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# A Mathematical Programming Model for Exam-Invigilator Assignment Problem

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At the end of each term or semester academic institutions must assign invigilators to exams as it is an important administrative activity that must be performed. To create a good exam invigilator schedule manually is a complex and time-consuming process as it must satisfy various requirements and constraints. Therefore, the aim of this study is to propose a mathematical programming model to solve the exam invigilator assignment problem at Universiti Teknologi Mara (UiTM) Pahang, Raub Campus. The model, which was formulated by using integer programming, assigns lecturers to time slots and rooms. The objectives of the proposed model are to fairly assign duties to the invigilators, chief invigilator and standby invigilators among the academic staff. This study also proposes new constraints, which are the chief invigilator in a large room should be a senior lecturer and chief invigilator's position can only be assigned to a lecturer once. These two constraints have never been considered in any studies. The model is sufficiently flexible to be used with various operational requirements in most academic institutions. Computational experiment was conducted by using real data from UiTM Pahang Raub Campus. Results from the experiment demonstrated that the proposed model can produce a feasible and optimal timetable that satisfies all the constraints within a reasonably short time as compared to the manual assignment procedure.

**Keywords:** examination timetabling; integer programming; invigilator assignment

## I. INTRODUCTION

Examination timetabling problems are among the most important and difficult tasks faced by many academic institutions worldwide. The problems are combinatorial optimisation problems in which a set of examinations are required to be scheduled within a fixed number of time slots or periods and rooms so that no student has to take multiple examinations on the same time slot (Carter, 1986). The examination timetabling problems are subjected to various types of hard constraints that must be met at all costs and soft constraints that may be violated but should be satisfied as many as possible. Both hard and soft constraints vary among institutions, depending on their particular needs and limited resources. Examples and detailed explanations of hard and soft constraints that are used in British universities can be found in Burke *et al.* (1996).

Invigilator assignment is one aspect of the examination timetabling problems. The problem deals with scheduling of invigilators to a given number of examinations and rooms so that there are no conflicts or clashes (Ozturk *et al.*, 2010). Invigilator assignment is often done separately prior to or after the examination timetabling phase. Constructing an invigilator's schedule manually is a difficult task and it cannot be done quickly. In addition, the manual process is vulnerable to errors and may require several corrections and amendments before a satisfactory solution is obtained. The complexity of this problem depends mainly on the number of available invigilators, number of examinations and number of examination rooms. Although several models have been proposed in

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literature to solve the invigilator assignment problems, it is difficult to find a model or solution that can solve this problem generally because the needs and requirements vary significantly across different institutions.

In practice, there are various constraints that should be overcome to solve the invigilator assignment problem. Although the constraints differ among institutions, there are some common requirements that serve as basis for the general model. Among the constraints are invigilators must not be scheduled for more than one room in a time slot, invigilators cannot invigilate their own exam papers, and invigilation duties must be assigned fairly among invigilators. Unfair invigilation duties may generate conflicts between invigilators and the administration. On top of that, a lecturer preference survey conducted by Cowling *et al.* (2002) suggested that invigilators preferred to have two to three invigilation duties with one or two days gap, and lecturers with other responsibilities, such as administrative or research work, should be given less invigilation duties.

This study is concerned with the invigilator assignment problem at Universiti Teknologi MARA (UiTM) Pahang, Raub Campus. Currently, the problem is manually solved by an examination timetable committee. The process requires two to three days of work and sometimes the obtained solution failed to adhere to some requirements imposed by the scheduler. To improve the assignment process, a mathematical programming model based on integer linear programming approach was developed. The model aims to satisfy a set of constraints, which are lecturers cannot invigilate their own subjects, no lecturer is scheduled to invigilate two or more examinations on the same time slot, only one chief invigilator is required in a room, chief invigilator in a large room should be a senior lecturer and a lecturer can only be assigned as the chief invigilator once. the proposed approach can be applied to invigilator scheduling issues at any other educational institutions which encounter the same type of problems.

The remaining part of this paper proceeds as follows: Section 2 reviews previous related works on the exam invigilator assignment problem. Section 3 presents a description of the invigilator scheduling problem at UiTM Pahang, Raub Campus, including the assumptions of the adopted model. The formulation of the integer linear programming model for the problem is discussed in Section 4, which also comprises a complete description of the hard and soft constraints imposed on the model. The objective function of the model is also presented in Section 4. Meanwhile, the implementation of the proposed model by using real data is presented in Section 5. Finally, conclusion and possible future research studies are highlighted in Section 6.

## II. LITERATURE REVIEW

Over the past several decades, a number of approaches were proposed to solve a variety of invigilator timetabling problems. A common approach to solve the problem is to formulate the problem by using mathematical programming. Among the most interesting studies which had used this approach were those by Kahar and Kendall (2014) and Marti *et al.* (2000). Kahar and Kendall (2014) formulated a mathematical programming model based on integer programming for Universiti Malaysia Pahang (UMP). The model considers three other hard constraints in addition to the ones presented in Cowling *et al.* (2002). The constraints are the chief invigilator must be a lecturer, all staff must invigilate not more than three examinations within the exam period, and the total number of invigilators who are assigned to each room has to equal the number of invigilators required for each room. On top of that, a constructive algorithm that can produce good quality solutions was also proposed as compared to the software used by UMP.

Marti *et al.* (2000) formulated an exam invigilator assignment problem as a multi-objective integer programming model with a weighted objective function that integrated a preference function with a workload-fairness function. The model used the concept of combining good solutions to obtain a better solution. To solve the formulated model, a solution technique was used based on a scatter search. Koide and Iwata (2014) formulated a mathematical programming model for the invigilator assignment problem at Konan University by employing a mixed integer programming approach. The authors also built a prototype system by using a spreadsheet tool to find the solution. Koide (2015) later extended and revised the work by Koide and Iwata (2014) to deal with several new practical conditions for invigilator assignment at Konan University. However, the proposed model was not able to yield a feasible solution in an acceptable time for the system users. Recently, Hanum *et al.* (2015) formulated an exam invigilator timetabling problem by using the non-pre-emptive goal programming approach. The model offers more fairness by incorporating several preferences related to the equity of invigilating task number. The proposed model was successfully applied to a simple case of exam invigilator assignment at the Department of Mathematics in Bogor Agricultural University.

Several researchers have employed metaheuristic algorithms to solve the exam invigilator assignment problem. Erden *et al.* (2016), for instance, used a genetic algorithm to find a solution that did not have overlaps in exams or invigilation duties, while the invigilator preferences were satisfied as much as possible. In Awad and Chinneck (1998), a basic genetic algorithm framework was combined with a simple user interface based on readily available software tools to develop a computer-based system for assigning invigilators. Pokudom *et al.* (2010) used the ant colony system to generate exam invigilator schedules for educational institutes. The aim of the study was to reduce the time for organising each staff's invigilating schedule. The proposed method was able to produce equal invigilation duties among staff during normal and extra workload times and avoid exam proctoring on weekends and on any staff's engaged hours.

There are also studies which focused on developing a decision support system for solving the invigilator assignment problem. Ozturk *et al.* (2010), for example, developed a user-friendly web-based automated system based on a multi-objective mixed integer programming model for exam invigilator assignment problem. The system optimised objectives related to assignment cost, total assignment on individual loads and total assignment on undesired timeslots. The system was tested with real data provided by the Industrial Engineering Department of Eskisehir Osamngazi University. Another computer-based system for the exam invigilator assignment was developed by Ong *et al.* (2009). The system which optimised lecturers' preferences allowed lecturers to view the examination timetable, choose their preferred invigilation timeslots, specify the examination date and time of their own subjects and view their individual schedules. In addition, the system enabled lecturers to give their feedback and any other relevant information to the invigilation scheduling committee.

## III. INVIGILATOR ASSIGNMENT AT THE UiTM PAHANG RAUB CAMPUS

UiTM Pahang Raub Campus is one of the UiTM campuses with almost 3,000 students. The university has three faculties which offer five diploma programmes in Business, Banking, Public Administration, Computer Science and Statistics. At the end of each semester, students must attend examinations for a couple of weeks. The examination timetable is prepared by the Examination Unit of UiTM main campus. Once the examination timetable is ready, all faculties and branch campuses need to assign examinations and invigilators to rooms and time slots. At UiTM Pahang Raub Campus this task is done manually by an examination timetable committee which consists of eight to nine members and normally it takes three to four days to complete.

To formulate a mathematical programming model for the exam invigilator assignment problem, the following assumptions were considered in this study.

*Assumptions:*

a) The examination timetable is already available.
b) There are two time slots per day, morning and afternoon.
c) Examination rooms with sufficient capacity are available.

The model is focused on scheduling academic staff for the exams.

## IV. MODEL FORMULATION

In this section, the formulation of the proposed integer programming model for invigilator-exam assignment problem is described in detail. Model by Kahar and Kendall (2014) constitutes the core of the study's model, although there are a few differences stemming from specific requirements in this study. To provide a better overview of the notations used, the following list contains all the sets, indices, parameters and decision variables.

*Sets:*

$N$ Set of all examinations

$S$ Set of academic staff

$S_p$ Set of academic staff with administrative post ($S_p \subseteq S$)

$S_b$ Set of academic staff without administrative post ($S_b \subseteq S$)

$R$ Set of rooms

$T$ Set of time slots

*Indices:*

$i$ Index for exams, $i \in \{1, \dots, N\}$

$s$ Index for staff, $s \in \{1, \dots, S\}$

$r$ Index for rooms, $r \in \{1, \dots, R\}$

$t$ Index for time slots, $t \in \{1, \dots, T\}$

*Parameters:*

$l_r$ The number of invigilators required in each room $r$.

$n_t$ The number of standby invigilators required in each time slot $t$.

$h_s$ 1 denotes a senior academic staff and 0 otherwise.

$a_{is}$ The exam-staff matrix. 1 denotes that the lecturer teaches the course in that semester, 0 otherwise.

$c_{it}$ 1 if examination $i$ is scheduled on time slot $t$, 0 otherwise.

$w_{ir}$ 1 if examination $i$ is assigned to room $r$, 0 otherwise.

$q_{rt}$ 1 if room $r$ is assigned to time slot $t$, 0 otherwise.

*Decision Variables:*

$x_{str}$ 1 if staff $s$ is assigned to invigilate time slot $t$ in room $r$ as an invigilator, and 0 otherwise.

$y_{str}$ 1 if staff $s$ is assigned to invigilate timeslot $t$ in room $r$ as a chief invigilator, and 0 otherwise.

$z_{st}$ 1 if staff $s$ is assigned as a standby invigilator for timeslot $t$, and 0 otherwise.

*Objective function:*

The objective function of the model is constructed with the aim of finding a schedule that balances the number of invigilation duties among the staff. It can be formulated as in Kahar and Kendal (2014):

$$\min \sum_{s=1}^{S} f(x_{str}, y_{str}) + g(z_{st}) \quad (1)$$

where

$$f(x_{str}, y_{str}) = \begin{cases} 0, & if\ \sum_{t=1}^{T}\sum_{r=1}^{R}(x_{str} + y_{str}) \le \frac{\sum_{t=1}^{T}\sum_{r=1}^{R} q_{rt} l_r}{S} \\ 1, & otherwise \end{cases} \quad (2)$$

and

$$g(z_{st}) = \begin{cases} 0, & if\ \sum_{t=1}^{T} z_{st} \le \frac{\sum_{t=1}^{T} n_t}{S_b} < 0 \\ 1, & otherwise \end{cases} \quad (3)$$

The objective function (1) can be rewritten as

$$\min \sum_{s=1}^{S}(u_s + v_s) \quad (4)$$

subject to

$$\sum_{t=1}^{T}\sum_{r=1}^{R}(x_{str} + y_{str}) - M u_s \le \frac{\sum_{t=1}^{T}\sum_{r=1}^{R} q_{rt} l_r}{S}\ ,\ \forall s \in S \quad (5)$$

$$\sum_{t=1}^{T} z_{st} - M v_s \le \frac{\sum_{t=1}^{T} n_t}{S_b}\ ,\ \forall s \in S_b \quad (6)$$

where $M$ is a large positive number, $u_s (s = 1,2, \dots, S)$ and $v_s\ (s = 1,2, \dots, S)$ are indicator variables restricted to be either zero or one.

*Constraints:*

The constraints for the examination invigilator assignment model are briefly listed as follows.

a) Invigilators or chief invigilators are not allowed to invigilate examinations of the courses they had taught.

$$\sum_{i=1}^{N}\sum_{t=1}^{T}\sum_{r=1}^{R}(a_{is} c_{it} w_{ir})(x_{str} + y_{str}) = 0\ ,\ \forall s \in S \quad (7)$$

b) The total number of staffs to invigilate room $r$ in time slot $t$ must be equal to the number of invigilators required for each room.

$$\sum_{s=1}^{S}(x_{str} + y_{str}) = q_{rt} l_r\ ,\ \forall r \in R, \forall t \in T \quad (8)$$

c) There is only one chief invigilator for room $r$ in time slot $t$.

$$\sum_{s=1}^{S} y_{str} = 1\ ,\ \forall r \in R\ , \forall t \in T \quad (9)$$

d) Staff without any administrative post are required to be a standby invigilator not more than once.

$$\sum_{t=1}^{T} z_{st} \le 1\ ,\ \forall s \in S_b \quad (10)$$

e) No staff is assigned to multiple rooms at the same time.

$$\sum_{r=1}^{R}(x_{str} + y_{str}) + z_{st} \leq 1 \ \ , \forall s \in S_b \ , \forall t \in T \quad (11)$$

f) Staff without any administrative post are required to invigilate exams not more than $k_b$ times within the exam period.

$$\sum_{t=1}^{T}\sum_{r=1}^{R}(x_{str} + y_{str}) \leq k_b \ \ , \ \forall s \in S_b \quad (12)$$

g) Staff with administrative post can only be assigned to one invigilation duty within the exam period.

$$\sum_{t=1}^{T}\sum_{r=1}^{R}(x_{str} + y_{str}) = 1 \ \ , \ \forall s \in S_p \quad (13)$$

h) Staff can only be assigned as a chief invigilator not more than once.

$$\sum_{t=1}^{T}\sum_{r=1}^{R} y_{str} \leq 1 \ \ , \ \forall s \in S \quad (14)$$

i) The chief invigilator in a large room must be a senior lecturer.

$$\sum_{t=1}^{T} q_{1t} y_{str} \leq h_s \ \ , \ \forall s \in S \quad (15)$$

j) The total number of standby invigilators in time slot $t$ must be equal to $n_t$.

$$\sum_{s=1}^{S} z_{st} = n_t \ \ , \ \forall t \in T \quad (16)$$

## V. MODEL IMPLEMENTATION

In this section, the researchers present the implementation of the model into a case of exam invigilator scheduling problem at Universiti Teknologi MARA (UiTM) Raub Campus. The problem (for March 2016) consisted of planning 87 different examinations within 19 days. Two time slots were reserved for examinations per day. The university has 105 academic staff who could serve as invigilators during the exams. Eight rooms of different capacities were available for examinations, and the total number of invigilators required in each room varied from a minimum of two to a maximum of five. In the experiment, the researchers set both $k_b$ in Equation (12) and $n_t$ in Equation (16) as 2.

The optimal solution was obtained in a few seconds by using the MATLAB R2017b software programme. The results are summarised in Table 1. The second column of this table shows the solution produced by the proposed model while the third column shows the solution obtained by solving the problem manually. It can be clearly seen from the table that the proposed exam invigilator assignment model could yield a solution that satisfies all constraints without any cost to the objective function. When the solution was compared to the manually prepared schedule, it was observed that the schedule from the proposed model was much better in satisfying all constraints. Note that in the manually prepared schedule, several important constraints were violated. For example, there were four cases, whereby staff without administrative post had to be on standby as invigilators and one case in which a staff had to invigilate more than two examinations. There were also three staff with administrative post who had to invigilate more than once during the exam period. In addition, the requirement of avoiding the chief invigilators or invigilators to invigilate the courses they taught was violated 6 times and the constraint of each staff can be assigned as an invigilator once was violated 7 times. However, by using the proposed model all these constraints were fully satisfied. Therefore, it can be concluded that the proposed model is able to produce a superior solution as compared to the manually prepared solution.

Table 1. Invigilator timetabling result

| Constraints | Proposed approach | Manual Process |
|---|---|---|
| Invigilators or chief invigilators are not allowed to invigilate examinations of the courses they taught | Yes | No (6) |
| Staff without administrative post must invigilate not more than two examinations within the exam period. | Yes | No (1) |
| Each staff can only be assigned as a chief invigilator not more than once | Yes | No (7) |
| Staff without administrative post are required to be a standby invigilator not more than once. | Yes | No (4) |
| Staff with administrative post can only be assigned to one invigilation duties within the exam period. | Yes | No (3) |
| Chief invigilator in a large room must be a senior lecturer. | Yes | Yes |
| Objective function value | 0 | - |

## VI. CONCLUSION

The process of assigning invigilators to examinations is a very challenging task. It is difficult to produce the optimum exam invigilator assignment schedule by solving the problem manually. In this paper, a mathematical programming model for the exam invigilator assignment problem was presented. The model was formulated by using integer linear programming approach and consisted of several common hard constraints that must be satisfied. This model was successfully tested by using real data from UiTM Pahang Raub Campus. The results demonstrated that the model could find an optimal solution that adhered to all requirements.

Concerning the model constraints, there are several practical requirements which can be included in the future such as the number of invigilators must be proportional to the number of students and invigilation duties on weekends should be fairly assigned among invigilators. However, the addition of these constraints may create certain complexities in terms of achieving an optimal solution. Therefore, further research is required into the use of heuristic or metaheuristic techniques to solve this problem.

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