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Cubic Trigonometric Spline for Preserving Positive Data

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A degree three, cubic trigonometric spline with three basic functions is presented in this paper. The presence of two shape parameters β_1 and β_2 provides flexibility in controlling the shape of the curve to develop a smooth and continuity spline without repositioning the control points. The spline formulation of geometric continuity of degree one, is then derived based on necessary conditions with the freedom of choosing values parameter to fit positive data. This result is an effective realistic approximation since it preserves the characteristics of the data it represents, and the curve display looks smooth and pleasant.

Keywords: shape parameters; continuity; interpolation; positive data; preserve positivity

I. INTRODUCTION

CAGD is about mathematical and computational method of describing entities geometrically, involved in so many different areas such as electronics, manufacturing, animations, robotics and scientific visualization. Many previous studies and recent developments in geometric design see the introduction of trigonometric terms in curve design. The functions help to improve the curve and surface capability which cannot be achieved by a polynomial for example in formation of circular arcs and conics which are the most basic geometrical entity in almost every modeling system. Positivity is an important aspect of many types of data like probability distribution, rainfall distribution, heart rate and discharge from some chemical process like fermentation. An approximation or interpolation which takes into consideration properties of the data will give a more realistic visualization and are even more computationally efficient. The result is to have a visualization that satisfies characteristics of the data.

In satisfying the characteristics of numerous of data, many researchers had developed schemes to provide splines that can preserve the positivity of the data. Schmidt and Heß (1987) studied the preserving problem using quadratic and rational quadratic spline. They discovered that the quadratic splines fail to retain the positivity data, but the problems can be solved through rational quadratic splines with the suitable selection of the parameters. Abbas *et al.* (2005) developed a rational cubic spline function with three shape parameters. An automated computation scheme for one shape parameter is derived to preserve positivity and the other two parameters are set free in adjusting the curve as desired. Sarfraz and Hussain (2006) presented sufficient conditions on piecewise rational cubics under C^1 continuity in preserving positivity by assigning automated values of the two shape parameters.

Karim *et al.* (2015), extended the C^1 rational cubic spline interpolant of Karim and Kong (2014) to the bi- cubic partially blended rational function. They built schemes that have 12 parameters and 8 of them are free to preserve the positivity of 3D curves and surfaces. This is similar to Abbas *et al.* (2012). They also extend the rational cubic function into bi-cubic function with six free parameters in each rectangular patch to preserve the shape of positive 3D data. The results of the scheme demonstrated not only local control but are computationally economical and also visually pleasant. Cubic Hermite spline scheme is derived by C^1piecewise rational quadratic trigonometric spline interpolation to preserve positivity data in paper by Dube and Rana (2014). Dube and Singh (2014) constructed a quadratic trigonometric beta-spline in preserving positivity

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and monotonicity of interpolating curve underGC^2 continuity. In Liu *et al.* (2014), the construction of bi-quartic rational interpolation spline surface over rectangular domain includes the classical bi- cubic Coons surface. To maintain the characteristics of positive and monotonic data, they chose suitable values of parameters on the spline using appropriate conditions. Bashir and Ali (2016) produced a piecewise C^1univariate rational quadratic trigonometric splinewith three positive shape parameters. A positive and monotone curve interpolation scheme is derived in order to sustain the respective shape features of the data. Qin *et al.* (2017) used Boolean sum of cubic kinds of quadratic/linear interpolation splines as the boundary functions to construct a class of C^1bi-cubic partially blended rational quartic/ linear interpolation splines with four families of local control parameters. Nur *et al.* (2018) proposed G^1quadratic trigonometric beta spline with a shape parameter that preserves the positivity curve of the data.

This paper is based on the generation of cubic trigonometric spline interpolation. By using a higher degree that is three will give more formative spline compared to spline with degree of two. In literature, there are numerous researchers provided cubic spline with four basic functions but in this paper, a cubic spline with only three basic functions is proposed which is simpler and not complicated. The purpose is to produce smooth and pleasant curve of G^1continuity with two shapes of parameters. The paper is arranged as follows. Firstly, cubic trigonometric spline basis functions with the shape parameters are introduced and the properties are proved. Secondly, the behaviors of the basic functions are tested by changing the values of shape parameters. The effect and role of shape parameters are discussed through the presented curves of proposed cubic trigonometric spline. Next, with different shape parameters, the figures are illustrated as open or closed curves. The scheme in preserving positivity of positive data for G^1 continuous conditions for joining constructed curves is also presented. Lastly, conclusion and future work are included.

II. METHODS

This study considers the theoretical aspect such as properties and characteristics of the proposed cubic trigonometric spline basis functions at first. The specialty of this cubic spline is it has three basic functions. The proposed curve enjoys and satisfies all the geometric properties needed. The spline curve is presented and tested by using the different values of shape parameters. To achieve the positivity preserving interpolant for the proposed cubic trigonometric spline function, the value of shape parametersβ_1,β_2 and γ are adjusted on a trial-and-error basis.

III. RESULTS AND DISCUSSIONS

A. Cubic trigonometric spline functions

Firstly, the cubic trigonometric spline basis functions are introduced, and all the properties are discussed.

Definition 1: The three cubic trigonometric spline basis functions are:

$$f_0(u) = (1-\sin(u))^2(1-\beta_1\sin(u));$$
$$f_1(u) = 1 - f_0(u) - f_2(u);$$
$$f_2(u) = (1-\cos(u))^2(1-\beta_2\cos(u)); \quad (1)$$

for$u \in \left[0,\frac{\pi}{2}\right]$, with two shape of parameters β_1,$\beta_2 \in [-2,1]$.

Theorem 1: The basic functions (1) above have all the following properties:

A. Non-negativity:

$$f_i(u) \geq 0, \qquad i = 0,1,2$$

B. Partition of unity:

$$\sum_{i=0}^{2} f_i(u) = 1, \qquad u \in \left[0,\frac{\pi}{2}\right]$$

C. Monotonicity: For the given value of the parameter, $\beta_1$andβ_2, $f_0(u)$ is monotonically decreasing and $f_2(u)$ is monotonically increasing.

D. Symmetry:

$$f_i(u,\beta_1,\beta_2) = f_{i-1}\left(\frac{\pi}{2} - u,\beta_1,\beta_2\right);$$
$$i = 0,12$$

Proof:

i) For $u \in \left[0, \frac{\pi}{2}\right]$,and $\beta_1, \beta_2 \epsilon [-2,1]$, then

$$(1-\sin(u))^2 \geq 0, \qquad (1-\beta_1 \sin(u)) \geq 0,$$

$$\sin u \ \geq 0, (1-\cos(u))^2 \geq 0,$$

$$(1-\beta_1 \cos(u)) \geq 0, \ \cos u \ \geq 0,$$

While$f_0(u) + f_2(u) < 1$

It is obvious that
$f_i(u) \geq 0, \qquad i = 0,1,2.$

ii) Partition of unity is obvious.

Forall u, β_1, β_2.

$$f(u) + \left(1 - f_0(u) - f_2(u)\right) + f_2(u) = 1$$

iii) For $u_0, u_1 \epsilon \left[0, \frac{\pi}{2}\right]$, such that $u_0 \leq u_1, f_0(u_0) \geq f_0(u_1)$ which shows that $f_0(u)$ is monotonically decreasing.

For $u_0 \leq u_1, f_2(u_0) \leq f_2(u_1)$which shows that $f_2(u)$ is monotonically increasing.

iv) For $i = 2$,

$$f_2(u; \beta_1, \beta_2) = (1-\cos(u))^2(1-\beta_2\cos(u));$$

$$= \left(1 - sin\left(\frac{\pi}{2} - u\right)\right)^2 \left(1 - \beta_1 sin\left(\frac{\pi}{2} - u\right)\right);$$

$$= f_0\left(\frac{\pi}{2} - u; \beta_1, \beta_2\right).$$

Figure 1 shows the curves of the cubic trigonometric spline basis function for $\beta_1 = \beta_2 = -1$ (blue dashed), $\beta_1 = \beta_2 = 0.5$ (red) and $\beta_1 = \beta_2 = 1$ (black). By comparing the cubic trigonometric spline with different values of β_1 and β_2, spline in Figure 1 (a) with lowest value of shape parameters β_1 and β_2, that is -1 shows curvier compared to Figure 1(b) and (c). Figure 1(d) shows the blending of three cubic trigonometric spline basis functions with various value of β_1 and β_2.

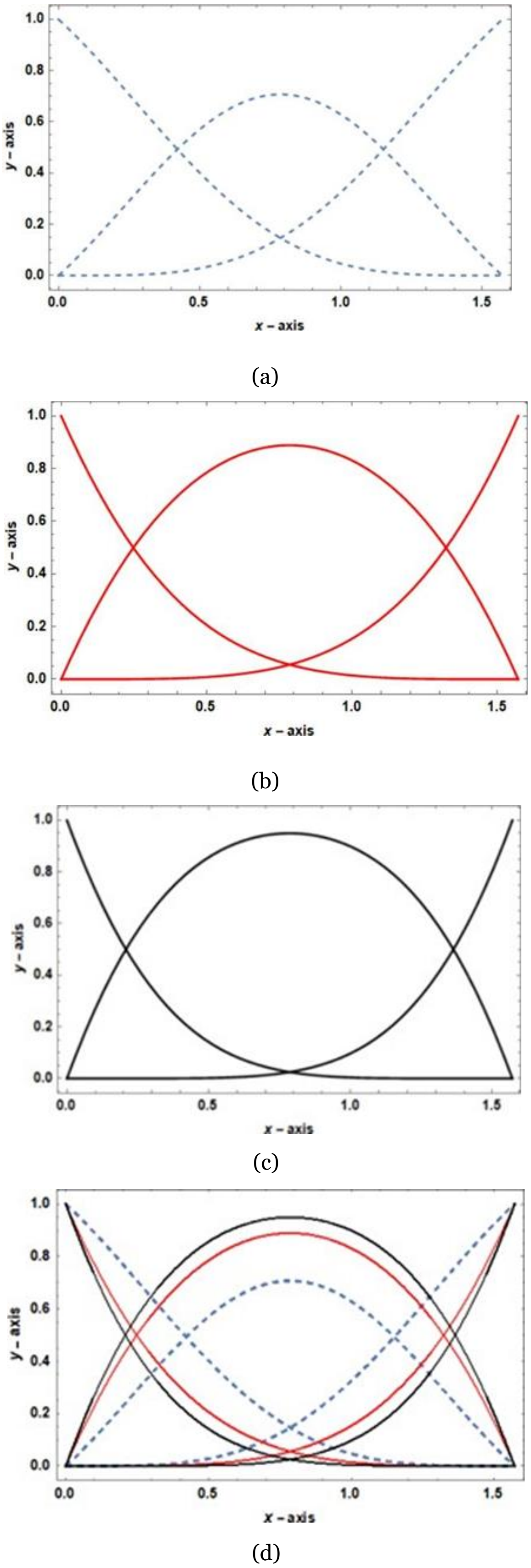
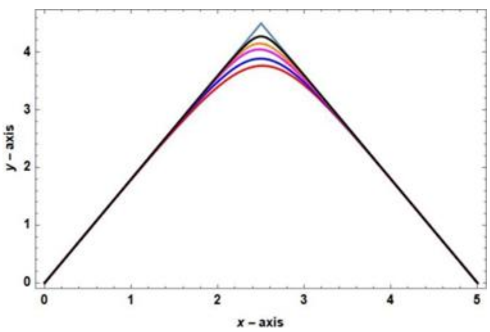


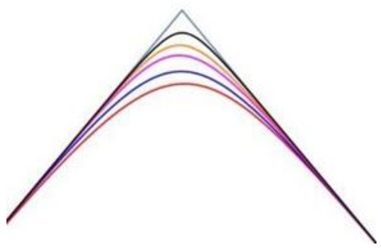
Figure 1. Cubic trigonometric spline basis functions varying values of $\beta_1$and β_2

B. Shape control of the cubic trigonometric spline

Cubic trigonometric curves are constructed with the chosen control points $V_0 = (0,0)$, $V_1 = (2.5,4.5)$,and $V_2 = (5,0)$. The cubic trigonometric curve is represented by $F(u)$ where $F(u) = \sum_{i=0}^{2} f_i(u) \cdot V_i$.The shape of the curve is controlled by changing the values of shape parameter β_1 and β_2.



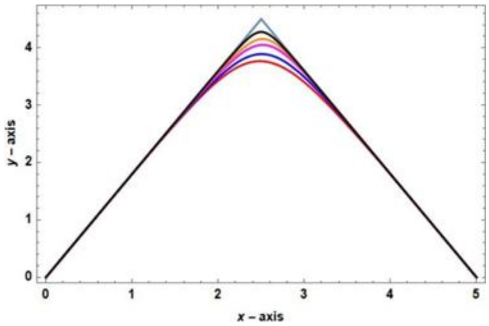
(a)



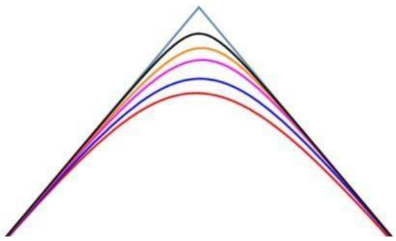
(b)

Figure 2. The effect on the shape of cubic trigonometric spline curve for $\beta_2 = 1$ and varying values of β_1

The parameters $\beta_1$and $\beta_2$adjust the shape and behavior of the curves in Figure 2 and 3. In Figure 2, by varying the values of shape parameters $\beta_1$keeping$\beta_2 = 1$, the cubic trigonometric Beta spline curve gets closer to control polygon as increases as clearly shown by the blowing of the image in 2(b). The curves are generated by setting the values $\beta_1 = -2$(red), $\beta_1 = -1$ (blue), $\beta_1 = 0$ (magenta), $\beta_1 = 0.5$ (yellow) and $\beta_1 = 1$(black). All the curves are in the convex hull.



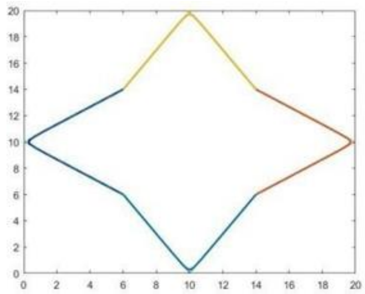
(a)



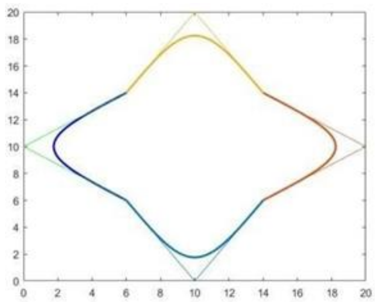
(b)

Figure 3. The effect on the shape of cubic trigonometric spline curve for $\beta_1 = 1$ and varying values of β_2.

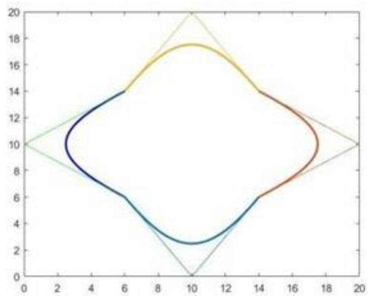
In Figure 3, the curves are generated by setting of $\beta_1 = 1$ and the values of $\beta_2$are varied. Thecurvesaregeneratedbysettingthe values $\beta_2 = -2$(red), $\beta_2 = -1$ (blue), $\beta_2 = 0$ (magenta), $\beta_2 = 0.5$ (yellow) and $\beta_2 = 1$ (black). This produced curves that are closer to the control polygon as the values of $\beta_2$increases and all the curves lean to the right of the control polygon as shown clearly in blow up image as shown in 3 (b).



a) $\beta_1 = \beta_2 = 1$



b) $\beta_1 = \beta_2 = -1$



c) $\beta_1 = \beta_2 = -2$

Figure 4. Cubic trigonometric spline curves with different shape parameters (star)

The presence of shape parameters provides an intuitive control on the shape of curves. This property allows generation of open and closed curves. Figure 4 shows the image of a four-pointed star produced by adjusting the values of $\beta_1$andβ_2. By decreasing the values of $\beta_1$andβ_2, the curves become smoother. The smoothest curve produced in 4(c) using $\beta_1 = \beta_2 = -2$.

C. Positive curve interpolation

The cubic trigonometric spline curve generated will not readily allow in preserving shape preserving of positive data. Suitable values of shape parameters are needed to have a positive curve.

Let $(x_i, f_i), i = 1,2, \dots, n$ be a given set of data points, where $x_1 < x_2 < \dots < x_n$. f_i is the function values and let d_i be at the endpoints of the derivatives. Let $\theta_i = \frac{x - x_i}{h_i}$where$h_i = x_{i+1} - x_i$.The$G^1$ cubic trigonometric spline function with two shape parameters β_1 and β_2, is defined over the subinterval $[x_i, x_{i+1}]$ as:

$$\begin{aligned} P_i(x) = (1 - \sin(u))^2(1 - \beta_1 \sin(u)) \cdot A_i + 1 - f_0(u) \\ - f_2(u) \cdot B_i \\ + (1 - \cos(u))^2(1 - \beta_1 \cos(u)) \cdot C_i \end{aligned} \quad (2)$$

that satisfy the following interpolation geometric continuity conditions.

$$\begin{aligned} P(x_i) &= f_i, & P(x_{i+1}) &= f_{i+1}, \\ P'(x_i) &= \gamma_i d_i, & P'(x_{i+1}) &= d_{i+1} \end{aligned} \quad (3)$$

After some calculations, the control points are:

$$\begin{aligned} A_i &= f_i; \\ B_i &= \frac{2h_i}{\pi\beta_1 + 2\pi} \gamma_i d_i + f_i; \\ C_i &= f_{i+1}; \end{aligned} \quad (4)$$

The determination of first d_1 using the arithmetic mean method by Malik and Muhammad (2009)

$$d_1 = \begin{cases} 0, & if\ \Delta_i = 0\ or\ sgn\,(d_1^*) \neq sgn(\Delta_i), \\ d_1^* = \Delta_i + \dfrac{(\Delta_1 - \Delta_2)h_1}{h_1 + h_2}, & otherwise \end{cases} \quad (5)$$

The other derivatives $d_{i+1}, i = 1. \dots, n$ are obtained by the derivation and calculation made up from (3).

$$d_{i+1} = \frac{\pi \cdot (\beta_2 + 2)}{2 \cdot h_i}(f_{i+1} - f_i) - \frac{\beta_2 + 2}{\beta_1 + 2} \cdot \gamma_i d_i \quad (6)$$

By using positive data set in Table 1 taken from Mridula and Meenal (2014), the proposed cubic trigonometric spline function is constructed with different values of shape parameters with G^1 continuity.

Table 1. Positive Data Set

i	x_i	f_i	γ	β_1	β_2	d_i
1	1	3.0	1.0	0.5	0.5	5.0
2	2	6.0	2.0	1.0	1.0	-14.7124
3	3	5.0	2.0	0.5	0.5	41.3058
4	4	8.0	1.0	0	0	-63.1969
5	5	1.0	5.0	1.0	1.0	-

The image produced is

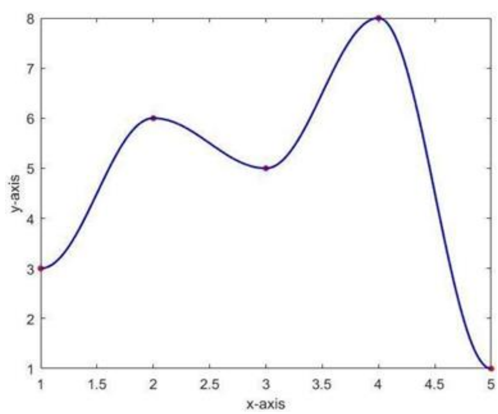


Figure 5. Positive graph of cubic trigonometric spline

A suitable value of shape parameters γ, β_1 and β_2 are assigned in order to satisfy positivity of data using the trial and error method. The curve produced in Figure 5 is G^1 continuous and preserves the positivity of the data. It represents a smooth and pleasant curve.

III. CONCLUSIONS

The cubic trigonometric spline proposed in this paper satisfies all the necessary properties needed. The presence of shape parameters allows adjusting the visual smoothness without changing the control points. Furthermore, the computations are less since the cubic has only 3 basis functions.

The presented cubic trigonometric spline manages to preserve the features of the real data and it gives a more realistic visualization besides forming a very flexible and pleasant curve. This research can be extended by using a higher continuity or by increasing the number of shape parameters to allow the formation of desired curve easily and flexibly.

IV. ACKNOWLEDGEMENT

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