)
Agree (L_4)	(6,8,10)
Strongly Agree (L_5)	(8,10,10)

Step 2: The number of respondents' opinion n_{ij} of each attribute i for each linguistic value j is given in Table 5.

Table 5. Frequency of students' opinion related to attributes and linguistic values

Attributes	L_1	L_2	L_3	L_4	L_5	**Total**
A_1	0	0	5	21	10	36
A_2	0	0	8	22	6	36
A_3	0	0	12	19	5	36
A_4	0	0	9	19	8	36
A_5	0	0	5	19	11	35
A_6	0	0	9	19	8	36

Based on Table 5, for attribute A_1, 10 students have chosen strongly agree (L_5), 21 students have selected agree (L_4) and 5 students have chosen indifferent (L_3). None of the students have chosen disagree (L_2) and strongly disagree (L_1). Thus, n_{11} = 0, n_{12} = 0, n_{13} = 5, n_{14} = 21 and n_{15} = 10.

Step 3: The fuzzy weight w_{ij} for attribute i related to linguistic j is computed using Eq. (2) and presented in Table 6.

Table 6. Fuzzy weight w_{ij}

Attributes	L_1	L_2	L_3	L_4	L_5
A_1	0	0	0.139	0.583	0.278
A_2	0	0	0.222	0.611	0.167
A_3	0	0	0.333	0.528	0.139
A_4	0	0	0.250	0.528	0.222
A_5	0	0	0.143	0.543	0.314
A_6	0	0	0.250	0.528	0.222

Step 4: The overall membership function of attribute i, R_i is computed using Eq. (3).

$R_1 = w_{11}L_1 + w_{12}L_2 + w_{13}L_3 + w_{14}L_4 + w_{15}L_5$

$= 0(0,0,2) + 0(0,2,4) + 0.139(3,5,7) + 0.583(6,8,10) + 0.278(8,10,10)$

$= (6.139, 8.139, 9.583)$.

In a similar manner, we obtained, $R_2 = (5.667, 7.667, 9.333)$, $R_3 = (5.278, 7.278, 9)$, $R_4 = (5.694, 7.694, 9.25)$, $R_5 = (6.2, 8.2, 9.571)$ and $R_6 = (5.694, 7.694, 9.25)$. Here, we can obtain the α-cut of each R_i for different value of $\alpha \in [0,1]$ as follows:

$$R_{1_\alpha} = [2\alpha + 6.139, 9.583 - 1.444\alpha],$$

$$R_{2_\alpha} = [2\alpha + 5.667, 9.333 - 1.666\alpha],$$

$$R_{3_\alpha} = [2\alpha + 5.278, 9 - 1.722\alpha],$$

$$R_{4_\alpha} = R_{6_\alpha} = [2\alpha + 5.694, 9.25 - 1.556\alpha] \text{ and}$$

$$R_{5_\alpha} = [2\alpha + 6.2, 9.571 - 1.371\alpha].$$

Step 5: The degree to which R_i and L_j are similar $S(R_i, L_j)$ is computed using Equation (1) and presented in Table 7.

Table 7. The degree of similarity $S(R_i, L_j)$

TrFN	L_1	L_2	L_3	L_4	L_5
R_1	0.299	0.434	0.718	0.988	0.863
R_2	0.338	0.472	0.757	0.960	0.823
R_3	0.373	0.508	0.792	0.924	0.789
R_4	0.338	0.473	0.757	0.959	0.824
R_5	0.295	0.430	0.714	0.987	0.868
R_6	0.338	0.473	0.757	0.959	0.824

From Table 7, the maximum amount of similarity degree (MAXSIM) of R_1 is computed as follows: MAXSIM(R_1) = max (0.299, 0.434, 0.718, 0.988, 0.863) = 0.988. Since 0.988 is the degree of similarity between R_1 and L_4, thus 0. 988 falls under linguistic value L_4 (agree). In a similar manner, Table 8 displays the maximum similarity and linguistic values for R_i with $i = 1,2, \ldots, 6$.

Table 8. The maximum similarity degree

TrFN	R_1	R_2	R_3	R_4	R_5	R_6
Maximum similarity degree	0.988	0.960	0.924	0.959	0.987	0.959
Linguistic value	Agree	Agree	Agree	Agree	Agree	Agree
Ranking	1	3	5	4	2	4

The degree of agreement that ADMLTI is positively beneficial for learning Mathematics recorded at 'agree' with 0.988 degrees of similarity. Students' also agreed at 0.960 degrees of similarity that ADMLTI will make the class more enjoyable. They also agreed at 0.924 degrees of similarity that regular usage of ADMLTI in class gives positive effect in education. They also agreed at 0.959 degrees of similarity that ADMLTI is useful in students' learning process and increases learning productivity. Students agreed at 0.987 degrees of similarity that ADMLTI enables to accomplish learning activities more quickly. Students agreed with all the attributes and they agreed on the attributes with ranking $A_1 \succ A_5 \succ A_2 \succ A_4 \approx A_6 \succ A_3$. This shows that ADMLTI gives benefit to students in learning mathematics.

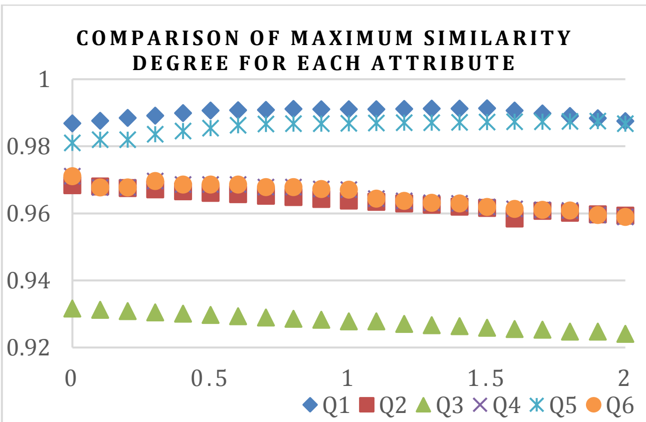

Figure 6. Comparison of maximum similarity degree for $\delta \in [0,2]$

Based on the degree of fuzziness δ from Tang and Lin (2011) the sensitivity analysis is used to measure the sensitivity of the FCM based on TFNs. Based on Figure 6, for $\delta \in [0,2]$, the ranking for the maximum degree of similarity remains unchanged for attribute A_1, A_3 and A_5. However, the ranking of the maximum degree for similarity between A_2 and A_4 have small changes with a difference in values less than 0.003. This indicates that when the degree of fuzziness δ changes for $\delta \in [0,2]$, students agreed on the attributes with ranking $A_1 \succ A_5 \succ A_2 \geq A_4 \approx A_6 \succ A_3$.

V. CONCLUSION

Various studies have evaluated the effectiveness of mathematics mobile applications, but the evaluation that considers the uncertainty of human judgement is largely understudied. This paper proposes an integrated fuzzy approach for evaluating the effectiveness of mathematics mobile application, namely as ADMLTI. The fuzzy evaluation is used due to the imprecise and vague human judgement and fuzziness condition. The pre and post-test results, which are obtained based on TFNs and DoC concept, show significant differences with a 10% increase in marks after learning Mathematics using ADMLTI. This indicates that ADMLTI gives a positive impact to students' performance which is consistent with the study by Supandi *et al.* (2017). The developed FCM based on TFNs can provide the α-cut for each fuzzy number for different values of $\alpha \in [0,1]$.

Based on the improvised FCM, the students agreed that ADMLTI gives positive impact in improving their learning activities which is consistent with Taleb *et al.* (2015). Furthermore, based on the degree of fuzziness δ, the sensitivity analysis shows that students agreed on the attributes with almost similar ranking for different values of $\delta \in [0,2]$. The degree of agreement on the satisfaction level of each attribute provides useful information in improving mathematics teaching and learning activities.

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