Development of Novel Subclasses for Bi-Univalent Functions

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This manuscript presents the development of new subclasses for bi-univalent functions and the subclasses are closely related to Chebyshev polynomials having Al-Oboudi differential operator. The functions contained in the subclasses were used to account for the initial coefficient estimates of \( |a_2| \) and \( |a_3| \).

**Keywords:** coefficient estimates; subordination; Chebyshev Polynomials; bi-univalent; Al-Oboudi Operator

### I. INTRODUCTION

Let \( \mathbb{C} \) be a set of complex numbers, \( \mathbb{R} = (-\infty, \infty) \), a set of real numbers and \( \mathbb{N} = \{1, 2, 3, \ldots \} = \mathbb{N}_0 \setminus \{0\} \) representing a set of positive integers. Then, let \( A \) be defined as \( A = \{ z \in \mathbb{C}, |z| < 1 \} \), and an open unit disc denoted as a class function expressed in (1).

\[
f(z) = z + \sum_{k=2}^{\infty} a_k z^k
\]

The subclass of \( A \) is represented by \( S \), which is a normalised analytic function as shown in \( f'(0) = 1 \) and \( f(0) = 0 \).

Given that \( K(\alpha) \) and \( S^*(\alpha) \) are expressed as the convex and starlike functions respectively in the order of \( \alpha (0 \leq \alpha < 1) \), they can be represented by the renowned subclasses of \( S \).

For \( f \in A \), Al-Oboudi (2004) represented the operator as thus:

\[
D^2_\delta f(z) = f(z),
\]

\[
D^2_\delta f(z) = (1 - \delta) f(z) + \delta f'(z): = D_\delta f(z) (\delta \geq 0) \quad (2)
\]

\[
D^2_\delta f(z) = D_\delta \left( D^{n-1}_\delta f(z) \right) \quad (n \in \mathbb{N}) \quad (3)
\]

As \( f \) is represented in (1), it can be seen from (2) and (3) that:

\[
D^n_\delta f(z) = z + \sum_{k=2}^{\infty} (1 + (k - 1) \delta)^n a_k z^k \quad (n \in \mathbb{N}_0) \quad (4)
\]

where \( D^n_\delta f(0) = 0 \). The Sălăgean’s differential operator is obtained when:

\[
\delta = 1. \quad (Sălăgean, 1983).
\]

A function \( f \in A \) is called univalent on \( \Delta \) (or schlicht or one-to-one) if \( f(z_1) \neq f(z_2) \) for all \( z_1, z_2 \in \Delta \) with \( z_1 \neq z_2 \). Based on Duren (1983), the theorem of Koebe’s one-quarter showed the images of \( f \) for every univalent function, \( f \in S \) enclosing \( 1/4 \) radius disc. Hence, every inverse function of \( f^{-1} \) in \( f \in A \) can be defined as:

\[
f^{-1}(f(z)) = z \quad (z \in U),
\]

\[
f^{-1}(w) = w
\]

\[
\left( |w| < r_0; r_0 \geq \frac{1}{4} \right).
\]

Furthermore, \( f^{-1} \) is shown as:

\[
g(w) = f^{-1}(w) = w - a_2 w^2 + (2a_2^2 - a_3)w^3 - (5a_2^3 - 5a_2a_3 + a_4)w^4 + \cdots
\]

\[
(5)
\]

A function \( f \in A \) is called bi-univalent in \( \Delta \), if every of \( f \) or \( f^{-1} \) is univalent. Thus, the notation of bi-univalent functions class is expressed as \( \Sigma \). The background and previous works on \( \Sigma \) can be found in Srivastava et al. (2010) and Brannan and Taha (1986; 1988). Indeed, the research

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findings of Srivastava et al. (2010) have been used as the basis of revitalisation for the research of numerous subclasses of bi-univalent functions class $\Sigma$. Several researchers that conducted similar studies included: Aldawish et al. (2020), Khan et al. (2020), Hern and Janteng (2020), Omar et al. (2019), Porwal and Darus (2013), Bulut (2013), Çağlar et al. (2013), Hayami and Owa (2012), Xu et al. (2012a), Xu et al. (2012b) and Frasin and Aouf (2011).

Next, the concept of subordination was used, given that $f(z) < g(z)$, with $f$ being a subordinant to $g$, and both functions taken to be analytic. This implies that $f(z) = g(w(z))$, where $w$ is taken as analytic in $\Delta$, which corresponds to $|w(z)| < 1$ and $w(0) = 0$.

Chebyshev polynomials applied in this study, is very relevant in numerical analysis. Chebyshev orthogonal polynomials are associated with important findings on $T_n(x)$ and $U_n(x)$ expressed as shown in (6).

$$
T_n(x) = \cos n \theta, \quad U_n(x) = \frac{\sin((n+1)\theta)}{\sin \theta} \quad (6)
$$

where $(-1 < x < 1)$ is expressed as $x = \cos \theta$, and subscript $n$ indicated the degree of the polynomial. With the use of the function: $H(z, t) = \frac{1}{1-2tzt+2z^2}$, we identified that when $t = \cos \alpha$, $\alpha \in \left(-\frac{\pi}{3}, \frac{\pi}{3}\right)$, thus, for all $z \in \Delta$,

$$
H(z, t) = 1 + \sum_{n=1}^{\infty} \frac{\sin((n+1)\alpha)}{\sin \alpha} z^n = 1 + 2 \cos \alpha z^2 + (3 \cos^2 \alpha - \sin^2 \alpha)z^4 + \ldots
$$

hence, we have:

$$
H(z, t) = 1 + U_1(t)z + U_2(t)z^2 + \ldots \\
(z \in \Delta, t \in (-1,1))
$$

for $U_{n-1} = \frac{\sin(n \arccos t)}{\sqrt{1-t^2}}$, $n \in \mathbb{N}$ and equally

$$
U_n(t) = 2tU_{n-1}(t) - U_{n-2}(t),
$$

where:

$$
U_1(t) = 2t; \\
U_2(t) = 4t^2 - 1; \\
U_3(t) = 8t^3 - 4t, \ldots
$$

Next, $T_n(t)$ that generated the function is represented as:

$$
\sum_{n=0}^{\infty} T_n(t)z^n = \frac{1-tz}{1-2tz+z^2} \quad (z \in \Delta),
$$

with $t \in [-1, 1]$. Nevertheless, $T_n(t)$ and $U_n(t)$ of the Chebyshev polynomials have the following relationships:

$$
\frac{dT_n(t)}{dt} = nU_{n-1}(t), \\
T_n(t) = U_n(t) - tU_{n-1}(t), \\
2T_n(t) = U_n(t) - U_{n-2}(t).
$$

More details on the applications of Chebyshev polynomials can be found in the studies of Doha (1994) and Mason (1967).

Inspired by the recent findings on the bi-univalent functions by Güney et al. (2017), Altinkaya and Yalcin (2016), Dziok et al. (2015), Murugusundaramoorthy et al. (2015), and Vijaya et al. (2014), we proposed the new subclasses of $\Sigma$ and determined the initial coefficients of $|a_{32}|$ and $|a_{33}|$ by the application of Chebyshev polynomials associated with Al-Oboudi differential operator.

II. METHODS

The main results of the subclasses were obtained by the application of the following definitions:

**Definition 1.**

For $0 \leq \lambda \leq 1$, $\delta \geq 0$, $n \in \mathbb{N}$ and $t \in (-1, 1), (f \in \Sigma)$ of the form (1) is said to be in the class of $N^\delta_2[n, \lambda, H]$, if:

$$
(1-\lambda) \frac{D_\delta^{n+1}f(z)}{D_\delta^n f(z)} + \lambda \frac{D_\delta^{n+2}f(z)}{D_\delta^{n+1} f(z)} < H(z, t),
$$

$$
(1-\lambda) \frac{D_\delta^{n+1}g(w)}{D_\delta^n g(w)} + \lambda \frac{D_\delta^{n+2}g(w)}{D_\delta^{n+1} g(w)} < H(w, t)
$$

where $D_\delta$ is the Al-Oboudi operator, $g$ as specified in (5) and $z, w \in \Delta$.

**Remark 1.**

The new subclasses of $\Sigma$ are introduced, by specifying the elements of $\lambda$ and $n$ in Definition 1, taking $t \in (-1, 1)$ and $f(z) \in \Sigma$.

(i) $N^\delta_2[n, 0, H] \equiv M^\delta_2(0, \Phi(z, t))$ (Güney et al., 2017)

(ii) $N^\delta_2[n, 1, H] \equiv M^\delta_2(1, \Phi(z, t))$ (Güney et al., 2017)

(iii) $N^\delta_2[0, \lambda, H] \equiv M^\delta_2(\lambda, \Phi(z, t))$ (Güney et al., 2017)

(iv) $N^\delta_2[0, 0, H] \equiv M^\delta_2(0, \Phi(z, t))$ (Güney et al., 2017)

(v) $N^\delta_2[0, 1, H] \equiv M^\delta_2(1, \Phi(z, t))$ (Güney et al., 2017)
Definition 2.
For $0 \leq \beta \leq 1$ and $t \in (-1, 1)$, a function, $f \in \Sigma$ as contained in (1) is said to be in the class $F^\beta_\Sigma[n, \beta, H]$, if the following subordinations apply:
\[
(1 - \beta) \frac{D^\beta f(z)}{z} + \beta \left( (1 - \beta) \frac{D^\beta g(w)}{w} + \beta \left( D^\beta g(w) \right)^t \right) < H(z, t),
\]
where $D^\beta_g$ is denoted as the Al-Oboudi operator, $g$ as expressed in (5) and $z, w \in \Delta$.

Remark 2.
Consequently, taking $t \in (-1, 1)$ and $f(z) \in \Sigma$, and $\beta$ and $n$ as expressed in Definition 2, we have the following subclasses of $\Sigma$ as listed below:

(i) $F^\beta_\Sigma[n, 0, H] \equiv F^k_\Sigma(0, \Phi(z, t))$ (Güney et al., 2017)
(ii) $F^\beta_\Sigma[n, 1, H] \equiv F^k_\Sigma(1, \Phi(z, t))$ (Güney et al., 2017)
(iii) $F^\beta_\Sigma[0, \beta, H] \equiv F^\beta_\Sigma(\beta, \Phi(z, t))$ (Güney et al., 2017)
(iv) $F^\beta_\Sigma[0, 0, H] \equiv F^\beta_\Sigma(0, \Phi(z, t))$ (Güney et al., 2017)
(v) $F^\beta_\Sigma[0, 1, H] \equiv F^\beta_\Sigma(1, \Phi(z, t))$ (Güney et al., 2017)

III. RESULTS

The main results of the subclasses are presented below:

Theorem 1.
Let $f$ expressed in (1) be in the class $M^\beta_\Sigma[n, \lambda, H]$ and $t \in (0, 1)$. Based on these:
\[
|a_2| \leq \sqrt{2t} \frac{t}{2\lambda + 2\lambda \delta} \left[ (1 + 2\lambda \delta) \left[ 1 + 2\lambda \right] \right]^n \left( 1 + 2\lambda \right)^{2n} \left( 1 + 2\lambda \delta \right)^n \left( 1 + 2\lambda \right)^{2n}
\]
and,
\[
|a_3| \leq \frac{4t^2}{\left( 1 + 2\lambda \delta \right)^2 \left( 1 + 2\lambda \right)^{2n}} + \frac{t}{\left( 1 + 2\lambda \right)^{2n} \left( 1 + 2\lambda \delta \right)^n \left( 1 + 2\lambda \right)^{2n}}
\]
where $0 \leq \lambda \leq 1$.

Proof.
Utilising (8), we have:
\[
(1 - \lambda) \frac{D^{\lambda + 1} f(z)}{D^\beta f(z)} + \lambda \left( (1 - \lambda) \frac{D^{\lambda + 2} g(w)}{D^\beta g(w)} + \lambda \left( D^{\lambda + 2} g(w) \right)^t \right) = H(u(z), t),
\]
\[
(1 - \lambda) \frac{D^{\lambda + 1} g(w)}{D^\beta g(w)} + \lambda \left( (1 - \lambda) \frac{D^{\lambda + 2} f(z)}{D^\beta f(z)} + \lambda \left( D^{\lambda + 2} f(z) \right)^t \right) = H(v(w), t).
\]

Use (11) and (12) in (9) and (10), respectively, we have:
\[
(1 - \lambda) \frac{D^{\lambda + 1} f(z)}{D^\beta f(z)} + \lambda \left( (1 - \lambda) \frac{D^{\lambda + 2} g(w)}{D^\beta g(w)} + \lambda \left( D^{\lambda + 2} g(w) \right)^t \right) = 1 + U_1(t) + U_2(t)u^2(z) + \cdots,
\]
\[
(1 - \lambda) \frac{D^{\lambda + 1} g(w)}{D^\beta g(w)} + \lambda \left( (1 - \lambda) \frac{D^{\lambda + 2} f(z)}{D^\beta f(z)} + \lambda \left( D^{\lambda + 2} f(z) \right)^t \right) = 1 + U_1(t) + U_2(t)u^2(w) + \cdots
\]

In consideration of (1), (4), (5), (7) and (14), we have:
\[
1 + \delta (1 + \lambda \delta)(1 + \delta)^n a_2 z^2 + \cdots
\]
\[
1 - \delta (1 + \lambda \delta)(1 + \delta)^n a_2 w^2 + \cdots
\]

Thus:
\[
\delta (1 + \lambda \delta)(1 + \delta)^n a_2 = U_1(t)c_1,
\]
\[
2\delta (1 + 2\lambda \delta)(1 + 2\delta)^n a_3
\]

In consideration of (1), (4), (5), (7) and (14), we have:
\[
1 + \delta (1 + \lambda \delta)(1 + \delta)^n a_2 z^2 + \cdots
\]
\[
1 - \delta (1 + \lambda \delta)(1 + \delta)^n a_2 w^2 + \cdots
\]

Thus:
\[
\delta (1 + \lambda \delta)(1 + \delta)^n a_2 = U_1(t)c_1,
\]
\[
2\delta (1 + 2\lambda \delta)(1 + 2\delta)^n a_3
\]

In consideration of (1), (4), (5), (7) and (14), we have:
\[
1 + \delta (1 + \lambda \delta)(1 + \delta)^n a_2 z^2 + \cdots
\]
\[
1 - \delta (1 + \lambda \delta)(1 + \delta)^n a_2 w^2 + \cdots
\]

Thus:
\[
\delta (1 + \lambda \delta)(1 + \delta)^n a_2 = U_1(t)c_1,
\]
\[
2\delta (1 + 2\lambda \delta)(1 + 2\delta)^n a_3
\]

In consideration of (1), (4), (5), (7) and (14), we have:
\[
1 + \delta (1 + \lambda \delta)(1 + \delta)^n a_2 z^2 + \cdots
\]
\[
1 - \delta (1 + \lambda \delta)(1 + \delta)^n a_2 w^2 + \cdots
\]

Thus:
\[
\delta (1 + \lambda \delta)(1 + \delta)^n a_2 = U_1(t)c_1,
\]
\[
2\delta (1 + 2\lambda \delta)(1 + 2\delta)^n a_3
\]
By utilising (15) and (17), we have:
\[ c_1 = -d_1, \tag{19} \]
\[ 2[\delta + \lambda \delta^2]^2[1 + \delta]^{2n}a_2 = U_1(t)[c_2^2 + d_2^2]. \tag{20} \]

By the addition of (16) and (18) and inputting into (20), gave rise to the below expression:
\[ a_2^2 = \frac{U_1^2(c_2 + d_2)}{2} \left( \frac{2\delta(1 + 2\lambda \delta)[1 + 2\delta]^n}{-\delta(\lambda^2 \delta^3 + 3 \lambda \delta^2 + 2 \lambda \delta + \delta + 1)[1 + \delta]^{2n}} U_1^2(t) \right) \]
\[ \frac{-\delta^2(1 + \lambda \delta)^2[1 + \delta]^{2n}}{4t^2} \sqrt{+\delta^2(1 + \lambda \delta)^2[1 + \delta]^{2n}} \]

By the subtraction of (18) from (16) and inputting into (19) and (20), resulted to:
\[ a_3 = \frac{U_1^2(t)(c_2^2 + d_2^2)}{2\delta^2(1 + \lambda \delta)^2[1 + \delta]^{2n}} + \frac{U_1(t)(c_2 - d_2)}{4\delta(1 + 2\lambda \delta)[1 + 2\delta]^n} \]

Using (7), and once again applying (13) to the coefficients \( c_1, c_2, d_1, \) and \( d_2, \) resulted to:
\[ |a_3| \leq \frac{4t^2}{\delta^2(1 + \lambda \delta)^2[1 + \delta]^{2n}} + \frac{t}{\delta(1 + 2\lambda \delta)[1 + 2\delta]^n} \]

Based on theorem 5, the following corollaries were obtained.

**Corollary 1.**

Let \( f \) be in the class \( \mathcal{C}_2^\delta[n, 0, H], \) then:
\[ |a_2| \leq \frac{2t \sqrt{2t}}{\sqrt{|\delta^2 - \delta(\delta - 1)4t^2|}}, \]
\[ |a_3| \leq \frac{4t^2}{\delta^2} + \frac{t}{\delta} \]

Taken \( \delta = 1, \) then, we had \( \mathcal{M}^2_2(0, \Phi(z, t)) \) as introduced in theorem 5 (Güney et al., 2017).

**Corollary 2.**

Let \( f \) be in the class \( \mathcal{C}_2^\delta[n, 1, H], \) then:
\[ |a_2| \leq \frac{2t \sqrt{2t}}{\sqrt{|\delta^2(1 + \delta)^2 - \delta(\delta + 3\delta^2 - 2\delta + \delta - 1)4t^2|}}, \]
\[ |a_3| \leq \frac{4t^2}{\delta^2(1 + \delta)^2} + \frac{t}{\delta(1 + 2\delta)} \]

Given \( \delta = 1, \) resulted to \( \mathcal{M}^2_2(1, \Phi(z, t)) \) as introduced in theorem 5 (Güney et al., 2017).

**Corollary 3.**

Let \( f \) be in the class \( \mathcal{C}_2^\delta[0, \lambda, H], \) then:
\[ |a_2| \leq \frac{2t \sqrt{2t}}{\sqrt{|\delta^2(1 + \lambda \delta)^2 - \delta(\delta + 3\lambda \delta^2 + 3\lambda \delta^2 - 2\lambda \delta + \delta - 1)4t^2|}}, \]
\[ |a_3| \leq \frac{4t^2}{\delta^2(1 + \lambda \delta)^2} + \frac{t}{\delta(1 + 2\lambda \delta)} \]

where \( 0 \leq \lambda \leq 1. \)

Given \( \delta = 1, \) resulted to \( \mathcal{M}^2_2(\lambda, \Phi(z, t)) \) as introduced in theorem 5 (Güney et al., 2017).

**Corollary 4.**

Taken \( f \) to be in the class \( \mathcal{C}_2^\delta[0, 0, H], \) then:
\[ |a_2| \leq \frac{2t \sqrt{2t}}{\sqrt{|\delta^2 - \delta(\delta - 1)4t^2|}}, \]
\[ |a_3| \leq \frac{4t^2}{\delta^2} + \frac{t}{\delta} \]

Taken \( \delta = 1, \) then, we had \( \mathcal{M}^2_2(0, \Phi(z, t)) \) as introduced in theorem 5 (Güney et al., 2017).

**Corollary 5.**

Taken \( f \) to be in the class \( \mathcal{C}_2^\delta[0, 1, H], \) then:
\[ |a_2| \leq \frac{2t \sqrt{2t}}{\sqrt{|\delta^2(1 + \delta)^2 - \delta(\delta^3 + 3\delta^2 - 2\delta + \delta - 1)4t^2|}}, \]

and
\[ |a_3| \leq \frac{4t^2}{\delta^2(1 + \delta)^2} + \frac{t}{\delta(1 + 2\delta)} \]

Taken \( \delta = 1, \) and \( t \neq 1/\sqrt{2}, \) resulted to \( \mathcal{M}^2_2(1, \Phi(z, t)) \) as introduced in theorem 5 (Güney et al., 2017).
Theorem 2.

Taken \( f \) as contained in (1) to be in the class \( F^S_L[n, \beta, H] \) and \( t \in (0,1) \) resulted to:

\[
|a_2| \leq \frac{2t\sqrt{2t}}{\sqrt{[(1+2\beta)[1+2\delta]^n-(1+\beta)^2][1+\delta]^{2n}}}\sqrt{1+\delta}^n  
\]  

(21)

and

\[
|a_3| \leq \frac{4t^2}{(1+\beta)^2[1+\delta]^{2n}} + \frac{2t}{(1+2\beta)[1+2\delta]n}.  
\]  

(22)

Proof.

According to the proofs obtained in theorem (5), we acquired the succeeding expressions below:

\[
(1+\beta)[1+\delta]^n a_2 = U_1(t)c_1,  
\]  

(23)

\[
(1+2\beta)[1+2\delta]^n a_3 = U_1(t)c_2 + U_2(t)c_i,  
\]  

(24)

\[-(1+\beta)[1+\delta] a_2 = U_1(t)d_1,  
\]  

(25)

\[
(21+2\beta)[1+2\delta]^n a_2 - (1+\beta)[1+2\delta]^n a_3 = U_1(t)d_2 + U_2(t)d_i.  
\]  

(26)

From (23) and (25), it was established that:

\[
c_1 = -d_1,  
\]  

(27)

\[
2(1+\beta)^2[1+\delta]^2a_2^2 = U_1^2(t)(c_1^2 + d_1^2).  
\]  

(28)

Then, the utilisation of (24), (26), and (28) resulted to:

\[
a_2^2 = \frac{U_1^2(t)(c_2 + d_2)}{2((1+2\beta)[1+2\delta]^nU_1^2(t) - (1+\beta)[1+\delta]^{2n}U_2(t))}.  
\]  

Then, by the use of (7) and (13) for the coefficients \( c_2 \) and \( d_2 \), we obtained the sought-after bound on \( |a_2| \) as stated in (21), and by subtracting (26) from (24), and inputting into (27) and (28), resulted to:

\[
a_3 = \frac{U_1^2(t)(c_1^2 + d_1^2) + U_1(t)(c_2 - d_2)}{2(1+\beta)^2[1+\delta]^{2n}} + \frac{U_1(t)(c_2 - d_2)}{2(1+2\beta)[1+2\delta]n}.  
\]  

Reiteratively, from the use of (7) and (13) for the coefficients \( c_1, c_2, d_1, \) and \( d_2 \), we got \( |a_3| \) as contained in (22).

Corollary 6.

Given \( f \) to be in the class \( F^S_L[n, 0, H] \), resulted to:

\[
|a_2| \leq \frac{2t\sqrt{2t}}{\sqrt{[(1+2\delta)^n - [1+\delta]^{2n}]}4t^2 + [1+\delta]^{2n}}  
\]  

\[
|a_3| \leq \frac{4t^2}{[1+\delta]^{2n}} + \frac{2t}{[1+2\delta]^{2n}}.  
\]  

(29)

(30)

\[
F^S_L[n,0,H] \equiv F^S_L(0,\Phi(z,t))  
\]  

(31)

Taken \( \delta = 1 \), we had \( F^S_L(0,\Phi(z,t)) \) as introduced in theorem 11 (Güney et al., 2017).

Corollary 7.

Given \( f \) to be in the class \( F^S_L[n, 1, H] \), then:

\[
|a_2| \leq \frac{2t\sqrt{2t}}{\sqrt{[3[1+2\delta]^n - 4[1+\delta]^{2n}]}4t^2 + [1+\delta]^{2n}}.  
\]  

\[
|a_3| \leq \frac{t^2}{[1+\delta]^{2n}} + \frac{2t}{3[1+2\delta]^{2n}}.  
\]  

(32)

(33)

\[
F^S_L[n,1,H] \equiv F^S_L(1,\Phi(z,t))  
\]  

(34)

Taken \( \delta = 1 \), resulted to \( F^S_L(1,\Phi(z,t)) \) as introduced in theorem 11 (Güney et al., 2017).

IV. SUMMARY

The bi-univalent functions subclasses of \( N^S_L[n,\lambda,H] \) and \( F^S_L[n,\beta,H] \) were developed using the subordinations of Chebyshev polynomials, defined by the Al-Oboudi differential operator and by the subclasses applications, the coefficient estimate of \( |a_2| \) and \( |a_3| \) were determined. The bounds obtained in Theorem 1 and Theorem 2 are the best possible.

V. REFERENCES


