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The Application of Fuzzy Weak Autocatalytic Set in Robots Kinematic Structures Evaluation

S.S. Mamat[1*], T. Ahmad[2], S.R. Awang[3] and N. Ahmad[1]

[1]*Centre of Foundation Studies, Universiti Teknologi MARA, Cawangan Selangor, Kampus Dengkil, Selangor, Malaysia*

[2]*Department Mathematical Sciences, Faculty of Science, Universiti Teknologi Malaysia, Johor, Malaysia*

[3]*Azman Hashim International Business School, Universiti Teknologi Malaysia, Johor, Malaysia*

Pairwise comparison of alternatives is common in decision-making procedures. A fuzzy weak autocatalytic set (FWACS) is a technique used in solving multi-criteria decision-making problems. A FWACS represented the comparison of a set of alternatives in form of a directed graph with fuzzy edges. This study discusses the implementation of ranking by FWACS on the evaluation of robots' kinematic structures. A comparison of results generated by FWACS is made with the results using Potential Method (PM). Sensitivity analysis verifies that the decision made by FWACS is stable and consistent, thus comparable to PM.

Keywords: Decision Making; Potential Method; Fuzzy Weak Autocatalytic Set; Robots

I. INTRODUCTION

Multi-Criteria Decision Making (MCDM) is a tool in decision making which concerned on pairwise comparisons of alternatives. Each alternative is needed to be evaluated and the best alternative is chosen. There are a lot of tools in MCDM such as Simple Additive Method (SAW), Analytic Hierarchy Process (AHP), Technique for the Order of Preference by Similarity to an Ideal Solution (TOPSIS), Preference Ranking Organization Method for Enrichment of Evaluations (PROMETHEE), Elimination and Choice Translating Reality (ELECTRE) (Rao, 2006), Potential Method (PM), etc. The Fuzzy Weak Autocatalytic Set (FWACS) is one of the tools that can be used in MCDM problems (Siti Salwana *et al.,* 2020).

The theory of FWACS is originally derived from a weak form of the Autocatalytic Set (ACS). ACS is a directed graph whose vertices have at least one incoming link from vertices belonging to the same subgraph (Jain and Krishna, 1998; 1999). A Weak Autocatalytic Set (WACS) represents a weak form of ACS. A WACS is a non-loop subgraph that contains a vertex without an incoming link (Siti Salwana *et al.,* 2018). The concept of WACS is extended to fuzziness which leads to the new concept of FWACS. Thus, an algorithm for ranking purposes namely ranking by FWACS is established as the result of integration between FWACS and Potential Method (PM) (Siti Salwana *et al.,* 2018).

In this paper, an implementation of FWACS in a MCDM problem is presented. The problem of robot evaluation from Ivandić *et al.* (2009) is used to illustrate the application of FWACS. The paper is organised as follows. Section 2 provides some basic concepts and definitions that are needed in this study. The fuzzification of WACS; i.e. FWACS as well as its algorithm are described in this section. Section 3 illustrates an implementation of the ranking procedure by FWACS. Finally, the conclusion is presented in Section 4.

II. PRELIMINARIES

This section presents on different structures of WACS and FWACS for decision-making purposes. Ranking by FWACS is also presented in this section.

A. Autocatalytic Set (ACS)

The concept of autocatalysis originated in chemistry for the description of catalytic interaction between molecules

*Corresponding author's e-mail: sitisalwana@uitm.edu.my

(Raphaël *et al.,* 2011). However, in 1998, Jain and Krishna formalised the concept of autocatalytic set (ACS) in form of a directed graph. The formal definition of an ACS is given as follows.

Definition 1: An ACS is a subgraph, each of whose vertices has at least one incoming link from vertices belonging to the same subgraph (Jain & Krishna, 1998).

Figure 1 illustrates some examples of ACS.

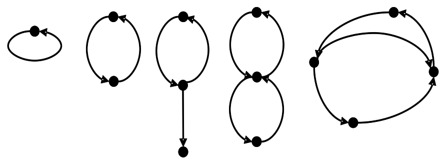

Figure 1. Some examples of ACS

A weak form of ACS has initiated a new structure of graph namely WACS. The following subsection described a WACS.

B. Weak Autocatalytic Set (WACS)

A WACS is defined as follows.

Definition 2: A WACS is a non-loop subgraph which contains a vertex with no incoming link (Siti Salwana *et al.,* 2020).

Figure 2 demonstrates some examples of WACS.

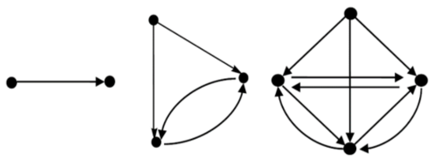

Figure 2. Some examples of WACS

The features of a WACS are listed as follows.

Theorem 1 Every WACS is a weakly connected graph (Siti Salwana *et al.,* 2020).

Theorem 2 Every WACS must have at least a path, which is not closed (Siti Salwana *et al.,* 2020).

A fuzzy graph is a replication of a crisp graph (Sabariah *et al.,* 2009). The following theorem proves that WACS is a special case of a fuzzy graph.

Theorem 3 Every WACS is a fuzzy graph (Siti Salwana *et al.,* 2020).

The emergence of fuzzy graphs and WACS has initiated a new concept called FWACS. The FWACS are presented in the following subsection.

C. Fuzzy Weak Autocatalytic Set (FWACS)

The integration of fuzzy graph and a WACS has led to the concept of FWACS by Siti Salwana *et al.* (2020). The formal definition of FACS is laid as follows.

Definition 3: A FWACS is a WACS such that every edge e_i has a membership value, $\mu(e_i) \in [0,1]$ for $e_i \in E$ (Siti Salwana *et al.,* 2020).

Figure 3 shows an example of a FWACS.

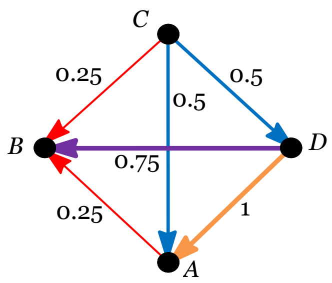

Figure 3. A FWACS

The edges have different 'strengths' which are determined by their membership values. The greater the membership value, the stronger the connection between the two vertices of the graph. Thus, the different thicknesses and colours of an edge represent the strength connection of its vertices.

1. Ranking by Fuzzy Weak Autocatalytic Set (FWACS)

An algorithm for ranking by FWACS is presented in this section. The inputs are the membership values of edges obtained by pairwise comparison (Siti Salwana *et al.,* 2018). The membership values of edges are the entries for a $m \times 1$

matrix F_μ. The edge's orientation is listed as an incidence matrix, *A*. The procedure of ranking with FWACS is outlined as follows.

Step 1: Build a FWACS, $G=(V,F_\mu)$ for a given problem and determine the membership values for its edges. Set *V* as a set of vertices and F_μ is the fuzzy flow matrix, which represents the membership value of edges.

Step 2: Construct the incidence matrix, *A*. A $m\times n$ incidence matrix is given by:

$$A_{\alpha,v}=\begin{cases}-1, & \text{if the edge } \alpha \text{ leaves } v\\ 1, & \text{if the edge } \alpha \text{ enter } v\\ 0, & \text{otherwise}\end{cases} \tag{1}$$

Step 3: Define Laplacian matrix, *L*. The Laplacian matrix is $L=A^TA$ with entries defined as:

$$L_{i,j}=\begin{cases}-1, & \text{if there is an edge } (i,j) \text{ or } (j,i),\\ \deg(i), & \text{if } i=j,\\ 0, & else.\end{cases} \tag{2}$$

such that $\deg(i)$ is the degree of vertex *i*. In other words, $\deg(i)$ represents the number of edges at the particular vertex *i*.

Step 4: Generate flow difference, D_μ. Let the flow difference be $D_\mu=A^TF$. The component of D_μ is determined as below:

$$\begin{aligned} D_\mu &= \sum_{\alpha=1}^{m} A^T_{v.\alpha}F_\alpha \\ &= \textstyle\sum_{\alpha \text{ enters } v} F_\alpha - \sum_{\alpha \text{ leaves } v} F_\alpha \end{aligned} \tag{3}$$

whereby D_μ is the difference between the total flow which enters *v* and the total flow which leaves *v*.

Step 5: Determine potential, *X*. The potential *X* is a solution of the Laplacian system:

$$LX=D_\mu \tag{4}$$

such that as its connected components.

Step 6: Check the consistency degree, $\beta<12°$. The measure of inconsistency is defined as:

$$\text{Inc }(F)=\frac{\|F-AX\|_2}{\|AX\|_2} \tag{5}$$

where $\|\cdot\|_2$ denotes 2-norm and $\beta=\arctan$ (Inc (F)) is the angle of inconsistency. The ranking is considered acceptable whenever $\beta<12°$. If the matrix is not consistent, then it needs to be revised (Saaty, 2008).

Step 7: Determine the weight, *w*. The following equation is used to obtain the weight.

$$w=\frac{a^X}{\|a^X\|_1} \tag{6}$$

where $\|\cdot\|_1$ represents l_1-norm and parameter a is chosen to be 2 suggested by Čaklović (2003). The *X* represents the potential value (see step 5).

Step 8: Rank the objects with respect to their associated weights. The object with the highest weight is given rank 1, followed by the object with the second highest weight, while the object with the lowest weight is ranked last.

Figure 4 represents the ranking procedure followed by its algorithm in Figure 5.

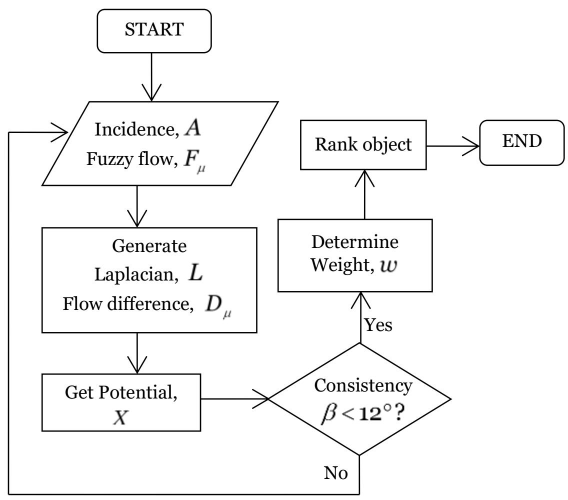

Figure 4. Flowchart for FWACS ranking

Algorithm 1 Ranking with FWACS

```
Begin
Input: A = (a_ij)_{m×n}
       F = (f_1, f_2, f_3, ..., f_m)
Output: w = (w_1, w_2, w_3, ..., w_n)
Procedure 1: [Define laplacian, L ]
L = (l_ij)_{n×n}
return L
Procedure 2: [Generate flow difference, D ]
D = (D_1, D_2, D_3, ..., D_n)
return D
Procedure 3: [Get potential, X ]
X = (x_1, x_2, x_3, ..., x_n)
return X
Procedure 4: [Consistency degree, β ]
β
return β
Procedure 5: [Determine weight, w ]
w = (w_1, w_2, w_3, ..., w_n)
return w
End
```

Figure 5. FWACS ranking algorithm

The following sections discussed a problem posted in Ivandić *et al.* (2009) as an implementation of ranking by FWACS.

III. APPLICATION

Ivandić *et al.* (2009) presented a new systematic approach to evaluating kinematic structures of the robot for welding purposes. The task is to select one kinematic structure of the robot for automated welding technology. The hierarchical structure of criteria for robot evaluation is illustrated in Figure 6.

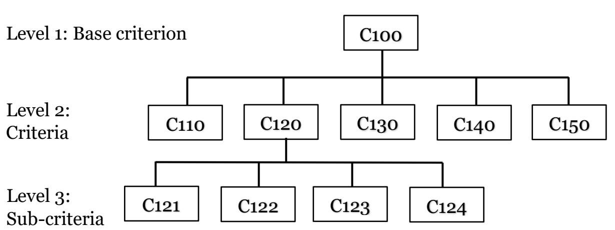

Figure 6. The hierarchical structure of criteria

Firstly, an analysis of five groups of welded structures at the conceptual level is performed. The groups are *C*110, *C*120, *C*130, *C*140, and *C*150. Then, the *C*120 represents the variant factors of kinematic structure which are *C*121, *C*122, *C*123, and *C*124. Finally, a set of solutions consisting of eight kinematic structures has to be evaluated to determine the best structure.

A. Criteria Weight Determination

The first task in evaluating the robots is to synthesis the welded structures. The matrix comparison of criteria is given in Table 1. Figure 7 shows the FWACS for the welded structures, whereby *C*120 is most preferred as arrows are pointing into *C*120. The arrows are all pointing out from *C*130 which interprets that the *C*130 is the least preferred.

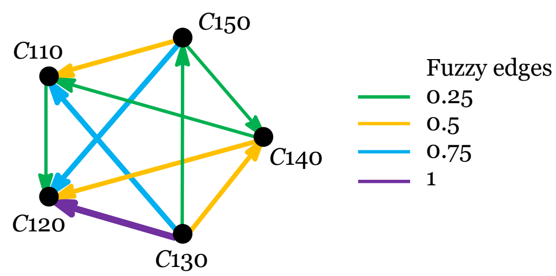

Figure 7. The FWACS for welded structures

The graph is transformed to incidence and flow matrices, A and F respectively. The incidence matrix, A represents the direction of edges for the graph in Figure 7. The flow matrix, F represents the fuzzy value of edges. The incidence and flow matrices are as follows:

$$A = \begin{bmatrix} -1 & 1 & 0 & 0 & 0 \\ 1 & 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & 0 & -1 \\ 0 & 1 & -1 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & 0 & 1 \\ 0 & 0 & 0 & 1 & -1 \end{bmatrix} \text{ and } F = \begin{bmatrix} 0.25 \\ 0.75 \\ 0.25 \\ 0.5 \\ 1 \\ 0.5 \\ 0.75 \\ 0.5 \\ 0.25 \\ 0.25 \end{bmatrix}$$

The weights are given in Table 1. The PM is adopted for FWACS. The PM weights are listed alongside the calculated weights obtained from FWACS. The criterion *C*120 is ranked first whereas the *C*130 is ranked last.

Table 1. The weights for welded structures

Welded structure	*C*110	*C*120	*C*130	*C*140	*C*150	PM		FWACS	
						weight	rank	weight	rank
*C*110	0	-1	3	1	2	0.258	2	0.231	2
*C*120	1	0	4	2	3	0.516	1	0.275	1
*C*130	-3	-4	0	-2	-1	0.032	5	0.137	5
*C*140	-1	-2	2	0	-2	0.129	3	0.194	3
*C*150	-2	-3	1	2	0	0.065	4	0.163	4

1. Weightage for factor C120

The *C*120 has four factors to be considered. Hence, the second task is to make comparisons for the set of factors. The comparison matrix with respect to factors of the *C*120 is given in Table 2, and is then transformed to FWACS. Figure 8 demonstrates the FWACS and its flows for factors with respect to *C*120. The FWACS depicts that *C*122 is the most preferred factor, whereas the least preferable is *C*124.

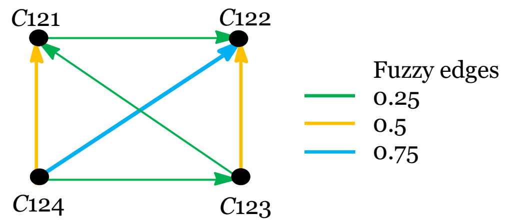

Figure 8. The FWACS factors with respect to *C*120

The weights are obtained and summarised in Table 2. The highest weight is given to *C*122 whereas the *C*124 has the lowest weight.

Table 2. The weights for factors with respect to *C*120

*C*120 factors	*C*121	*C*122	*C*123	*C*124	PM		FWACS	
					weight	rank	weight	rank
*C*121	0	-1	1	2	0.26667	2	0.267579	2
*C*122	1	0	2	3	0.53333	1	0.318207	1
*C*123	-1	-2	0	-2	0.13333	3	0.225006	3
*C*124	-2	-3	2	0	0.06667	4	0.189207	4

2. *Weights for solution set*

The matrix of preferences with respect to solution set is given in Table 3. It consists of eight kinematic structures which are $R1, R2, R3, R4, R5, R6, R7$, and $R8$. Figure 9 demonstrates the FWACS for the solution set. There are 24 comparisons in the solution set in the graph.

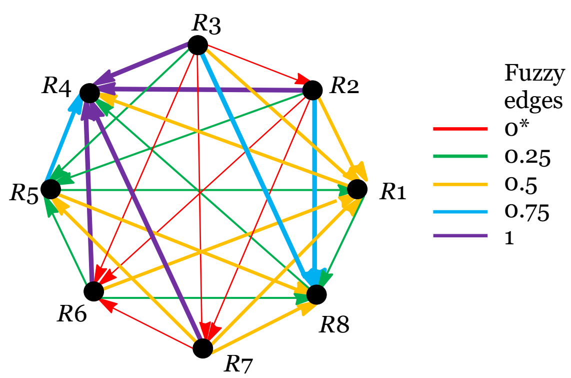

Figure 9. The FWACS for solution set

Table 3 lists the weights for the solution set. The FWACS and PM yield $R4$ as the highest rank whereas the lowest rank are $R2$ and $R3$.

Table 3. The weights for the solution set

Solution set	$R1$	$R2$	$R3$	$R4$	$R5$	$R6$	$R7$	$R8$	PM		FWACS	
									Weight	Rank	Weight	Rank
$R1$	0	2	2	-2	1	2	2	1	0.1200	3	0.1377	3
$R2$	-2	0	0	-4	-1	0	0	-3	0.0300	7	0.0974	7
$R3$	-2	0	0	-4	-1	0	0	-3	0.0300	7	0.0974	7
$R4$	2	4	4	0	3	4	4	1	0.4500	1	0.1947	1
$R5$	-1	1	1	-3	0	1	2	-2	0.0601	4	0.1158	4
$R6$	-2	0	0	-4	-1	0	1	-1	0.0304	6	0.0977	6
$R7$	-2	0	0	-4	-1	-1	0	-2	0.0327	5	0.0995	5
$R8$	-1	3	3	-1	2	2	2	0	0.2169	2	0.1597	2

The expected utility of an act is a weighted average of the utilities of each of its possible outcomes, where the utility of an outcome measures the extent to which that outcome is preferred, or preferable to the alternatives. The utility of each outcome is weighted according to the probability that the act will lead to that outcome.

The value of the utility function of the solution set and the rank of importance and weights for the solution set with respect to preference utility is summarised in the Table 4.

Table 4. The solution set with respect to preference utility

Solution set	$R1$	$R2$	$R3$	$R4$	$R5$	$R6$	$R7$	$R8$	PM		FWACS	
									Weight	Rank	Weight	Rank
$R1$	3	1	1	5	4	5	3	3	0.1402	4	0.1245	4
$R2$	1	1	2	2	4	4	3	3	0.0817	8	0.0921	8
$R3$	1	4	3	5	1	3	4	4	0.1372	5	0.1218	5
$R4$	5	5	5	5	5	5	4	4	0.1693	1	0.1783	1
$R5$	2	4	3	3	4	4	3	2	0.0991	6	0.1133	6
$R6$	1	1	4	5	4	5	4	4	0.1439	3	0.1349	3
$R7$	1	1	4	2	4	4	3	3	0.0839	7	0.0995	7
$R8$	4	4	3	5	2	4	3	3	0.1446	2	0.1356	2

The weights in Table 4 indicate that $R4$ is the largest weight as the appropriate kinematic structure using PM and FWACS. The PM and FWACS rank the solution in order of $R4 \succ R8 \succ R6 \succ R1 \succ R3 \succ R5 \succ R7 \succ R2$.

B. Sensitivity Analysis

The final weights of the alternatives are highly dependent on the weights attached to the main criteria. Small changes in the relative weights can therefore cause major changes in the final ranking. Since these weights are usually based on highly subjective judgments, the stability of the ranking under varying criteria weights has to be tested (Irfan, 2013). For this purpose, sensitivity analysis can be performed by increasing or decreasing the weight of individual criteria, hence, the resulting changes in the weights and the ranking of the alternatives can be observed.

The solution set weights are varied by using Expert Choice 11 Software. The performance graph in Figure 10 indicates how the kinematic structures perform with respect to the change in the scenario for all criteria.

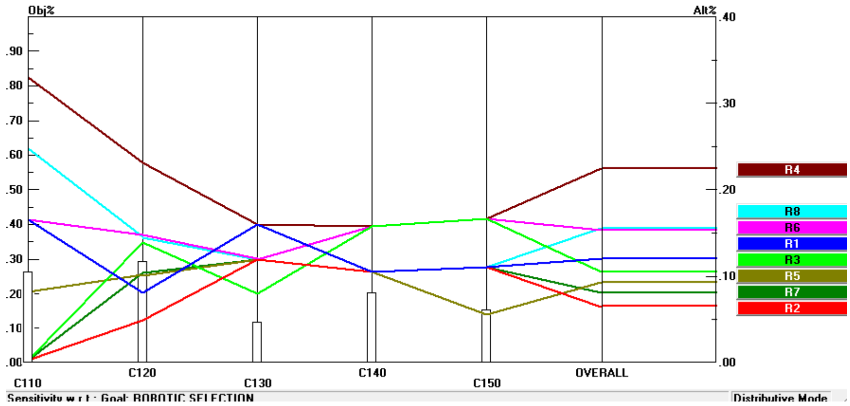

Figure 10. Performance sensitivity of the solution set

Performance sensitivity of the solution set is analysed when *C*120 is increased to 40%. Increasing *C*120 did not have any effect would not result in any changes to the ranking as given in Figure 11.

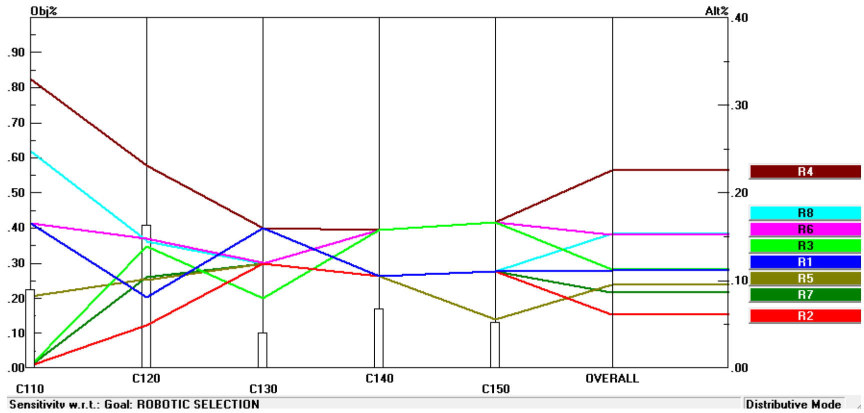

Figure 11. Performance sensitivity for the solution set when *C*120 increased to 40%

Performance sensitivity of solution set has been analysed when *C*120 is decreased to 5%, would decreases the weight of *R*3 and *R*7 from 0.1218 to 0.1209 and from 0.0995 to 0.0945, respectively (Figure 12).

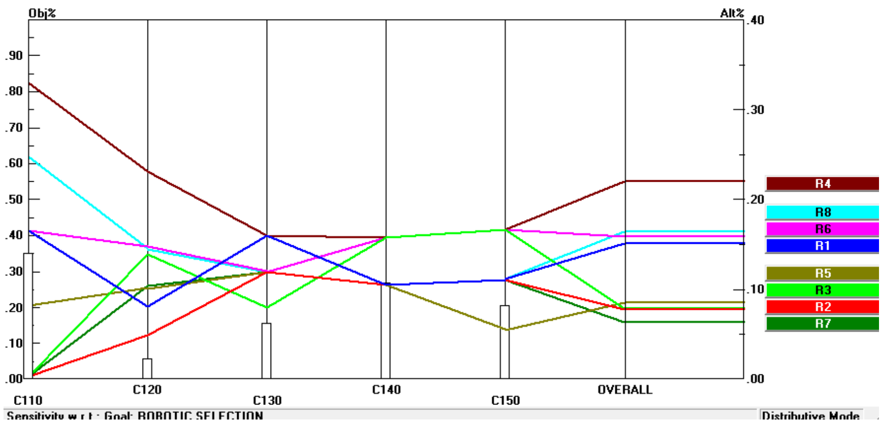

Figure 12. Performance sensitivity for the solution set when *C*120 decreased to 5%

Overall, based on sensitivity analysis, it can be concluded that the final decision is consistent and reliable.

IV. CONCLUSION

A sample of welding robot kinematic structure evaluation is discussed using FWACS. The results by FWACS are compared with the results obtained by PM taken from Ivandić *et al.* (2009). The result shows that FWACS is able to accommodate the uncertainty in a fuzzy environment.

V. ACKNOWLEDGEMENT

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VI. CONFLICT OF INTERESTS

The authors declare no conflict of interest in the subject matter or materials discussed in this manuscript.

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