Assessing Reaeration Rate Equations for Modelling Dissolved Oxygen of Pusu River in Malaysia

A.A. Mamun1∗, M. Nuruzzaman2 and M.N. Salleh1

1Department of Civil Engineering, Kulliyyah of Engineering, International Islamic University Malaysia (IIUM), 53100 Kuala Lumpur, Malaysia.
2Department of Civil Engineering, Curtin University, Kent Street, Bentley, WA 6102, Australia.

Many authors reported high variability in the prediction of reaeration rates by various equations, leading to uncertainty in the estimation of the reaeration rate for a river. Due to this uncertainty, it is essential to identify a suitable equation to predict the dissolved oxygen (DO) concentration of the river in concern. Pusu River in Malaysia receives sewage discharges and suffers from land-clearing activities and stormwater-related pollution. Pusu River is a small river, but highly important in terms of demography and geographic location. As such, it is required to identify a suitable reaeration rate equation for predicting its DO concentration, which indicates the overall health of a river. The purpose of this study is to assess the suitability of reaeration rate equations to predict Dissolved Oxygen (DO) concentrations of the Pusu River. The water quality analysis simulation program (WASP) model was employed to model the DO of the Pusu River. Reaeration rates calculated from the available 31 equations were given input in the model, and errors in prediction were calculated in terms of Root Mean Square (RMS) error and R² for every equation. It was revealed that Neguluscu and Rojanski (1969) equation using depth and velocity as the variables performed best among all the equations. It produced a minimum RMS error of 0.17 and 0.09 mg/L in calibration and validation data, respectively. R² values for predicted-observed plots were 0.98 and 0.97 in these two data sets using the equation. Based on overall Performance Indicator Values (PIVs), reaeration rate equations with depth and velocity as the variables performed better than the other equations with more variables for Pusu River. This study provided important information to accurately model the DO of the Pusu River for future simulation of different scenarios.

Keywords: reaeration rate; water quality modelling; uncertainty; dissolved oxygen; Pusu River

I. INTRODUCTION

Rivers are getting polluted with the onslaught of urbanisation and the dumping of huge amounts of waste into the river systems (Bayram et al., 2013; Yuan et al., 2005). Rivers have their own assimilative capacity for the waste discharged into the streams, otherwise known as a self-purification system (Demars & Manson, 2013). The system functions through several mechanisms, e.g., biodegradation, absorption, sedimentation, atmospheric reaeration, adsorption, dilution, etc. (Bahadur & Samuels, 2014; González et al., 2014; Menezes et al., 2015). When the waste discharge rate exceeds the assimilative capacity of the river, it is degraded.

Determining the reaeration rate of natural streams accurately is crucial (Chu & Jirka, 2003) for Total Maximum Daily Load (TMDL) calculation. TMDL is the maximum waste load that a river can receive without degrading its existing water quality, and it also depends on the river conditions. TMDL varies according to existing water-quality level, desired level and pollutant load allocations (Mukundan et al., 2012). TMDL is expressed by the following equation:

\[ TMDL = \sum LA + \sum WLA + MOS \]
where TMDL is the total maximum loading capacity of the river, LA is nonpoint sources of pollutant load allocations, WLA is point sources pollutant load allocations and MOS is the margin of safety expressed as a fraction of the load capacity.

Reaeration rate (Ka) is defined as the rate of transfer of oxygen from air to flowing water and can be expressed in the following form (Thomann & Mueller, 1987):

$$\frac{dc}{dt} = K_a(C_s - C)$$  \hspace{1cm} (2)

where, $\frac{dc}{dt}$ is the rate of change of oxygen concentration, t is the time, $K_a$ is the reaeration rate, C is the DO concentration of the stream and $C_s$ is the saturation concentration of Dissolved Oxygen (DO) at corresponding temperature.

Two theories known as (i) Two-film theory and (ii) Surface renewal theory have been developed to describe the flux of oxygen from air to water. Regardless of the available theories, a vast majority of the studies followed the empirical method to determine the reaeration rate equations (Haider & Ali, 2010; Haider & Ali, 2013) due to difficulties in estimating theoretical parameters. Both field and laboratory investigations were carried out to develop reaeration rate equations. Some suggested that laboratory investigation method performs poorly compared to field method (Benson et al., 2014; Raymond et al., 2012). The equations were developed using several variables, including Froude number, shear stress velocity, slope, depth, flow, velocity, Reynolds number and molecular diffusion coefficient (Cox, 2003). No less than 31 equations exist till now to predict reaeration rate of flowing water including small, medium and large rivers. However, most of them are applicable within a very narrow range (Demars & Manson, 2013; Demars et al., 2015; Wallin et al., 2011). For Ravi River in Pakistan, in which flow fluctuates to an extreme range, reaeration rate equations using velocity and depth as the variables performed better than equations having other variables (Haider et al., 2013). Nevertheless, many other studies reported that having more variables in reaeration rate equation enhances the prediction accuracy (Palumbo & Brown, 2013).

Reaeration rate equations available till now are listed in Table 1. The equations have been divided into five groups based on variables considered to develop the equations. The first group of the equations considered only depth and velocity as the variables to predict reaeration rate. The second group incorporated one more parameter, i.e., slope as an influencing parameter on reaeration rate. The second group incorporated one more parameter, i.e., slope as an influencing parameter on reaeration rate. In many cases, it has been observed that addition of slope improves prediction capability. Equations of group three assumed shear stress, Froude number and depth of stream as the factors to estimate reaeration rates. There are only two equations in group four and the main feature of this group is inclusion of dispersion coefficient as a variable. The latest development of reaeration rate equation is the equation given by Gualtieri and Gualtieri (2004) listed in group five. This equation considered several new variables such as gas-transfer Reynolds number, gravitational acceleration, kinematic velocity of water and molecular diffusion coefficient. This equation demonstrated acceptable performance on several occasions.

Table 1. Available reaeration rate equations.

<table>
<thead>
<tr>
<th>No.</th>
<th>Equation</th>
<th>Applicability</th>
<th>Type of Study</th>
<th>Ref.</th>
<th>Code</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Group 1</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>$K_a = 5.792 \frac{U^{0.5}}{H^{0.25}}$</td>
<td>-</td>
<td>-</td>
<td>(Jha, 2001)</td>
<td>JH</td>
</tr>
<tr>
<td>2</td>
<td>$K_a = 1.923 \frac{U^{1.325}}{H^{2.006}}$</td>
<td>Mountainous rivers</td>
<td>Mountainous rivers</td>
<td>(Baecheler, 1999)</td>
<td>BL</td>
</tr>
<tr>
<td>3</td>
<td>$K_a = 4.1528 \frac{U^{0.6}}{\sqrt{H}}$</td>
<td>Large and medium rivers</td>
<td>Large and medium rivers</td>
<td>(Bansal, 1973)</td>
<td>BA</td>
</tr>
<tr>
<td>4</td>
<td>$K_a = 5.772 \frac{U^{0.607}}{H^{1.689}}$</td>
<td>Large and small rivers</td>
<td>Large and small rivers</td>
<td>(Bennett, 1972)</td>
<td>BR</td>
</tr>
<tr>
<td>5</td>
<td>$K_a = 4.54 \frac{U^{0.703}}{H^{1.854}}$</td>
<td>$K_a$ range: 9.8 – 28.8 per day</td>
<td>Regression Analysis</td>
<td>(Padden, 1971)</td>
<td>PG</td>
</tr>
<tr>
<td>No.</td>
<td>Equation</td>
<td>Applicability</td>
<td>Type of Study</td>
<td>Ref.</td>
<td>Code</td>
</tr>
<tr>
<td>-----</td>
<td>----------</td>
<td>---------------</td>
<td>---------------</td>
<td>------</td>
<td>------</td>
</tr>
<tr>
<td>6</td>
<td>$K_a = 4.05 \frac{U}{H^{1.5}}$</td>
<td>-</td>
<td>Recirculating flume</td>
<td>(Eloubaidy, 1969)</td>
<td>EL-1</td>
</tr>
<tr>
<td>7</td>
<td>$K_a = 10.9 \left(\frac{U}{H}\right)^{0.85}$</td>
<td>Velocity: 0.2 – 1.2 m/s Depth: &lt; 0.5 m</td>
<td>Recirculating flume</td>
<td>(Negulescu, 1969)</td>
<td>NR-1</td>
</tr>
<tr>
<td>8</td>
<td>$K_a = 3.6 \frac{U}{H^{1.5}}$</td>
<td>-</td>
<td>Recirculating cylindrical flume</td>
<td>(Isaacs, 1969)</td>
<td>IS</td>
</tr>
<tr>
<td>9</td>
<td>$K_a = 4.7531 \frac{U}{H^{1.5}}$</td>
<td>Velocity: 0.18 – 0.5 m/s Depth: 0.15 – 0.46 m</td>
<td>Recirculating cylindrical flume</td>
<td>(Isaacs, 1968)</td>
<td>IG</td>
</tr>
<tr>
<td>10</td>
<td>$K_a = 5.134 \left(\frac{U}{V^{1.5}}\right)$</td>
<td>Velocity: 0.14 - 1.52 m/s Depth: 0.3 - 9.15 m</td>
<td>Large rivers</td>
<td>(Langbein, 1967)</td>
<td>LD</td>
</tr>
<tr>
<td>11</td>
<td>$K_a = 5.32 \frac{U^{0.67}}{H^{1.85}}$</td>
<td>Velocity: 0.03 - 1.52 m/s Depth: 0.12 – 3.35 m</td>
<td>Small and Large Rivers</td>
<td>(Owens, 1964)</td>
<td>OW</td>
</tr>
<tr>
<td>12</td>
<td>$K_a = 5.026 \frac{U}{H^{0.67}}$</td>
<td>Velocity: 0.46 - 1.52 m/s</td>
<td>Large rivers (Under a dam)</td>
<td>(Churchill, 1962)</td>
<td>CH</td>
</tr>
<tr>
<td>13</td>
<td>$K_a = 3.93 \frac{U^{0.5}}{H^{1.5}}$</td>
<td>Velocity: 0.14 - 0.5 m/s Depth: 0.3 - 9.15 m</td>
<td>Conceptual model</td>
<td>(O'Connor, 1956)</td>
<td>OD</td>
</tr>
<tr>
<td>14</td>
<td>$K_a = 596 \left(\frac{U S^{0.528}}{Q^{0.136}}\right)$</td>
<td>Large rivers and streams (pool and riffle)</td>
<td>Large rivers and streams (pool and riffle)</td>
<td>(Melching, 1999)</td>
<td>MF</td>
</tr>
<tr>
<td>15</td>
<td>$K_a = 1740\left(U^{0.465}S^{0.79}H^{0.728}\right)$</td>
<td>-</td>
<td>-</td>
<td>(Moog, 1998)</td>
<td>MJ</td>
</tr>
<tr>
<td>16</td>
<td>$K_a = 543 \frac{U^{0.5225}S^{0.6236}}{H^{0.728}}$</td>
<td>-</td>
<td>-</td>
<td>(Smoot, 1988)</td>
<td>SM</td>
</tr>
<tr>
<td>17</td>
<td>$K_a = 8784 \frac{U^{0.734}S^{0.93}}{H^{0.42}}$</td>
<td>Small Streams</td>
<td>Small Streams</td>
<td>(Thyssen, 1987)</td>
<td>TS</td>
</tr>
<tr>
<td>18</td>
<td>$K_a = 22700SU$</td>
<td>Reaeration rate: 2.1 – 55 day$^{-1}$ Flow rate: 0.0085 – 1.05 m$^3$/s</td>
<td>Small Streams</td>
<td>(Grant, 1976)</td>
<td>GR</td>
</tr>
<tr>
<td>19</td>
<td>$K_a = 3170S$</td>
<td>Flow rate: 0.028 - 0.28 m$^3$/s</td>
<td>Radioactive tracer method on 24 different streams Multivariate Analysis</td>
<td>(Tsivoglou, 1976)</td>
<td>TN</td>
</tr>
<tr>
<td>20</td>
<td>$K_a = 186 \left(\frac{SU}{H}\right)^{0.5}$</td>
<td>-</td>
<td>-</td>
<td>(Cadwallader, 1969)</td>
<td>CM</td>
</tr>
<tr>
<td>21</td>
<td>$K_a = 173 \left(\frac{SU}{H}\right)^{0.404}$</td>
<td>-</td>
<td>Recirculating Flume</td>
<td>(Krenkel, 1962)</td>
<td>KO</td>
</tr>
</tbody>
</table>

**Group 3**

<table>
<thead>
<tr>
<th>No.</th>
<th>Equation</th>
<th>Applicability</th>
<th>Type of Study</th>
<th>Ref.</th>
<th>Code</th>
</tr>
</thead>
<tbody>
<tr>
<td>22</td>
<td>$K_a = 0.000025\left(1 + 9F^{0.25}\right) \frac{U}{H}$</td>
<td>Large rivers</td>
<td>Large rivers</td>
<td>(Thackston, 2001)</td>
<td>TD</td>
</tr>
<tr>
<td>23</td>
<td>$K_a = 23000 \frac{(1 + F)^{2.66}U^{0.76}S^{1.13}}{H^{0.6}}$</td>
<td>Small streams</td>
<td>Small streams</td>
<td>(Thyssen, 1980)</td>
<td>TJ</td>
</tr>
<tr>
<td>24</td>
<td>$K_a = 123 \frac{U}{H}$</td>
<td>Subcritical and Turbulent flow</td>
<td>Recirculating Flume</td>
<td>(Alonso, 1975)</td>
<td>AL</td>
</tr>
<tr>
<td>25</td>
<td>$K_a = 2506.7 \frac{U}{H} \left(\frac{U}{U}\right)^3$</td>
<td>Depth: 0.61 – 3.35 m Velocity: 0.46 - 1.52 m/s</td>
<td>Reanalysis of Reaeration Data</td>
<td>(Lau, 1972)</td>
<td>LA</td>
</tr>
<tr>
<td>26</td>
<td>$K_a = 23.04 \frac{(1 + 0.17F^2)(SU)^{0.375}}{H}$</td>
<td>-</td>
<td>Sewers and Natural Streams</td>
<td>(Parkhurst, 1972)</td>
<td>PP</td>
</tr>
</tbody>
</table>
The Pusu River is small (4.1 km in length) and flows through the Gombak campus of the International Islamic University Malaysia (IIUM). Wastewater generated by an approximate population of about 40,000 (including students and staffs of the university) is discharged into the river. Though the wastewater is partially treated, the river water is generally considered polluted and categorised as Class III water (DOE, 1994); mainly due to low DO and ammoniacal nitrogen (Nuruzzaman et al., 2017; Mamun et al., 2016; Nuruzzaman et al., 2015; Zainudin et al., 2014). Wastewater contains organic compounds, which are oxidised by oxygen consuming micro-organisms and the process causes depletion of dissolved oxygen concentration. However, shallow depth and fast-moving water of Pusu River should help to recover dissolved oxygen concentration of the river. For TMDL calculation of the river, it is very much essential to estimate reaeration rates of the river in dry days with great precision. In rainy season, storm-water dilution helps to elevate DO concentration of Pusu River and hence is not critical. It is well known from the published literatures and among water quality modelers that the predictions by many reaeration equations vary by a great extent (Palumbo & Brown, 2013; Melching & Flores, 1999; Moog & Jirka, 1998). Reaeration is a very important phenomenon for calculating assimilative capacity of a river, it is essential to assess performance of available reaeration equations in predicting DO concentration of Pusu River as predicted by different equations greatly differ with each other. As such, this study aims at evaluating performance of 31 available reaeration equations in predicting DO of Pusu River.

II. MATERIALS AND METHOD

A. Pusu River Segmentation and Data Collection

Water quality Analysis Simulation Program (WASP) was employed to model Pusu River for a length of 2.05 km within the IIUM Gombak campus area. The river could be modelled using water quality equations or any other computer model. Nonetheless, WASP was used to avoid tedious calculations in Microsoft Excel. Secondly, due to existence of a few ponds in the river system, WASP model was preferred as ponding of water can be simulated well by the model. Ponding of water improves river water quality by increasing the detention time and retaining the pollutants. Therefore, WASP was used in this study to simulate the effect of ponding on the water quality. Pusu River was segmented into seven segments within the IIUM campus boundary along its main stem. The segments were divided based on the location of Point Sources (PS) and tributaries and the differences of hydraulic conditions, which affect the reaeration rate. Location of the Point Source (PS) pollutions, tributaries and segmentation of the river are shown in Figure 1.
Two sets of data both in dry season were used to calibrate and validate Pusu River WASP model. Physical data, e.g., length, width, slope, channel roughness, pollution sources and all the kinetic coefficients were used same as adopted by Nuruzzaman et al. (2017).

B. Dissolved Oxygen Model

Once all the data were entered into the model, calibration and validation of DO were performed using the available field data. WASP has a DO mass balance system containing all the parameters affecting DO. Only relevant portion of the DO mass balance system to the Pusu River was used to evaluate performance of available reaeration rate equations. This mass balance equation can be represented by the modified Streeter-Phelps method (Equation 3) as used by Thomann and Mueller (1987):

\[
D = D_0 e^{-K_a t} + \frac{K_d + K_a}{K_a - K_r} (e^{-K_r t} - e^{-K_a t}) + \frac{K_d + K_n}{K_n - K_r} (e^{-K_n t} - e^{-K_a t}) \tag{3}
\]

where, \(D_0\) is the initial oxygen deficit (mg/L), \(D\) is the oxygen deficit (mg/L) after travel time ‘t’, \(t\) is the travel time (day), \(K_a\) is the reaeration rate coefficient, \(L_{NBOD}\) is the ultimate NBOD (mg/L) in the river after mixing, \(K_r\) is the NBOD or Ammonia Nitrogen (AN) decay rate coefficient (per day), \(K_n\) is the ultimate CBOD (mg/L) in the river, \(K_d\) is the CBOD decay rate coefficient (per day), \(K_r\) is the CBOD removal rate coefficient (per day).

C. Parameter Estimation for Reaeration Rate (Ka)

Velocity (U) and depth (H) of the river were directly measured by using current meter and tape, respectively as the river was easily navigable due to its shallow depth (0.05 - 0.17 m). Slope (S) was calculated by measuring the difference of Reduce Level (RL) of the starting and ending points of each segment of the river and dividing by the length (L) of corresponding segment. Froude number (F) was calculated from the following equation (Giles et al., 2014):

\[
F = \frac{U}{\sqrt{gH}} \tag{4}
\]

where, g is the gravitational acceleration.

Shear velocity (u*) was determined using the following form of equation as suggested by Gualtieri and Gultieri (Gualtieri et al., 2002).

\[
u^* = \sqrt{gHS} \tag{5}
\]
The values of gas-transfer Reynolds number \((R_g-t)\) and molecular diffusion coefficient \((D_m)\) and were assumed as 0.750 and \(1.8 \times 10^{-9}\) m²/s at 20 °C and, respectively (Haider & Ali, 2010; Gualtieri & Gualtieri, 2004) and kinematic velocity of water \((v)\) was assumed as \(1.003 \times 10^{-6}\) m²/s (Crittenden et al., 2012) at 20 °C.

The following equation is used to apply temperature correction for reaeration rate:

\[
K_a = K_{a20} \theta^{T-20}
\]

where, \(K_a\) is Reaeration rate at \(T\) °C; \(K_{a20}\) is the Reaeration rate at 20 °C; \(T\) is temperature; \(\theta\) is the temperature correction factor; \(\theta = 1.0241\) (Alonso et al., 1975).

**D. Performance Evaluation**

Reaeration rates were calculated for each segment of Pusu River by using all the 31 equations. These reaeration rates were given input in the model individually and the errors in prediction against the observed DO concentration of Pusu River were measured in terms of RMS error and \(R^2\) values.

RMS error was calculated by using Equation 7:

\[
RMSE = \sqrt{\frac{\sum_{i=1}^{n}(X_o - X_p)^2}{n}}
\]

where, \(X_o\) is the observed values, \(X_p\) is the predicted values, \(n\) is the number of data points.

Apart from RMS and \(R^2\) values, residual analysis was also performed. Overall performance of the equations was measured and ranked based on the performance indicator value \((PIV)\). The \(PIV\) was measured using the following equation:

\[
PIV = \frac{Rc^2 + Rv^2}{RMSEC^2 + RMSEV^2}
\]

where, suffix C and V denotes Calibration and Validation data, respectively for corresponding parameter.

The equation was used based on the fact that accuracy of model prediction is proportional to \(R^2\) value and inversely proportional to RMS error. Hence, a higher \(PIV\) indicates good performance, and a lower \(PIV\) refers to poor performance.

**III. RESULT AND DISCUSSION**

Figure 2 shows the range of estimated reaeration rates by different equations. Reaeration rates calculated by Thyssen and Jeppesen (1980) occupy both the extreme points in terms of maximum and minimum rates. It implies that the equation is highly sensitive to variation in the parameters considered. Krenkel and Orlob (1969) equation resulted in high reaeration rate. Since, the range is not too wide, it is not as much sensitive as Thyssen and Jeppesen (1980) equation. It is noteworthy that Tackston and Krenkel (1969) equation predicts extremely low reaeration rate (maximum is 0.00031 per day). The fact is that it was developed for large rivers where depth is usually high having low velocities, whereas Pusu River is a small river with medium velocity (around 0.5 m/s) and shallow depth (highest is 0.17 m) during dry season. Several equations of Group 1 were predicting maximum and minimum reaeration rate around 50 – 100 per day and 5 – 25 per day, respectively. A huge variation is visible among the predictions of equations of other groups. One of the reasons of variation is that all these equations were not developed in identical hydraulic conditions of Pusu River. Moreover, variation in predictions is mentioned by several authors for reaeration equations, which were developed in similar hydraulic conditions (Palumbo & Brown, 2013).

Figure 3 demonstrates RMS error estimations and \(R^2\) values of observed and predicted data yielded by the reaeration rate equations for Pusu River. Negulescu and Rojanski (1969) equation – 1 (NR -1) resulted in RMS error of 0.17 mg/L and 0.09 mg/L in calibration and validation data, respectively, which are the lowest errors produced by any of the equations. Hence, NR-1 equation is the best performer to predict DO of Pusu River in terms of RMS error. On the other hand, both Takston and Dawson (2001) , and Tackston and Krenkel (1969) yielded 2.03 mg/L RMS error in calibration data, the highest error among all the equations. In case of validation, Thyssen and Jeppesen (1980) equation produced RMS error of 1.88 mg/L, which is the maximum error. Equations of Group 1 resulted in the lowest RMS errors in terms of median values both in calibration (0.49 mg/L) and validation (0.51 mg/L). Median values of RMS errors of Group 2 were found to be 0.65 mg/L and 0.73 mg/L for calibration and validation data, respectively. Median RMS errors for the remaining groups were nearly 1 mg/L and higher.
Figure 2. Calculated maximum and minimum reaeration rates by different equations.
Therefore, it is evident that Group 1 equations performed comparatively better than other groups in terms of RMS error and NR-1 was the top performer both in calibration and validation data.

It is remarkable again that NR-1 equation resulted in highest R² values of 0.97 and 0.98 in calibration and validation data, respectively. Thyssen and Jeppesen (1980), and Krenkel and Orlob (1962) equations produced lowest R² values of 0.01 and 0.21 in calibration and validation data, respectively. Group 1 equations performed better than other groups by yielding maximum median R² of 0.93 and 0.90 in calibration and validation data, respectively. Median R² of other groups were less than 0.9 except Group 5. R² values of the only equation of Group 5 were slightly less than the median value of Group 1. Thus, Group 1 equations are more suitable than other groups in terms of R² and NR-1 equation.
is the best one for Pusu River. Group 1 equations were also performing better than the other groups and O’connor and Dobbins (1958) equation was found to be most suitable in case of Ravi River in Pakistan (Haider et al., 2013). However, other studies suggest that incorporating slope, shear stress and other variables improves DO predictability (Palumbo & Brown, 2013).

Many authors have criticised R² estimation for reliability analysis as it fails to reflect good agreement between predicted and observed values in many cases (Ewen, 2011; Gupta & Kling, 2011; Omole et al., 2015; Ritter & Muñoz, 2013). It is also evident in the case of Gualtieri and Gualtieri (2004) equation, which yielded impressive R² values and performed very poor in terms of RMS error (around 1 mg/L). Though performance of the equations is already substantiated by RMS error, nevertheless, residual plots are also shown to confirm the findings, which verifies observed-predicted agreement. Figure 4 demonstrate residual plots of best performing equations from each group in terms of R² for calibration and validation data, respectively. Distance of residuals from zero corresponds to the amount of error for prediction on that point.

![Residual plots](image)

Figure 4. Residual plots for top performing equations from each group.
It is visually evident that residuals of NR-1 equation predictions are very close to zero. In most of the data points, residual was less than 0.06 for NR-1 equation, whereas for other equations it was more than 0.1 and as high as 0.5. Hence, residual plots substantiate better agreement between observed and predicted data for NR-1 equation than other equations. In both the datasets, $R^2$ values were impressive for Gualtieri and Gualtieri (2004) equation (0.92 and 0.90). However, it is evident from the residual plots that residuals deflected too much from zero implying false agreement between observed and predicted data.

Overall performance of each of the equations are shown in Figure 5 and ranked in descending order. It is clear from the figure that NR-1 equation performed best amongst all the equations, scoring a PIV of around 7.7. Among the top 5 equations for Pusu River, there are Melching and Flores (1999), Langbein and Durum (1967), Isaacs and Chaulavachana (1969), and Padden and Gloyna (1971) equations. It is notable that four equations are from Group 1 out of the top five equations for Pusu River. Top 10 equations for Pusu River DO modelling consists of 7 equations (54%) from Group 1, only 2 equations (29%) from Group 3 and only 1 equation (12.5%) from Group 2. Hence, it is evident that Group 1 equations are performing comparatively better than other groups. Only 10 equations out of 31 were underpredicting DO concentration of Pusu River. Moreover, top performing equations from each group were underpredicting DO of Pusu River. In contrast, four equations out of the top five equations were overpredicting. Hence, further research is recommended to investigate on this issue.

Figure 5. Ranking of reaeration rate equations for Pusu River based on performance indicator value.

Figure 6 illustrates how the predicted DO concentrations of Pusu River by the best performing equations from each group fit in the observed data sets. It is already substantiated from RMS errors, $R^2$ values, PIV residual plots and that NR-1 equation performed best. From the following figures, observed-predicted agreement of the equations is distinguishable visually that NR-1 equation predicts DO of Pusu River very closely. Sensitivity of DO model of Pusu River.
using NR-1 equation is shown in Figure 7. Parameters affecting DO were changed by 20% in both directions to observe the effect of change in average DO of Pusu River. The effect of DO coming from the point sources is most significant followed by AN temperature coefficient. Around 0.28 mg/L DO concentration is possible to improve by increasing the DO concentrations of the point source pollutions of Pusu River. This graph provides a useful information that DO from the point sources is the crucial parameter to elevate DO of Pusu River. A full-scale simulation is recommended to measure the impact of various scenarios on DO; and to do that with accuracy, this study has already identified the suitable reaeration rate equation for Pusu River.

Figure 6. DO predictions of Pusu River by the best performing equations from each group.

(a) Calibration Graph
(b) Validation Graph

Figure 7. Sensitivity analysis of Pusu River DO concentration.
Determination of kinetic rates of various water quality parameters is essential for water quality modelling of a river. DO is the most important water quality parameter of a river as it indicates the overall health of a river. Estimation of reaeration rate is crucial to accurately predict DO concentration. As shown by previous literature that estimation of reaeration rate is highly unpredictable and requires investigation on different available equations to find out suitable equations for the river in concern. This study has identified the suitable equations and their performance in predicting DO of Pusu River. The outcome of this study can be applied to estimate reaeration rates of Pusu River and predict future water quality scenarios of the river. This will allow precise estimation of TMDL for Pusu River and prevent deterioration of water quality and river health.

IV. CONCLUSIONS

Following conclusions were drawn from the investigations of this study.

1. Neguluscu and Rojanski (1969) -1, i.e., NR-1 equation is the most suitable equation for Pusu River DO modelling in dry periods, which has been substantiated by comparing with prediction errors of other equations.

2. Equations of Group 1, where velocity and depth are the only considered variables, outperformed other equations where more variables are included. Similar result was also observed for Ravi River in Pakistan, but it contrasts with the findings of other studies. Further studies are recommended to investigate the reason.

3. All the best performing equations from each group under-predicted the DO concentration of Pusu River. In contrast, four equations (overall) out of best five were over-predicting DO of Pusu River. Therefore, future researches are suggested to investigate on this matter.

4. DO coming from the point sources is the most influential parameter impacting on DO of Pusu River.

5. This study has facilitated simulating DO of Pusu River for different scenarios in dry season, which will be beneficial to elevate Pusu River water quality.

V. ACKNOWLEDGEMENT

The Ministry of Science, Technology and Innovation (MOSTI), Malaysia funded this study (Grant # SF-14-009-0059).

VI. REFERENCES


removal-case of a university campus’, Asian Res. Public
Nuruzzaman, M, Mamun, AA & Salleh, MNB 2017, ‘A
Modified Laboratory Approach to Determine Reaeration
Rate for River Water’, Arab. J. Sci. Eng., vol. 4, pp. 2037-
2051.
O’Connor, DJ & Dobbins, WE 1956, ‘The mechanics of
reaeration in natural streams’, J. Sanit. Eng. Div., vol. 82,
pp. 1–30.
Omole, DO, Ndambuki, JM, Musa, GA, Longe, EO, Badejo,
AA & Kupolati, WK 2015, ‘A Model Selection Procedure for
Stream Re-Aeration Coefficient Modelling’, Mod. Appl. Sci.,
vol. 9, pp. 138–160.
Owens, M, Edwards, RW & Gibbs, JW 1964, ‘Some reaeration
studies in streams’, Air Water Pollut., vol. 8, p. 469.
Padden, TJ & Gloyna EF 1971, ‘Simulation of stream
processes in a model river: Austin, University of Texas’,
Palumbo, JE & Brown LC 2013, ‘Assessing the performance
140, p. 4013013.
Parkhurst, JD & Pomeroy, RD 1972, ‘Oxygen absorption in
Raymond, PA, Zappa, CJ, Butman, D, Bott, TL, Potter J,
Mulholland, P, Laursen, AE, McDowell, WH & Newbold, D
2012, ‘Scaling the gas transfer velocity and hydraulic
geometry in streams and small rivers. Limnol. Oceanogr’,
Fluids Environ., vol. 2, pp. 41–53.
of hydrological models: Statistical significance for reducing
480, pp. 33–45.
Smoot, JL 1988, ‘An examination of stream reaeration
effects and hydraulic conditions in a pool-and-riffle
stream’ [doctoral dissertation], Virginia Polytechnic
Institute and State University.
Thackston, EL & Dawson, JW 2001, ‘Recalibration of a
Thackston, EL & Krenkel, PA 1969, ‘Reaeration prediction in
Thomann, RV & Mueller, JA 1987, ‘Principles of surface water
Thyssen, N & Jeppesen, E 1980, ‘Reaeration in small water
Thyssen, N, Erlandsen, M, Jeppesen, E & Ursin, C 1987,
‘Reaeration of oxygen in shallow, macrophyte rich streams:
I–determination of the reaeration rate coefficient’, Int. Rev.
Hydrobiol., vol. 72, pp. 405–429.
Tsivoglou, EC & Neal, LA 1976, ‘Tracer measurement of
reaeration: II. Predicting the reaeration capacity of inland
Wallin, MB, Öquist, MG, Bifam, I, Billett, MF, Nisell, J &
Bishop, KH 2011, ‘Spatiotemporal variability of the gas
transfer coefficient (KCO2) in boreal streams: Implications
for large scale estimates of CO2 evasion’, Global
Yuan, W, Yang, K, Tang M & Xu, Q 2005, ‘Stream structure
characteristics and their impact on storage and flood
control capacity in the urbanized plain river network’ J.
‘Pollution load management to improve water quality of