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# Chaotic Existence Analysis on Short Term Traffic Flow in Urban Network

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Traffic flow is a continuous phenomenon. The irregular patterns in the traffic flow data show the complexity of the system under a variety of internal and external factors restrict and influence. The possibility of making short range forecasting of traffic flow using chaos approach is by investigating the presence of chaotic behaviour. Traffic flow data of four stations located in Selangor, Malaysia were analysed. There were three methods employed in this analysis; (1) phase space plot, (2) Cao method and (3) Lyapunov exponent. The phase space plot can be constructed by phase space. There were two parameters needed in phase space reconstructed; (1) time delay, $\tau$ that is calculated by using average mutual information (AMI) and (2) embedding dimension, $m$ that is obtained from Cao method. The traffic flow data were analysed to reveal the existence of chaotic behaviour. Therefore, short range forecasting of traffic flow using chaos approach can be applied to show the suitability of chaos approach to forecast the traffic flow time series data in Malaysia.

**Keywords:** Cao method; chaotic; Lyapunov exponent; phase space plot; traffic flow

## I. INTRODUCTION

A huge number of vehicles are one of the factors that impact the traffic problem in urban area that can affect daily life. Furthermore, traffic can lead to vehicles wasting more fuel which causes pollution (Xie & Choi, 2017). This situation is significantly related with the important role of traffic management (Jieni & Zhongke, 2008; Priambodo & Ahmad, 2018). Therefore, efficient traffic flow forecasting may help to analyse the traffic flow and monitor the need in urban area as well as provide citizens with a better choice of public transport (Vlahogianni *et al.*, 2004). The possibility of making short range forecasting of traffic flow using chaos approach is by analysing the existence of chaotic behaviour in the traffic flow data. Hence, the main point for this research is to determine the existence of chaotic behaviour on short range traffic flow data series in forecasting traffic flow using chaos approach.

Referring to complex dynamics of traffic flow, it may face challenges to analyse the dynamics of certain traffic flow data. Some methods that refer to nonlinear analysis can analyse the nonlinear dynamics. However, systems that are very complex can be analysed using chaos approach (Shang *et al.*, 2005). Furthermore, chaos theory can distinguish between deterministic and random systems. Shang *et al.* (2005) stated that chaos approach can be applied to highly nonlinear systems.

There are many literatures that have been published on chaos approach. It started with the "discoverer" of chaos by Lorenz and Martin (1993) who provided the ideas and Abarbanel (1996) who gave outstanding analysis techniques of chaotic data. It continued with research from Hilborn (2000) that gave details on determination of chaos. Meanwhile, chaotic behaviour in traffic that gives impact to human reaction was studied by Safonov *et al.* (2002). Recently, an analysis has been done on traffic data for incident detection performed by recurrence plot that was based on phase space. Fragkou *et al.* (2018) and Cheng *et al.* (2017) proposed various sources and many methods based on traffic flow forecasting based on chaos approach and supported with the regression method.

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## II. CHAOTIC EXISTENCE ANALYSIS

The main advantage of identifying chaotic behaviour is for short range forecasting (Vlad, 2010). There are many methods in order to determine chaotic behaviour in a time series. Table 1 shows the methods to determine chaotic existence in time series data. In this research, phase space plot, Cao method and Lyapunov exponent are involved in chaotic existence analysis.

Table 1. List of methods for detecting chaos

| Method | Data |
|---|---|
| Phase space plot | Ozone (Hamid, 2018) |
| Cao method | River flow (Adenan & Noorani, 2016) |
| Lyapunov method | Traffic flow (Cheng *et al.*, 2017) |
| Kolmogorov entropy | Optical signal (Peng *et al.*, 2017) |
| Correlation dimension | River discharge (Albostan & Önöz, 2015) |

### A. Phase Space Plot

An appropriate phase space reconstruction may reveal the presence of an attractor in the phase space trajectories. The Takens' theorem is used to reconstruct the phase space. Through phase space plots, the phase space is plotted in plane by using computed parameters. If there is an attractor in the plotted phase, the system is chaotic (Sivakumar, 2002). In doing this, a dynamic univariate time series, $X$ is applied:

$$X=\left(x_1,x_2,...,x_{N-1},x_N\right). \tag{1}$$

The phase space reconstruction can be reconstructed according to:

$$Y_j=\left(x_j,x_{j+\tau},...,x_{j+(m-1)\tau}\right), \tag{2}$$

with $j=1,2,...,N-(m-1)\tau$ where $Y_j\in\mathbb{R}^m$, $\tau$ is the time delay and $m$ is the embedding dimension.

The $\tau$ can be calculated when the average mutual information (AMI) reaches the first minimum (Frazier *et al.*, 2004). AMI can be computed by:

$$I(T)=\frac{1}{N}\sum_{a=1}^{N}\omega\left(u_a,u_{a+T}\right)\log_2\left[\frac{\omega\left(u_a+u_{a+T}\right)}{\omega\left(u_a\right)\omega\left(u_{a+T}\right)}\right] \tag{3}$$

with $\omega\left(u_a\right)$ and $\omega\left(u_{a+T}\right)$ are the probabilities to find a given value of $u_a$ and $u_{a+T}$ of the time series $X$. Meanwhile, $\omega\left(u_a+u_{a+T}\right)$ is joint probability referring to $\omega\left(u_a\right)$ as well as $\omega\left(u_{a+T}\right)$.

An optimal embedding dimension can suggest the number of possible factors to show the low-dimensional chaos in a certain system (Sivakumar, 2002). Hence, Cao method has been used to calculate the dimension in order to determine a possible optimal dimension. Optimal dimension from Cao method can be calculated by:

$$\beta1\left(m\right)=\frac{\beta1\left(m+1\right)}{\beta\left(m\right)} \tag{5}$$

$$\beta\left(m\right)=\frac{1}{N-m\tau}\sum_{n-1}^{N-m\tau}\frac{\left\|Y_n^{m+1}-Y_{jj}^{m+1}\right\|}{\left\|Y_n^m-Y_{jj}^m\right\|} \tag{6}$$

with $\|\bullet\|$ referring to Euclidean distance and $Y_{jj}^m$ is nearest neighbour of $Y_n^m$. The $m+1$ is the optimal dimension, if $\beta1\left(m\right)$ saturated when $m$ increases (Zaim & Hamid, 2017).

### B. Cao Method

Before this, we have discussed that Cao method can give optimal dimension. This method can also distinguish whether the dynamic of time series is chaotic or random. If random time series, $\beta1\left(m\right)$ may not be saturated by increasing $m$. $\beta2\left(m\right)$ can be calculated by:

$$\beta2\left(m\right)=\frac{\beta^*\left(m+1\right)}{\beta^*\left(m\right)}, \tag{7}$$

$$\beta^*\left(m\right)=\frac{1}{N-m\tau}\sum_{i=1}^{N-m\tau}\left|x_{i+m\tau}-x_{i+m\tau}^{NN}\right|. \tag{8}$$

There exist chaotic behaviour if $\beta2\left(m\right)$ is not equal to 1 which denotes chaotic behaviour exists in the time series (Hamid *et al.*, 2017).

### *C. Lyapunov Exponent*

The rate of divergence or convergence for every dimension of a time series can be measured by Lyapunov exponent. The maximum Lyapunov exponent can be referred to as the exponential growth between nearby trajectories. Meanwhile, the positive value Lyapunov exponent ( $\lambda$ ) represent the chaotic behaviour of phase space. Hence, it showed the chaotic character of the certain attractor *(Shang et al.*, 2009). Definition of Lyapunov exponent is:

$$\lambda = \frac{1}{N} \ln \frac{d_n}{d_0}, \tag{9}$$

$$d_0 = \left| x_j - x_j^{NN} \right| \text{ and } d_N = \left| x_{j+N} - x_{j+NN}^{NN} \right| , \tag{10}$$

with $x_j$ as the time series data and $x_j^{NN}$ as the nearest neighbour of $x_j$ (Lan *et al.*, 2003). In this study, a method from Rosenstein *et al.* (1993) is used to calculate the value of Lyapunov exponent ( $\lambda$ ). The first step is to compute algorithm of the average distance $Y_{n_0}$ in phase space for all points in $Y_n$ in neighbourhood. The phase space reconstruction is using parameter $\tau$ and $m$ that have been pre-determined. $N$ refers to the number of points. This algorithm is repeated for each $N$ in the trajectories. As such, the stretching factor ($S$) can be calculated as follows:

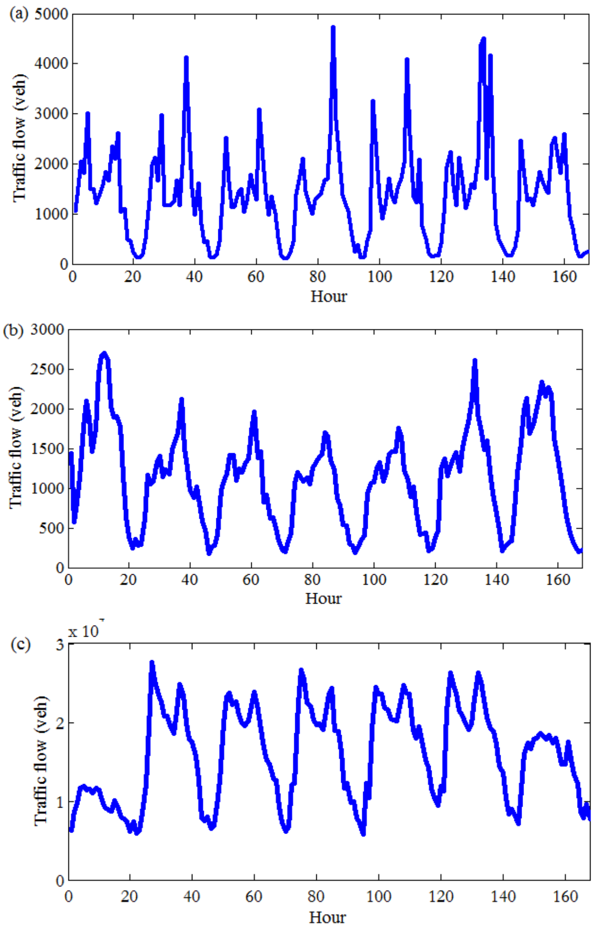
$$S = \frac{1}{N} \sum_{n_0=1}^{N} \ln \left[ \frac{1}{\left| u_{Y_{n_0}} \right|} \left| Y_{n_0} - Y_n \right| \right], \tag{11}$$

where $\left| u_{Y_{n_0}} \right|$ is the number of neighbour near $Y_{n_0}$. If we plot the $S$ with the separation time, we can get an estimate value of Lyapunov exponent from the gradient of line. The positive value of the slope line can be the indicator of chaotic behaviour of the time series.

## III. DATA

Traffic flow variables that are often observed include volume, speed, density, travel time and headways (Kumar *et al.*, 2013). In this study, volume (vehicle per hour) is considered. Daily activities can contribute to traffic congestion in high population distribution areas. Hence, this research is conducted in high population distribution areas. According to Department of Statistics Malaysia, Selangor has a high population distribution. Therefore, this study is conducted in Selangor which involves four stations of Banting-Kajang (BR204), Klang-Sabak Bernam (BR501), Klang-Kuala Lumpur (BR807) and Kuala Lumpur-Karak (BR902).

After reviewing process is made with Highway Department Division, Ministry of Work, Malaysia (MoW), only Average Daily Traffic (ADT), Vehicle Classification (VC), Peak Hour Volume (PHV) can be obtained from MoW. ADT is used to forecast the traffic volume and pavement thickness design. ADT is obtained from manual classical traffic count (MCC) which is carried out twice a year. For each counting implement, the counting is done 7 consecutive days for data collection. Second counting implement is carried out six months after the first counting. Hence, ADT used in this research refers to vehicle per hour time series data.



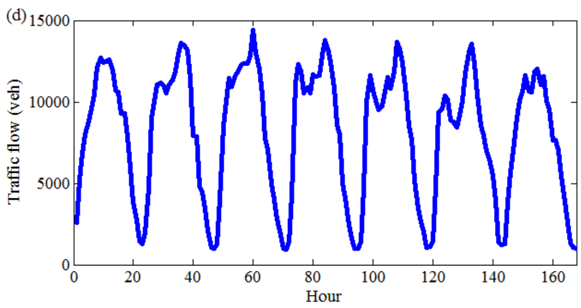



Figure 1. Time series data of traffic flow at (a) BR204, (b) BR501, (c) BR807 and (d) BR902

Data from four stations that have been used in this research can be referred to in Figure 1. Data that have been used in this study were retrieved from Highway Department Division, Ministry of Work, Malaysia (MoW) which involved the first counting data collected in March 2014 for 7 consecutive days. Hence, only 168 data that referred to total vehicles per hour were analysed. Based on Figure 1, overall data from four stations showed that there were different volumes. Station BR807 stated the highest volume with more than 25000 vehicles using Klang-Kuala Lumpur path while the highest volume at BR501 did not reach 3000 vehicles per hour. Hence, it is important to determine the existence of chaotic behaviour before short range forecasting using chaos approach can be done.

## IV. RESULTS AND DISCUSSION

To plot phase space, the delay time $\tau$ was computed using the AMI method for each station (Figure 2). The first minimum in AMI can be considered as the optimal $\tau$ . Hence, the optimal $\tau$ is chosen as 4, 3, 4 and 4 hours for station BR204, BR501, BR807 and BR902, respectively.

The $\beta 1(m)$ from Cao method is calculated for embedding dimensions using the calculated $\tau$ . Figure 3 shows the relationship between $\beta 1(m)$ and increasing $m$ for each station. $\beta 1(m)$ stops changing and the values of $m$ can be determined. Hence, the optimal $m$ is chosen as 6, 5, 4 and 6 for stations BR204, BR501, BR807 and BR902, respectively and all time series were been analysed as chaotic behaviour. Therefore, we can conclude that a minimum 4 of variables influence traffic flow in Selangor based on the embedding dimension from each station.

Reconstruction of phase space uses the combination of parameter that has been calculated before. Then, the phase space is plotted to reveal the existence of attractor in three dimensional plots. However, all phase spaces plotted in Figure 4 do not show any identifiable pattern. Hence, we have to observe that it might be possible due to the complexity of the system. This complexity of the system also happens when determining the chaotic behaviour in Euro-Leu exchange rate (Vlad, 2010). As such, analysis of chaotic existence has to be continued using other methods.

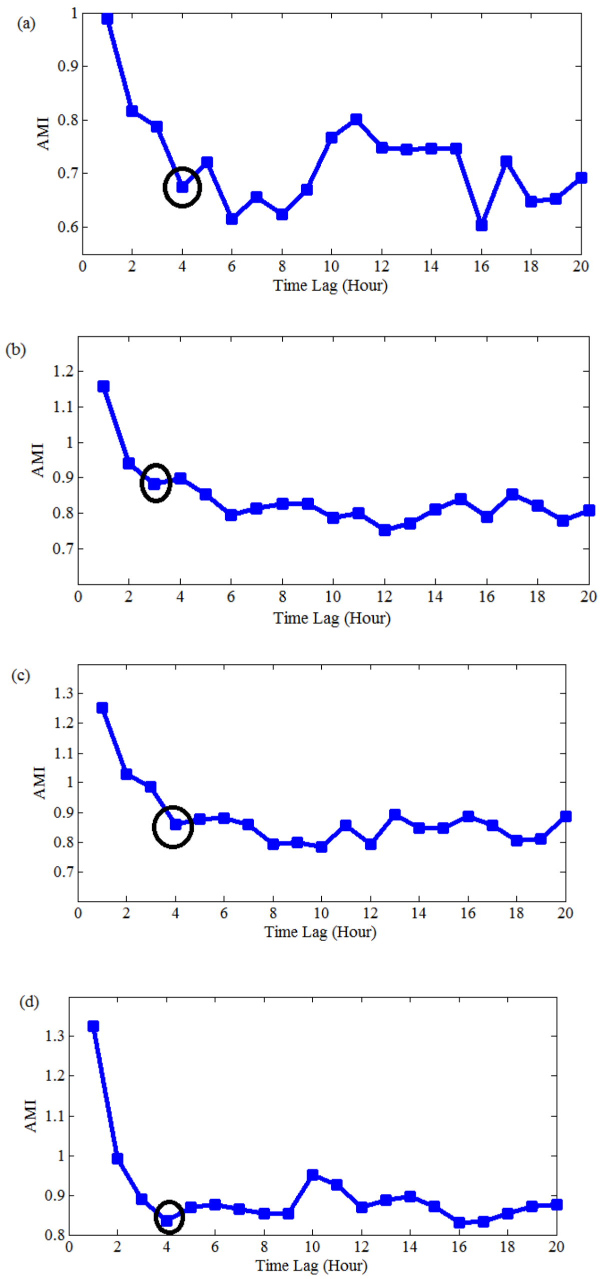



Figure 2. Selection of the delay time $\tau$ for the hourly traffic flow data using AMI at (a) BR204, (b) BR501, (c) BR807 and (d) BR902

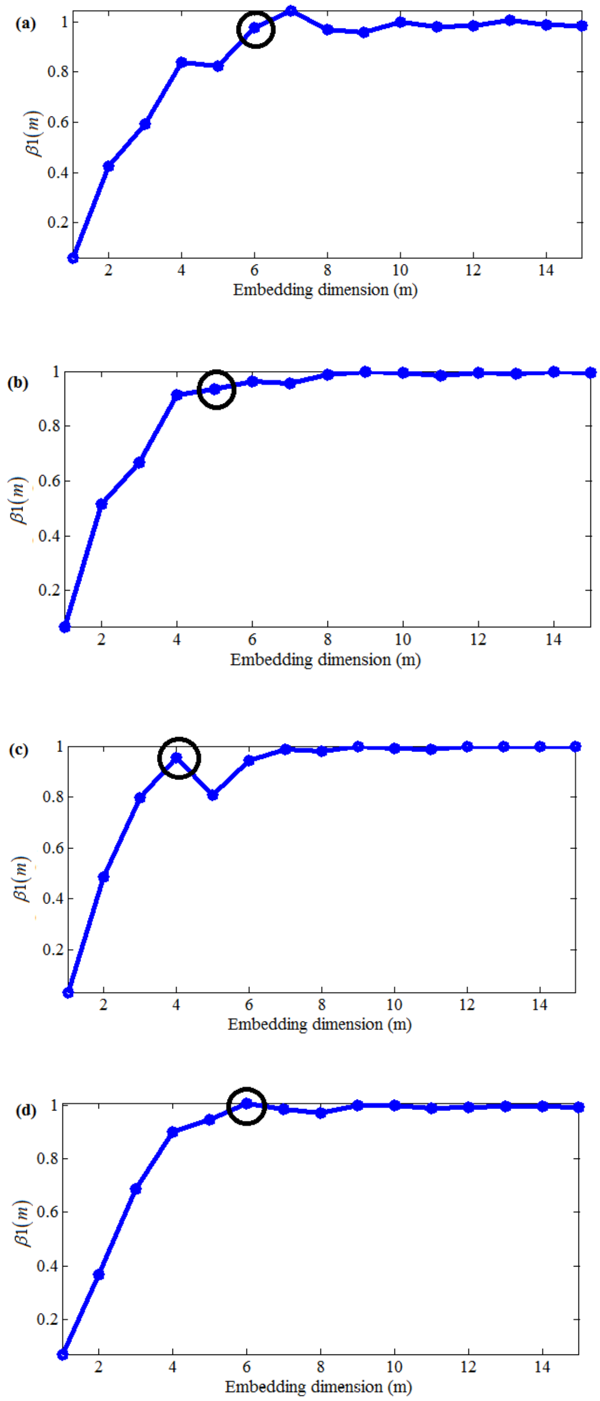



Figure 3. Selection of the embedding dimension $m$ delay time $\tau$ for the hourly traffic flow data using AMI at (a)BR204,(b) BR501, (c) BR807 and (d) BR902

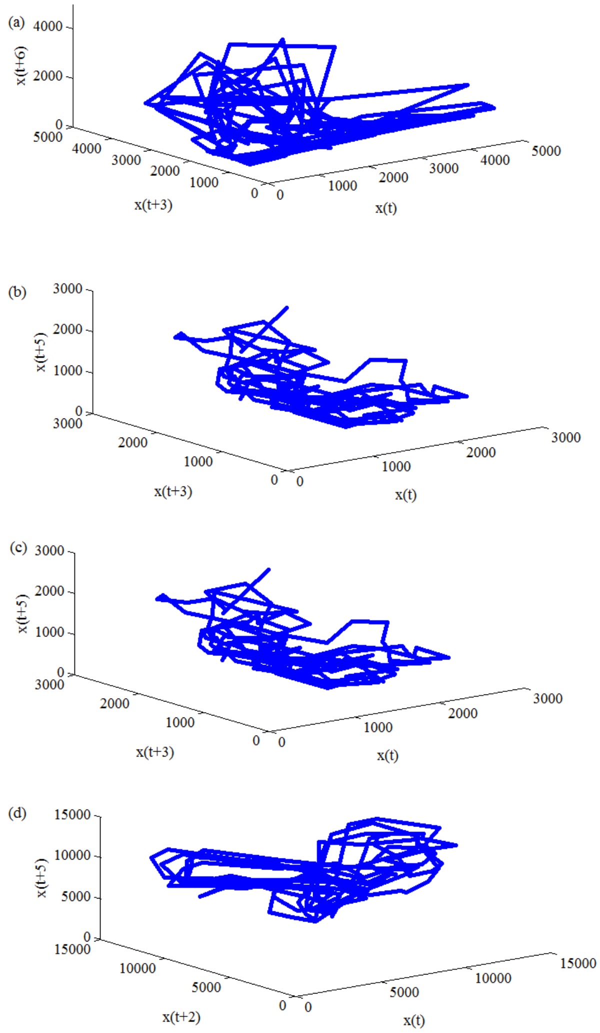
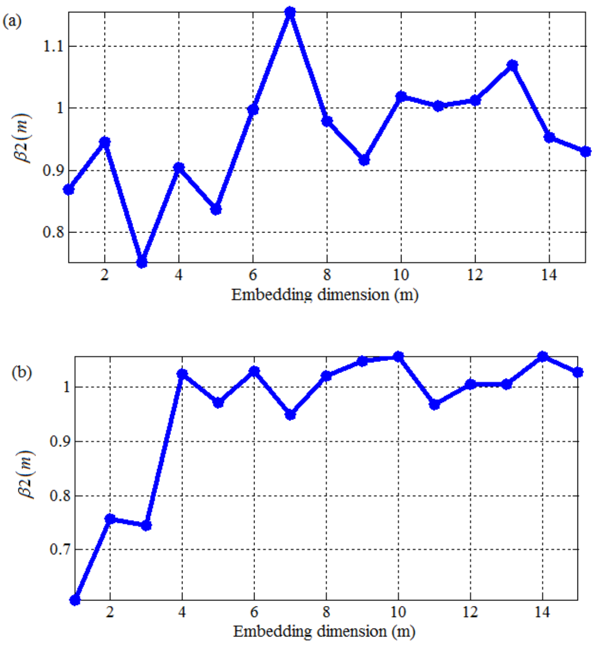



Figure 4. Phase space plot of traffic flow data at (a)BR204,(b) BR501, (c) BR807 and (d) BR902



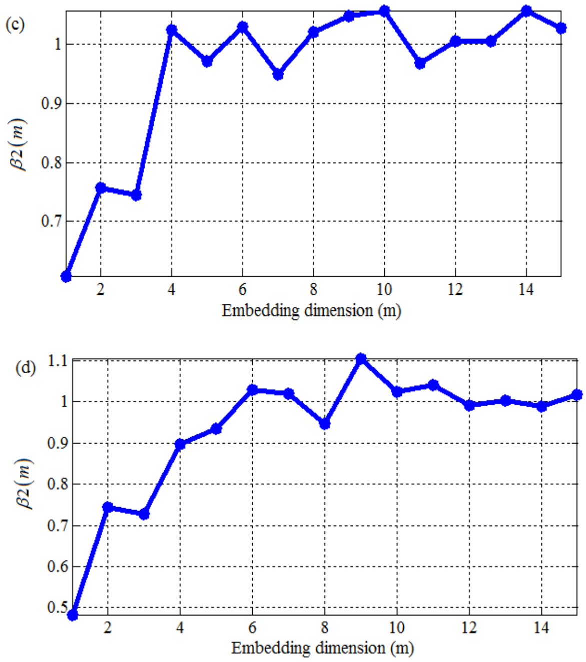
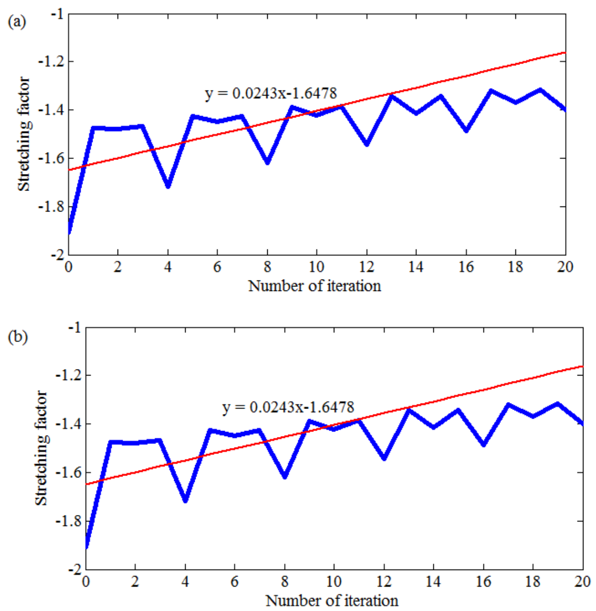



Figure 5. Chaotic analysis existence using Cao method at (a)BR204,(b) BR501, (c) BR807 and (d) BR902

The existence of chaotic behaviour can also be revealed using $\beta 2(m)$. By analysing all plots in Figure 5, there exists at least one value where $\beta 2(m)$ is not equal to 1 for all data at different station. Hence, there exists chaotic behaviour based on Cao method for all time series that have been analysed.



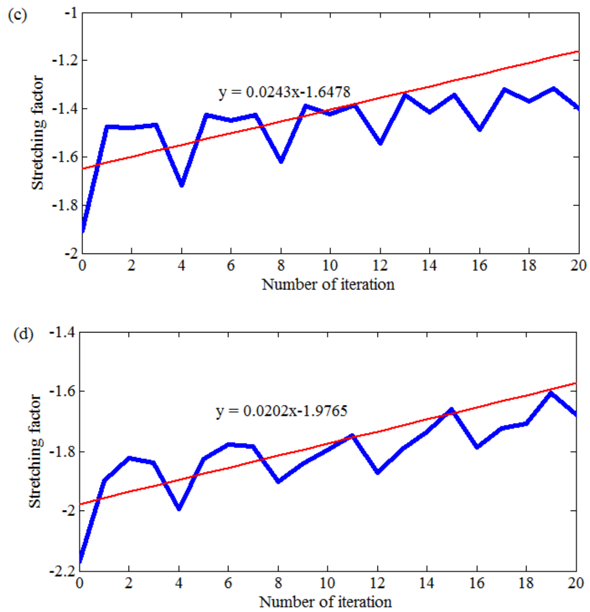




Figure 6. Chaotic analysis existence using Lyapunov exponent at (a)BR204,(b) BR501, (c) BR807 and (d) BR902

Figure 6 shows the curve for the stretching factor versus the number of iterations. The gradient is related to the positive Lyapunov exponent. It is obtained after the least-squares line fit has been analysed for each data. The positive value of the largest Lyapunov exponent exhibits an exponential divergence of the trajectories and consequently, the existence of chaotic behaviour for all stations can be revealed.

## V. CONCLUSION

Traffic flow (total vehicle per hour) in urban area located at four stations in Selangor were analysed and reconstructed. The possibility to determine the chaotic behaviour is related to deterministic dynamics. Hence, chaos theory was implemented in the analysis in order to forecast traffic flow using chaos approach. The analysis was based on nonlinear dynamic methods: (1) phase space plot, (2) Cao method and (3) Lyapunov exponent to determine the chaotic existence. The result shows: (1) All phase space plot does not show any specific pattern due to complexity of traffic flow systems; (2) $\beta 1(m)$ is saturated by increasing $m$ and there exists at least one value where $\beta 2(m)$ is not equal to 1 for all data at different stations; and (3) the positive value of the largest Lyapunov exponent is exhibited. In conclusion, all time series that have been analysed showed chaotic behaviour based on analysis using Cao method and Lyapunov exponent. Hence, all time series data that have been analysed were ready to be forecasted using chaos approach.

## VI. ACKNOWLEDGEMENT

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