

Measuring the Efficiency of Water Supply Service: Application of Charnes, Cooper Rhodes and Slack-Based Measure Models

N.R.M. Rashid¹, B.A. Halim², I.S.M. Zawawi^{3*}, N.F. Ibrahim⁴, F.S. Ahmad⁵, L.H.M. Isa⁶ and N.A.S. Hishamudin⁷

^{1,5,6,7}*Mathematical Sciences Studies, College of Computing, Informatics and Mathematics, Universiti Teknologi MARA, Negeri Sembilan Branch, Seremban Campus, Persiaran Seremban Tiga/1, 70300 Seremban, Negeri Sembilan, Malaysia*

²*Mathematical Sciences Studies, College of Computing, Informatics and Mathematics, Universiti Teknologi MARA, Melaka Branch, Alor Gajah Campus, 78000 Alor Gajah, Melaka, Malaysia*

³*School of Mathematical Sciences, College of Computing, Informatics and Mathematics, Universiti Teknologi MARA, 40450 Shah Alam, Selangor, Malaysia*

⁴*Universiti Kebangsaan Malaysia, 45600 UKM Bangi, Selangor, Malaysia*

Measuring the efficiency of water supply services is very important, as the demands for water supply are increasing as the year goes by due to increasing population and development. Improving the efficiency of water supply services is also an economically viable way for water conservation. This paper examines the efficiency of water supply services using Charnes, Cooper, and Rhodes (CCR) output-oriented and Slack-Based Measure (SBM) models. SBM provides the input excesses and output shortfalls for the concerned decision-making units (DMUs). Meanwhile, the CCR output-oriented model interprets the efficiency by maximising the output produced by the DMUs whilst maintaining the input used. The results indicate that the optimal efficiency value for SBM model is lower than the optimal efficiency value of the CCR model. Furthermore, the SBM model provides the water supply providers with values for input excess and output shortfalls for future improvements.

Keywords: Water supply efficiency; slack-based measure; Charnes, Cooper and Rhodes.

I. INTRODUCTION

In recent years, the availability of water has emerged as a pressing global issue, particularly due to the escalating demand for water supply driven by population growth. Asia, home to two-thirds of the world's population, is experiencing a significant rise in water requirements across various sectors, emphasising the critical need for accessible water supplies (Ahmed *et al.*, 2014). Water is not only a crucial resource for the survival of human beings, but it is also a prime resource for the development of society. Due to the rapid growth of the human population and socio-economic, residential, industrial, and agricultural development, the demand for water has also increased, so it is imperative to enhance the quality of water supply and ensure efficient services that meet

society's demands. The initial process towards achieving this goal is to measure the performance of water supply services (Saad & Harun, 2017).

Measuring water efficiency holds value in various contexts beyond water security, making it a significant undertaking for environmental, economic, cultural, spiritual, and aesthetic reasons. Implementing such measurements not only enhances water availability but also serves as intrinsic purposes in these diverse aspects (Park *et al.*, 2022). In Malaysia, the National Water Services Industry Act was introduced in 2006 to oversee the performance of utility organisations or water operators. The National Water Services Commission (NWSC) and *Pengurusan Aset Air Berhad* (PAAB) have been established under this act to coordinate, manage, and control the water services industry,

*Corresponding author's e-mail: iskandarshah@uitm.edu.my

including sewerage services. NWSC is a technical and economic regulator whose function is to oversee and regulate water and sewage treatment services in Malaysia.

Assessing the performance of water supply services is essential for determining the effectiveness of water supply service providers in each region. This evaluation allows providers to meet the rising demands of consumers, considering the continuous increase in population throughout the year, while also striving to enhance the quality of their services (Ong Boon *et al.*, 2007). There are several quantitative methods available for measuring the efficiency of water supply service performance, including Data Envelopment Analysis (DEA) model, stochastic frontier analysis (SFA), index-based approach, performance benchmarking, econometric model, and mathematical programming.

DEA is a widely used method in empirical studies that compares the relative efficiencies of multiple decision-making units (DMUs) by analysing their input-output relationships (Wang *et al.*, 2018). It is suitable to measure efficiency as well as set specific goals for improvement purposes (Kamarudin *et al.*, 2014). DEA has also been applied across a wide range of industries such as the evaluation efficiency of public state libraries (Guccio *et al.*, 2018), the banking sector (Othman *et al.*, 2016) and sustainable supplier evaluation (Zarbakshnia & Jaghdani, 2018). DEA is a nonparametric method in operation research that has been used for assessing the relative efficiency of homogeneous DMUs (Charnes *et al.*, 1978). The DMUs consider two groups of factors, which are input and output, that provide the efficiency score for each DMU with a range between zero and one.

Furthermore, the analysis identifies potential adjustments in inputs and outputs that could lead to an increase or decrease in the efficiency of the DMUs (Ng *et al.*, 2014). Less productive units, or inefficiencies, are identified with an efficiency score that is less than one. In true efficiency score distributions, the upper bound is always set at one, but the lower bound can be customised (Zarrin & Brunner, 2023).

The Slack-Based Measure (SBM) (Tone, 2001) and Charnes, Cooper, and Rhodes (CCR) (Charnes *et al.*, 1978) are specific approaches within the framework of DEA. SBM is an extension of DEA that focuses on identifying and measuring

inefficiencies in the utilisation of inputs. This model directly addresses input excesses and output deficiencies of the DMUs involved as it takes into account the underutilised resources, referred to as "slack", during the evaluation process. Consequently, SBM enables the establishment of specific targets for improvement by calculating the amount of unused slack that can be utilised to increase efficiency.

CCR, on the other hand, is a pioneering linear programming model within DEA (Adna *et al.*, 2019). It measures the relative efficiency of DMUs by comparing their input-output relationships. The CCR model determines efficiency scores by optimising the weights for inputs and outputs. Thus, this guarantees that no other DMUs can achieve greater efficiency without adversely affecting the efficiency of at least one DMU. CCR comprises the Constant Scale Returns (CSR) radial model, which can be categorised into two variations: input-oriented and output-oriented. The input-oriented model aims to minimise input usage while maintaining output levels, while the output-oriented model focuses on maximising output production while maintaining constant input usage (Kamarudin *et al.*, 2016).

Therefore, the objective of this study is to evaluate the efficiency of 14 regions of water service providers by using the CCR and SBM models. The primary contribution of this paper is to demonstrate the practicality and effectiveness of the proposed models in assessing water supply efficiency using actual data from the Malaysian Water Association (MWA).

II. METHODOLOGY

This section describes the steps in methodology for the efficiency measurement of the water providers, by employing the applications of CCR and SBM models.

A. Collection of Data

The secondary data for this study are collected from the reports published by the MWA in cooperation with the NWSC and the Ministry of Water, Land, and Natural Resources, known as the Malaysian Water Industry Guide (Malaysian Water Association (MWA), 2018), where the report is published on a year-to-year basis for researchers or any institutions to obtain data about water supply service as well as sewerage services for 14 states in Malaysia. The data used

in this study are from the year 2017, as this is the latest data that could be obtained.

B. Case Study: Input and Output Data

The selection of representative indicators as inputs and outputs is both very important and challenging in the

efficiency assessment process, as a lack of data might produce errors and inaccurate results. Therefore, the efficiency measurement in this study considers two inputs and two outputs that affect the services that are being provided by each state. The selection of input and output is shown in Table 1 (Kamarudin & Ismail, 2016).

Table 1. Input and output data of water supply operators.

States	Input Data		Output Data	
	Number of Workers	Operation Cost/ OPEX (RM)	Volume of Water Use (MLD)	Total Revenue (RM)
Johor	2220	587929	1320	1139807
Kedah	1418	296451	719	307299
Kelantan	816	101170	240	115153
Labuan	134	25608	48	33034
Melaka	805	156985	413	230956
Negeri Sembilan	1079	212455	519	275543
Pulau Pinang	1332	219923	826	336472
Pahang	1655	298411	582	175122
Perak	1075	227784	907	392244
Perlis	138	42480	89	33650
Sabah	1064	465119	582	335633
Sarawak	2399	194552	870	270875
Selangor	4569	2596082	3243	2094242
Terengganu	458	101333	427	134961
Total	19162	5526282	10785	5874991

In the present study, the input data refers to total operating expenses and the number of employees, whereas the output data refers to the total volume of water distributed to consumers and the total revenue. The volume of water is calculated in units of Million Liters per Day (MLD) in order to represent the number of litres (million) of water in a day.

To determine the reference set or benchmarks for inefficient DMUs, the optimum intensity vector, λ is obtained from the optimal solution. The reference set for DMUs is $R^o = \{j | \lambda^* > 0\}$ for $\{1, \dots, n\}$ (Tone, 2001). This reference set is used after each DMU has been appointed. The frequency of the efficient DMUs is used as a reference set for inefficient DMUs, which can elevate the importance of efficient DMUs and then, the rank of each

DMUs can be determined (the rank of each DMU can be determined by utilising the frequency of the efficient DMUs as a benchmark for the inefficient DMUs, which can increase the importance of efficient DMUs).

C. Mathematical Model

In this study, there are two models used to measure the efficiency of water supply services in Malaysia. The models are Charnes, Cooper and Rhodes (CCR), and Slack-Based Measure (SBM).

Minimise

$$\sum_{i=1}^m v_i X_{i0}$$

subject to,

$$\sum_{r=1}^s u_r Y_{r0} = 1,$$

$$\sum_{i=1}^m v_i X_{ij} - \sum_{r=1}^s u_r Y_{rj} \geq 0, \quad j = 1, \dots, n,$$

$$u_r, v_i \geq \varepsilon, \quad r = 1, \dots, s, \quad i = 1, \dots, m.$$

2. SBM model

Minimise

$$\rho = \frac{1 - \left(\frac{1}{m}\right) \sum_{i=1}^m \frac{S_i^-}{X_{i0}}}{1 + \left(\frac{1}{s}\right) \sum_{r=1}^s \frac{S_r^+}{Y_{r0}}}$$

subject to,

$$X_{i0} = \sum_{j=1}^n X_{ij} \lambda_j + S_i^-, \quad i = 1, \dots, m,$$

$$Y_{r0} = \sum_{j=1}^n Y_{ij} \lambda_j - S_r^+, \quad r = 1, \dots, s.$$

$$\lambda \geq 0, \quad j = 1, \dots, n.$$

$$S_i^- \geq 0, S_r^+ \geq 0,$$

where X_{ij} is input, Y_{rj} is output, j is DMUs or state, m is number of inputs, s is number of outputs, n is number of DMUs or states, ρ is objective function, S_r^+ is default output variable, S_i^- is default input variable and λ_j is dual weight for state j .

D. Process of Data

All the inputs and outputs are coded and executed under LINGO 20.0 software for both CCR and SBM models to obtain the efficiency scores.

E. Analysis of Data

The efficiency scores are measured on a scale of 0 to 1, where a value of 1 indicates the DMU is relatively efficient and a value of less than 1 indicates the unit is inefficient.

III. RESULTS AND DISCUSSION

The results of CCR and SBM models are analysed as presented in Table 2.

Table 2. Efficiency scores of 14 states in Malaysia using CCR and SBM models.

DMUs	States	Efficiency Scores		Reference Sets		Reference Frequencies		Ranks	
		CCR	SBM	CCR	SBM	CCR	SBM	CCR	SBM
1	Johor	1	1.0000	1	1	6	2	2	2
2	Kedah	0.6073	0.6015	9,12,14	9	0	0	14	10
3	Kelantan	0.6464	0.4967	1,9	9	0	0	13	13
4	Labuan	0.6868	0.5498	1,9	9	0	0	11	12
5	Melaka	0.8110	0.7155	1,9	9	0	0	10	7
6	Negeri Sembilan	0.7219	0.6523	1,9	9	0	0	12	8
7	Pulau Pinang	0.9225	0.8140	9,12,14	9	0	0	7	6
8	Pahang	0.4591	0.3720	12,14	9	0	0	5	14
9	Perak	1	1.0000	9	9	9	11	1	1
10	Perlis	0.7305	0.6020	9,14	9	0	0	8	9
11	Sabah	0.7320	0.5501	1,9,13	9	0	0	9	11
12	Sarawak	1	1.0000	12	12	4	1	3	3
13	Selangor	1	1.0000	13	1,9,13	2	1	6	3
14	Terengganu	1	1.0000	14	14	5	1	3	3
Average Score		0.8084	0.7396						

DMUs are classified as efficient if their score is one. However, those with an efficiency score less than one are considered inefficient (Kamarudin & Ismail, 2016). Therefore, inefficient DMUs are urged to enhance their efficiency by closely examining their utilisation of inputs and outputs. Both models yield congruent results, where five states are deemed efficient, while nine states display inefficiency. In conclusion, the ranking of DMUs can serve as a benchmarking guide, where inefficient DMUs can draw insight from the top-ranked DMUs to drive future improvements. Nevertheless, the SBM model provides a more comprehensive and precise depiction of the inefficiency

scores, notably in terms of input redundancy and output insufficiency.

As in Table 3, the negative values in input variables indicate that the states should reduce a certain amount of the input, while on the other hand, the positive values in output values represent an addition to the output value. For instance, Kelantan indicates an inefficient score where Kelantan should reduce 41.49% of the number of workers (Input 1). Meanwhile, Kelantan must increase the output of 67.85% of the volume of water used (Output 1) and 51.29% (Output 2) for the total revenue.

Table 3. Result of score efficiency with input excess and output shortfall of each state.

States	Efficiency Scores	Slack Variables			
		s_1^-	s_2^-	s_1^+	s_2^+
Johor	1	0	0	0	0
Kedah	0.6015	0	18.93411 (6.39x10 ⁻³ %)	461.4212 (+64.18%)	203189.6 (+66.12%)
Kelantan	0.4967	338.5400 (-41.49%)	0	162.8430 (+67.85%)	59061.72 (+51.29%)
Labuan	0.54980	42.33032 (-31.59%)	6183.920 (-24.15%)	29.34363 (+61.13%)	414.2624 (+1.25%)
Melaka	0.7154740	76.24124 (-9.47%)	2566.780 (-1.64%)	201.8690 (+48.88%)	34952.14 (+15.13%)
Negeri Sembilan	0.6522752	76.34343 (-7.08%)	0	326.9623 (+63%)	90304.47 (+32.77%)
Pulau Pinang	0.8140236	294.0991 (-22.08%)	0	49.69874 (+6.02%)	42235.36 (+12.55%)
Pahang	0.3719790	246.6900 (-14.91%)	1.242239 (-0.0004%)	606.2206 (+104.16%)	338739.5 (+193.43%)
Perak	1	0	0	0	0
Perlis	0.6019886	0	13238.89 (-31.16%)	27.43349 (+30.82%)	16703.18 (+49.94%)
Sabah	0.5500632	0	239665.8 (-51.53%)	315.7191 (+54.25%)	52597.34 (+15.67%)
Sarawak	1	0	0	0	0
Selangor	1	0	1.234568 (-5 × 10 ⁻⁵ %)	0	0
Terengganu	1	0	0	0	0

IV. CONCLUSION

This study utilises the CCR and SBM models for efficiency measurement and analysis. Both models enable a full evaluation of the efficiency for water supply services provided for each state in Malaysia. The findings demonstrate that these innovative models allow water providers to identify deficiencies and excesses in input and output so that the service providers can improve their annual performance

through a benchmarking process. Moreover, the evaluation of the performance can also act as an indicator to the service providers to aim for the best in the future, as improving water efficiency service is an economically viable way to conserve water.

Finally, future research could include undesirable input and output as performance measures for those water service providers. Other than that, future studies should include the

outcomes of the efficiency measurement as practiced by NWSC with the DEA model to determine the suitability of the DEA model as an alternative performance indicator for water supply services in Malaysia using any correlation test.

V. REFERENCES

- Adna, N, Mohamad, SNF, Alias, S, Mohd Yusoff, NS & Hashim, H 2019, 'An Efficiency Tool: Application of CCR and SBM models in measuring commercial banks' performance in Malaysia', in *2nd Kelantan International Learning and Innovation Exhibition 2019 (KILIEx2019)*.
- Ahmed, F, Siwar, C, Begum, RA & Selangor, DE 2014, 'Water resources in Malaysia: Issues and challenges', *Journal of Food Agriculture & Environment*, vol. 12, pp. 1100–1104.
- Charnes, A, Cooper, WW & Rhodes, E 1978, 'Measuring the efficiency of decision making units', *European Journal of Operational Research*, vol. 2, no. 6, pp. 429–444.
- Guccio, C, Mignosa, A & Rizzo, I 2018, 'Are public state libraries efficient? An empirical assessment using network Data Envelopment Analysis', *Socio-Economic Planning Sciences*, vol. 64, pp. 78–91.
- Kamarudin, N & Ismail, WR 2016, 'Establishment of performance indicators for Malaysian water utilities with the presence of undesirable output', *Jurnal Teknologi*, vol. 78, no. 4-4.
- Kamarudin, N, Ismail, WR & Abd Kader, NH 2014, 'Assessing Efficiency and Effectiveness of Malaysian Water Supply Services: A Data Envelopment Analysis Approach', *Journal of Quality Measurement and Analysis*, vol. 10, no. 2, pp. 1–13, retrieved May 22, 2023, from <https://www.ukm.my/jqma/v10_2/jqma-10-2-paper1.pdf>.
- Kamarudin, N, Ismail, WR & Ramli, NA 2016, 'Malaysia water utilities performance with the presence of undesirable output: a directional distance function approach', *Jurnal Teknologi*, vol. 78, no. 4-3.
- Malaysian Water Association (MWA) 2018, *Malaysian Water Industry Guide 2018*.
- Ng, K, Wong, S, Yap, P & Khezzimotlagh, D 2014, 'A Survey on Malaysia's Banks Efficiency: Using Data Envelopment Analysis', vol. 1, pp. 586–592.
- Ong Boon, K, Suhaimi, AT & Ghufran Redzwan 2007, 'Establishment of Performance Indicators for Water Supply Services Industry in Malaysia', *Malaysian Journal of Civil Engineering*, vol. 19, no. 1, pp. 78–83.
- Othman, FM, Mohd-Zamil, A, Zaleha, S, Rasid, A, Vakilbashi, A & Mokhber, M 2016, 'Data Envelopment Analysis: A Tool of Measuring Efficiency in Banking Sector', *International Journal of Economics and Financial Issues*, vol. 6, no. 3, pp. 911–916, retrieved from <<http://www.econjournals.com>>.
- Park, S-Y, Kim, J-S, Lee, S & Lee, J-H 2022, 'Appraisal of Water Security in Asia: The Pentagonal Framework for Efficient Water Resource Management', *Applied Sciences*, vol. 12, no. 16, p. 8307.
- Saad, NM & Harun, A 2017, 'Restructuring to Improve Water Services in Malaysia', *Institutions and Economies*, vol. 9, no. 3, pp. 21–49.
- Tone, K 2001, 'A slacks-based measure of efficiency in data envelopment analysis', *European Journal of Operational Research*, vol. 130, no. 3, pp. 498–509.
- Wang, S, Zhou, L, Wang, H & Li, X 2018, 'Water Use Efficiency and Its Influencing Factors in China: Based on the Data Envelopment Analysis (DEA)—Tobit Model', *Water*, vol. 10, no. 7, p. 832.
- Zarbakhshnia, N & Jaghdani, TJ 2018, 'Sustainable supplier evaluation and selection with a novel two-stage DEA model in the presence of uncontrollable inputs and undesirable outputs: a plastic case study', *The International Journal of Advanced Manufacturing Technology*, vol. 97, no. 5–8, pp. 2933–2945.
- Zarrin, M & Brunner, JO 2023, 'Analyzing the accuracy of variable returns to scale data envelopment analysis models', *European Journal of Operational Research*, vol. 308, no. 3, pp. 1286–1301.