

A Framework for Diagnosing Dyscalculia

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This paper outlines the development of mathematical items which evolves from making sense of mathematics through perception, operation, and reason. There are three main constructs in this framework, and it involves seven tests namely simple reaction time, short term memory, number sense, matching Items, dot enumeration, number comparison and arithmetic. This study is concerned with how a group of students struggle with basic numerical skills which may be caused by dyscalculia. One of the most important contributions of this study, in addition to its wider theoretical and practical applications, is that it has come out with a set of standardized instruments which is called the Malaysian Dyscalculia Instrument (MDI) for primary school students. This tool could be further used widely in the fields of psychology and education, especially special education and particularly research studies related to education.

Keywords: Perception, Operation, Reason, Dyscalculia.

I. INTRODUCTION

This study described the development of a framework for diagnosing dyscalculia. It is based on a fundamental theory of how humans learn to think mathematically from early childhood to adult including mathematicians. In general, this framework is developed based on the concept of numerosity (Butterworth, 2002), theories of cognitive development in mathematical thinking (Tall, 1995; Tall, 2007) and symptoms and causes proposed by several researchers such as Murphy (2006), Geary (2006), Gersten *et. al.*, (2008), and Shalev & Von Aster (2007). Consequently, the researchers concluded four constructs of MDI as shown in Table 1.

II. MATERIALS AND METHODS

There are not many studies about diagnosing Dyscalculia. A Dyscalculia screener developed by Butterworth (2003) is widely used to diagnose and identify students who suffer from Dyscalculia. The development of the Dyscalculia screener is based on the concept of numerosity proposed by Butterworth (2005). However, it does not provide a clear theoretical framework to explain in more detail about the cognitive development of students in mathematical thinking. A framework for diagnosing Dyscalculia in this study evolves from the work of Butterworth (2003), Tall (2004), Geary (2006), and others. In this case, various theories of mathematical thinking are presented in order to show how this framework evolves from previous work.

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The study focuses on 7-9 years old children (primary 1-3) who experience difficulties in learning mathematics. In this study, the first objective is to design and develop a framework for the Malaysian Dyscalculia instrument (MDI) that can be used to screen and measure the extent of dyscalculia among primary school students. In order to develop the MDI in this study, the researcher proposed several constructs that are relevant for identifying dyscalculia. Referring to Table 1, the MDI has four main constructs and four sub-constructs as shown in the table. Each of the constructs has a specific role and the order of these constructs is based on the theory of cognitive development in mathematical thinking proposed by Tall (2007), that is to classify the learners into the appropriate level of basic numeracy skills.

Simple Reaction Time is a test to measure psychomotor response time. Recorded response time will take into account to identify the actual cognitive processing time (Butterworth, 2003). Figure 1 displays the chronology of a screenshot for the items in simple reaction time. At first, the screen will blank for a few seconds and then black dots will appear on any area of the screen. Once the black dots appeared, the pupils have to press a particular button as soon as possible to obtain the best response time. This process was repeated for ten times for both left and right hands respectively. The response times for the next six tests were adjusted to take this measure into account.



Figure 1. Simple Reaction Time

It is commonly believed that most individuals can hold between five and nine items of information in the short-term memory span at one time (Westwood, 2004). Working memory is the short-term memory (Bandeley, 2002) and the concept of working memory evolved from earlier concepts of Short-Term Memory (Lervag & Hulme, 2013) Figure 2 shows the chronology of items in short term memory. Students are asked to memorise the pictures which will appear then disappear in a few seconds on the left and then to the right of the screen in a row (shown in Figure 2). After that, they have to give an answer to indicate whether the right screen or the left screen has more black dots.

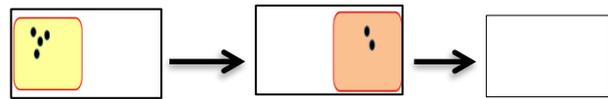


Figure 2. Short Term Memory

The idea of numerosity (Butterworth, 1999) involves familiar consequences such as two sets of things might have the same numerosity as the other, or a smaller or larger numerosity. This innate ability that we are born with a sense of numbers is called Number Sense (Butterworth, 2002). Students with learning disabilities seem to have problems in many aspects of basic number sense, such as the difficulty in understanding quantity (Gersten et al., 2008). They are slower in completing very simple quantity tasks. Figure 3 unveils a sample item of number sense which was used to evaluate whether a student possesses the concept of quantity and the scientists called this concept as numerosity (Santos-Sousa, 2007). Pupils must provide their answers by identifying which diagram has more black dots.

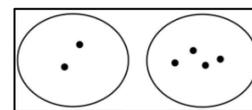


Figure 3. Number Sense

Butterworth (2002) proposed that two sets would have the same numerosity if and only if the members of each set can be put in the form of one-to-one correspondence with none left over. It involves the principle of matching every item of one set to the items of the other set. Hence, the researchers included **Matching Items** (see Figures 4) in order to test whether a student understands the one-to-one correspondence principle or not.



Figure 4. Matching Items

Dot Enumeration is a test (see Figures 5) that requires better skills in counting numerals and using symbols (Butterworth, 2002). Learning the basic counting sequence, “one, two, three and four ...” is not difficult and almost all children including dyscalculic students can learn this (Geary, 2006). However, it is not only about the

sequence but also involves the ability to assign each counted object and represents the quantity of items in the counted set. Figures 5 shows an item of Dot Enumeration in this study. This is a very straightforward task and researchers expect all the pupils will be able to get most of the correct answers. The critical factor is how long they need to get the correct answer for each problem.

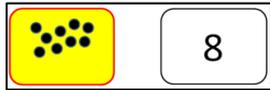


Figure 5. Dot Enumeration

According to Geary (2006), dyscalculic students often do not know basic number names (e.g. “7” = “Seven”), and they have difficulties in discriminating which number is larger or smaller. They have difficulties in completing simple quantity tasks such as comparing two numbers (e.g. Which is bigger, 6 or 8?). Hence, Number Comparison (Figure 6) becomes a crucial construct to test the brain areas (affected by dyscalculia) which are specialised to deal with the idea of quantity. This is known as sense of ordered numericities (Butterworth, 2002). This task asks pupils to select the larger of two numbers which requires a fluent understanding of numerals.



Figure 6. Number Comparison

Many children with dyscalculia have difficulties in remembering basic arithmetic facts, such as the answer for $3+2$ (Geary, 2006). They have a great difficulty in memorising simple addition, subtraction, and multiplication facts. Thus, the Arithmetic test is required (Butterworth, 2003) to measure the basic arithmetic skills. This task consists of number sentences that involve addition and subtraction only. The number sentence is presented on the screen with an answer (e.g. $3+2=4$ in Figure 7). The pupil has to judge as quickly as possible whether 4 is the correct answer or not.

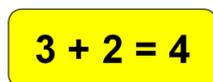


Figure 7. Arithmetic

III. RESULTS AND DISCUSSIONS

The numerosity concept (Butterworth, 1999), identified in a systematic literature review, coupled with the cognitive development in mathematical thinking framework (Tall, 2007) served as the theoretical foundation for the framework development process. The results indicated that the instrument constructed from the numerosity construct and the mathematical thinking framework provided valid and reliable measures of dyscalculia and basic numeracy skills among primary school students. As a result of this study, Figure 8 shows the framework for diagnosing Dyscalculia. This framework evolves from the concept of numerosity (Butterworth, 2002), theory of making sense of mathematics through *perception*, *operation* and *reason* (Chin & Tall, 2012) and symptoms and causes of Dyscalculia

This framework used the analysis of different forms of representation to show how they feature in various mathematical tests. It outlined the development of visuo spatial to verbal in short term memory and number sense, perceptual development in the dot enumeration, number comparison and arithmetic, and the relationship between them in matching items. At the top of the figure was the basic numeracy skills which are the basic requirements to advanced mathematical thinking. All these require significant cognitive reconstructions (Tall, 2008).

One of the most important contributions of this study, in addition to its wider theoretical and practical applications, is that it has come out with a set of standardized instruments which is called the Malaysian Dyscalculia Instrument (MDI) for primary school students. This tool could be further used widely in the fields of psychology and education, especially special education and particularly research studies related to education. This study illustrates show the constructs of MDI were used, as well as the interactions among themselves and with the basic numeracy skills. The findings would help policy makers especially the Malaysia Ministry of Education to develop a more robust theoretical framework for understanding the way in which the components are interrelated, and for conceiving the relative roles of the components of numeracy as applied to primary school students.

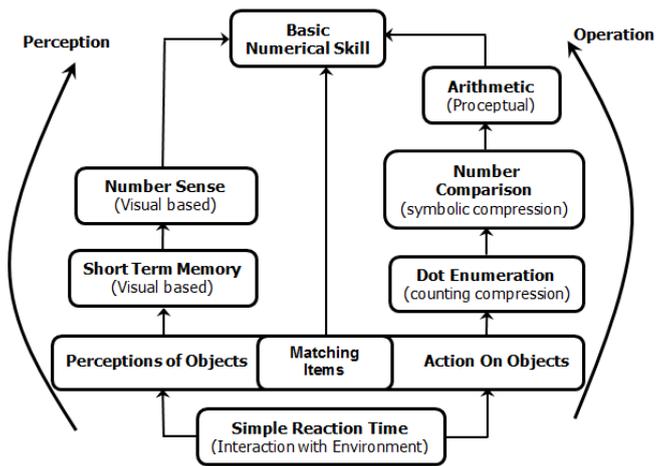


Figure 8. Framework for Diagnosing Dyscalculia

IV. SUMMARY

This framework exhibits the analysis of different forms of representation to illustrate the features of various mathematical tests. It outlines the development of visuospatial to verbal in number sense, the proceptual development in dot enumeration, number comparison, and arithmetic, as well as the relationships between them in matching items. The top of the framework depicts the basic numeracy skills, which are the fundamental requirements to advanced mathematical thinking. This requires significant cognitive reconstructions (Tall, 2008). In addition, this framework functions as a basis for the construction and development of a dyscalculia classification model that was used to identify dyscalculia.

Generally, this framework was developed based on the sense of numerosity (Butterworth, 2003), coupled with the theory of making sense of mathematics (Chin & Tall, 2012). The framework basically consists of two parts namely perceptions of objects and actions on objects (operation). *Perception* refers to humans learn mathematics through their perception or senses. Meanwhile, *operation* refers to physical actions, such as counting, which involves the compression of actions to thinkable concepts (see Figure 8).

V. ACKNOWLEDGEMENT

The authors would like to express sincere gratitude to the Malaysia Ministry of Education and Sabah Education Department for providing the financial means and laboratory facilities. We would like to acknowledge the financial support of the Fundamental Research Grant Scheme (FRG0312-SS1-1/2012) and the Malaysia Ministry of Education for the full scholarship of one of the authors in this study.

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Appendix: Tables

Table 1. Constructs for Diagnosing Dyscalculia

No.	Construct	Description of items	Capacity/Test	Source
1.	Simple Reaction Time	10 items for left, 10 items for right.	Response Time	Butterworth (2003), Murphy (2006)
2	Short Term Memory	10 items	Short Term Memory	Gersten et. Al (2008)
3.	Numerosity			
	i. Number sense	10 items	Sense of Numerosity	Butterworth (1999), Geary (2006), Gersten et al. (2008), Von Aster and Shalev (2007)
	ii. Matching Items	10 items	Numerosity as a property of sets	Butterworth (2002b), Geary (2006)
	iii. Dot Enumeration	10 items	Enumeration (counting)	Butterworth (2002b)
	iv. Number Comparison	10 items	Sense of ordered numerosities	Butterworth (2002b)
4.	Arithmetic Test	10 items	Arithmetic	Butterworth (2003), Geary (2006)