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Elasticity Effects on Hydromagnetic Convective Instability in Viscoelastic Nanofluid Layer

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Convective fluid flow instability is sensitive to disturbance forces and its inherent rheological properties. In this study, we examined the effects of viscous elasticity and magnetic field on convective instabilities in a deep horizontal viscoelastic nanofluid layer. Linear stability theory was used to determine the onset of stationary and oscillatory instabilities. Closed form solution for the critical Rayleigh number was obtained using the Galerkin-type weighted residuals method. The effects of the scaled stress relaxation parameter scaled strain retardation parameter and Chandrasekhar number on the stability of the system were investigated. Magnetic field also delays the stationary and oscillatory convective instabilities in viscoelastic nanofluids.

Keywords: convective instability; Oldroyd-B; stress relaxation; strain retardation; magnetic field

I. INTRODUCTION

Convective instabilities are caused by the gradients in surface tension, density and concentration. Pearson (1958) was the first to study theoretically the steady Marangoni convection in a horizontal fluid layer. Surface tension gradients are dominant in thin fluid layer and in microgravity, but in deep fluid layer gravitational forces overcome the surface tension forces. Buoyancy creates density gradient in fluids and can lead to natural convection flows in the presence of gravitational field. Gravitational and magnetic fields are ubiquitous in our environment and the convective flows is highly sensitive to the direction of these two disturbance forces.

The stability of Marangoni convection has been considered by Arifin *et al.* (2007) and Allias *et al.* (2017). Nield & Kuznetsov (2010), Nield & Kuznetsov (2012) and Yadav *et al.* (2014) considered the onset of convective instability in nanofluids. Studies has shown that nanoparticles and magnetic fields can enhance or attenuate the instability of an originally unstable flow (Dastvareh & Azaiez, 2017; Perez *et al.*, 2011). Magnetic field is found to stabilize the fluid layer for both stationary and oscillatory convection. The convective flow is also influenced by the inherent rheological properties of the fluid. Shivakumara *et al.* (2015) and Tahir *et al.* (2016) investigated the stability in the viscoelastic nanofluids. Narayana *et al.* (2013) investigated the stabilizing effect of an external magnetic field on double-diffusive convection in a weakly electrically conducting viscoelastic fluid. Stronger fluid's elasticity of viscoelastic fluid can overcome the stabilizing effect of magnetic field.

The purpose of this paper is to examine the influence of the magnetic field on the convective instability in a deep viscoelastic nanofluid layer. The linear stability theory is employed by scaling, regular perturbation, linearization and method of normal modes. The eigen value problem is solved analytically using the Galerkin-type weighted residuals method to determine the critical value of the thermal Rayleigh number.

II. PROBLEM FORMULATION

Consider an infinite horizontal layer of incompressible viscoelastic nanofluid of Oldroyd-B type of depth $z = H$ with weak-electrical conductivity subject to vertical temperature gradient. The gravity $\mathbf{g} = -g\hat{\mathbf{e}}_z$ is assumed to act vertically downwards. A uniform magnetic field, $\mathbf{H} = H_0\hat{\mathbf{e}}_z$ is applied parallel to the z-axis. The magnetic Reynolds number is assumed small so that the induced

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magnetic field is negligible. The conservation equations for flow in the nanofluid layer are (Perez *et al.*, 2011; Narayana *et al.*, 2013; Chandrasekhar, 2013)

$$\nabla \cdot \mathbf{v} = 0, \tag{1}$$

$$\left(1+\lambda_1 \frac{\partial}{\partial t}\right)[\rho_{f_0}\left(\frac{\partial \mathbf{v}}{\partial t}+\mathbf{v}\cdot\nabla\mathbf{v}\right)+\nabla p-\rho\mathbf{g} +\sigma u \mu_m^2 H_0^2 \hat{\mathbf{e}}_x + \sigma v \mu_m^2 H_0^2 \hat{\mathbf{e}}_y] = \mu\left(1+\lambda_2\frac{\partial}{\partial t}\right)\nabla^2\mathbf{v}, \tag{2}$$

$$(\rho c)_f\left(\frac{\partial T}{\partial t}+\mathbf{v}\cdot\nabla T\right)=k\nabla^2 T +(\rho c)_p\left[D_B\nabla\phi\cdot\nabla T+\left(\frac{D_T}{T_c}\right)\nabla T\cdot\nabla T\right], \tag{3}$$

$$\frac{\partial \phi}{\partial t}+\mathbf{v}\cdot\nabla\phi = D_B\nabla^2\phi+\left(\frac{D_T}{T_c}\right)\nabla^2 T, \tag{4}$$

where $\mathbf{v}=(u,v,w)$ is the nanofluid velocity, λ_1 is the relaxation time, λ_2 is the retardation time, t is the time, p is the pressure, σ is the electrical conductivity, μ_m is the magnetic permeability, μ is the coefficient viscosity, ϕ is the nanoparticle volume fraction, T is the temperature, $(\rho c)_f$ is the effective heat capacity of the fluid, $(\rho c)_p$ is the effective heat capacity ofthe nanoparticle, k is the thermal conductivity, D_B is the Brownian diffusion coefficient and D_T is the thermophoretic diffusion coefficient. The nanofluid density, ρ is

$$\rho \cong \phi\rho_p + (1-\phi)\left[\rho_{f_0}-\rho_{f_0}\beta(T-T_c)\right], \tag{5}$$

where ρ_p is the nanoparticle mass density, ρ_{f_0} is the nanofluid density at reference temperature T_c and β is the volumetric coefficient of thermal expansion. The boundary conditions are a rigid lower boundary and free upper boundary given by

$$w=0,\ \frac{\partial w}{\partial z}=0, T=T_h,\ \phi=\phi_0 \text{ at } z=0, \tag{6}$$

$$w=0,\ \frac{\partial^2 w}{\partial z^2}=0, T=T_c,\ \phi=\phi_1 \text{ at } z=H. \tag{7}$$

III. LINEAR STABILITY THEORY

The system (1) – (7) will be subjected to linear stability theory in order to examine the onset of the stationary and oscillatoryinstabilities. The introduction of the scaling quantities Nield & Kuznetsov (2010) for the length, velocity, time, pressure, nanoparticles volume fraction and temperature give the nondimensional variables

$$\left(x^*,y^*,z^*\right)=\frac{(x,y,z)}{H},\left(u^*,v^*,w^*\right)=(u,v,w)\frac{H}{\alpha_f}, \quad t^*=\frac{\alpha_f}{H^2}t,\ p^*=\frac{H^2}{\mu\alpha_f}p,\phi^*=\frac{\phi-\phi_0}{\phi_1-\phi_0}, \quad T^*=\frac{T-T_c}{T_h-T_c}, \tag{8}$$

where $\alpha_f=\dfrac{k}{(\rho c)_f}$ is the thermal diffusivity of the fluid and asterisks denote dimensionless quantities. The non-dimensional equations after dropping the asterisks are

$$\nabla\cdot\mathbf{v}=0, \tag{9}$$

$$\left(1+\Lambda_1\frac{\partial}{\partial t}\right)[\frac{1}{P_r}\left(\frac{\partial\mathbf{v}}{\partial t}+\mathbf{v}\cdot\nabla\mathbf{v}\right)+\nabla p+Rm\hat{\mathbf{e}}_z -RaT\hat{\mathbf{e}}_z+Rn\phi\hat{\mathbf{e}}_z+Q\left(u\hat{\mathbf{e}}_x+v\hat{\mathbf{e}}_y\right)] =\left(1+\Lambda_2\frac{\partial}{\partial t}\right)\nabla^2\mathbf{v}, \tag{10}$$

$$\frac{\partial T}{\partial t}+\mathbf{v}\cdot\nabla T=\nabla^2 T+\frac{N_B}{Le}\nabla\phi\cdot\nabla T +\frac{N_A N_B}{Le}\nabla T\cdot\nabla T, \tag{11}$$

$$\frac{\partial\phi}{\partial t}+\mathbf{v}\cdot\nabla\phi=\frac{1}{Le}\nabla^2\phi+\frac{N_A}{Le}\nabla^2 T, \tag{12}$$

subject to the dimensionless boundary conditions

$$w=0,\ \frac{\partial w}{\partial z}=0, T=1,\ \phi=0 \text{ at } z=0, \tag{13}$$

$$w=0,\ \frac{\partial^2 w}{\partial z^2}=0, T=0,\ \phi=1 \text{ at } z=1. \tag{14}$$

The nondimensional parameters are

$$\Lambda_1 = \lambda_1 \frac{\alpha_f}{H^2}, \Lambda_2 = \lambda_2 \frac{\alpha_f}{H^2}, P_r = \frac{\mu}{\rho_{f_0}\alpha_f},$$
$$Le = \frac{\alpha_f}{D_B}, Q = \frac{\sigma\mu_m^2 H_0^2 H^2}{\mu},$$
$$N_A = \frac{D_T\left(T_h^* - T_c^*\right)}{D_B T_c^*\left(\phi_1^* - \phi_0^*\right)}, N_B = \frac{(\rho c)_p}{(\rho c)_f}\left(\phi_1^* - \phi_0^*\right),$$
$$Ra = \frac{\left[\rho_{f_0}\beta\left(T_h^* - T_c^*\right)\right] gH^3}{\mu\alpha_f}, \tag{15}$$
$$Rn = \frac{\left[\left(\rho_p - \rho_{f_0}\right)\left(\phi_1^* - \phi_0^*\right)\right] gH^3}{\mu\alpha_f},$$
$$Rm = \frac{\left[\rho_p\phi_0^* + \rho_{f_0}\left(1-\phi_0^*\right)\right] gH^3}{\mu\alpha_f},$$

where Λ_1 is the scaled stress relaxation parameter, Λ_2 is the scaled strain retardation parameter, P_r is the Prandtl number, Le is the Lewis number, Q is the Chandrasekhar number, N_A is the modified diffusivity ratio, N_B is the modified particle density increment, Ra is the thermal Rayleigh number, Rn is the concentration Rayleigh number and Rm is the basic density Rayleigh number.

A. Perturbation Solution

The fluid is at rest in the reference steady basic state varying in the *z*-direction (Nield & Kuznetsov, 2014). The basic state is perturbed by infinitesimal perturbations

$$\mathbf{v} = \mathbf{v}',\ p = p_b + p',\ T = T_b + T',\ \phi = \phi_b + \phi', \tag{16}$$

where prime denotes the perturbation quantity and $p_b = p_0,\ T_b = 1 - z,\ \phi_b = z$ where p_0 is the constant reference pressure. The linearized perturbed system of (9) – (14) is,

$$\left(1 + \Lambda_1\frac{\partial}{\partial t}\right)\left[\frac{1}{P_r}\frac{\partial}{\partial t}\nabla^2 w' - \nabla_H^2 RaT' + \nabla_H^2 Rn\phi' + Q\nabla_z^2 w'\right] = \left(1 + \Lambda_2\frac{\partial}{\partial t}\right)\nabla^4 w', \tag{17}$$

$$\frac{\partial T'}{\partial t} - w' = \nabla^2 T' + \frac{N_B}{Le}\left(\frac{\partial T'}{\partial z} - \frac{\partial \phi'}{\partial z}\right) - 2\frac{N_A N_B}{Le}\frac{\partial T'}{\partial z}, \tag{18}$$

$$\frac{\partial \phi'}{\partial t} + w' = \frac{1}{Le}\nabla^2\phi' + \frac{N_A}{Le}\nabla^2 T', \tag{19}$$

subject to boundary conditions,

$$w' = 0, \frac{\partial w'}{\partial z} = 0, T' = 0, \phi' = 0 \text{ at } z = 0, \tag{20}$$

$$w' = 0, \frac{\partial^2 w'}{\partial z^2} = 0, T' = 0, \phi' = 0 \text{ at } z = 1, \tag{21}$$

where ∇^4 is the three-dimensional biharmonic operator, ∇^2 is the three-dimensional Laplacian operator and ∇_H^2 is the one-dimensional Laplacian operator in the horizontal plane.

B. Normal Modes

The non-dimensional system constitutes a linear boundary-value problem that can be solved using the method of normal modes given by

$$\left(w', T', \phi'\right) = \left[W(z), \Theta(z), \Phi(z)\right] e^{\left(st + i\alpha_x + i\alpha_y\right)}, \tag{22}$$

where $W(z), \Theta(z)$ and $\Phi(z)$ are the amplitudes of the velocity, temperature and nanoparticle's volume fraction, respectively. $\alpha = \left(\alpha_x^2 + \alpha_y^2\right)^{1/2}$ is the total wave number and s is a dimensionless complex growth rate. The system (17) – (21) in form of the normal modes (22) are

$$(1 + \Lambda_1 s)\left[\frac{s}{P_r}\left(D^2 - \alpha^2\right)W + QD^2W + Ra\alpha^2\Theta - Rn\alpha^2\Phi\right] = (1 + \Lambda_2 s)\left(D^2 - \alpha^2\right)^2 W, \tag{23}$$

$$W + \left(D^2 + \frac{N_B}{Le}D - 2\frac{N_A N_B}{Le}D - \alpha^2 - s\right)\Theta - \frac{N_B}{Le}D\Phi = 0, \tag{24}$$

$$W - \frac{N_A}{Le}\left(D^2 - \alpha^2\right)\Theta - \left[\frac{1}{Le}\left(D^2 - \alpha^2\right) - s\right]\Phi = 0, \tag{25}$$

with boundary conditions

$$W = 0, DW = 0, \Theta = 0, \Phi = 0 \text{ at } z = 0, \tag{26}$$
$$W = 0, D^2W = 0, \Theta = 0, \Phi = 0 \text{ at } z = 1, \tag{27}$$

where $D \equiv \frac{d}{dz}$. The Galerkin-type weighted residuals method is used to obtain an approximation of the closed form solution to the system (23) – (27). The functions W, Θ and Φ are in form of

$$W = \sum_{n=1}^{N} A_n W_n, \Theta = \sum_{n=1}^{N} B_n \Theta_n, \Phi = \sum_{n=1}^{N} C_n \Phi_n, \qquad (28)$$

where A_n, B_n and C_n are unknown coefficients and $n = 1, 2, 3, \ldots, N$. Equations (23) – (25) are multiplied by W, Θ and Φ, respectively of Equation (28). Performing integration by parts with respect to z from 0 to 1 will lead to a system of $N-1$ equation with $N-1$ unknown. For rigid-free boundaries, the trial functions are

$$W_1 = z^2(1-z)(3-2z), \Theta_1 = z(1-z), \quad \Phi_1 = z(1-z), \qquad (29)$$

where W_1, Θ_1 and Φ_1 are trial functions that satisfying the boundary conditions (26) and (27).

IV. RESULTS AND DISCUSSIONS

The thermal Rayleigh number Ra determines the thresholds for the transition from stationary patterns to weakly chaotic evolution to highly turbulent state. For $s = 0$, we will obtain the solution for the case of stationary convection

$$Ra_{stat} = \frac{28(10+\alpha^2)}{507\alpha^2}[4536 + 432\alpha^2 + 19\alpha^4 + 216Q] - (N_A + Le)Rn. \qquad (30)$$

The stationary instability does not depend on the Λ_1 and Λ_2. For the case of Newtonian fluid where $\Lambda_1 = \Lambda_2 = 0$ (no scaled stress relaxation and scaled strain retardation parameters) and in the absence of magnetic field, Equation (30) reduces the result of Nield & Kuznetsov (2010) for stationary convection. Setting $s = i\omega$ we obtain the solution for Ra in form of $Ra = Ra_r + iRa_i$. Setting Ra_i will give the corresponding ω for oscillatory instability and the oscillatory Rayleigh number is

$$Ra_{osc} = \frac{28}{507\alpha^2}\{\frac{4536 + 432\alpha^2 + 19\alpha^4}{1+\Lambda_1^2\omega^2}[(10+\alpha^2)(1+\Lambda_1^2\omega^2) - \omega^2(\Lambda_2 - \Lambda_1)] - \frac{\omega^2}{P_r}(12+\alpha^2) + 216Q(10+\alpha^2)\} - \frac{(10+\alpha^2)^2(N_A + Le) + \omega^2 Le^2}{(10+\alpha^2)^2 + \omega^2 Le^2}Rn. \qquad (30)$$

Figure 1 shows the neutral stability curves for the variations of thermal Rayleigh number, Ra as a function of the wave number α for various values of Λ_1. Stress relaxation occurs when the applied deformation rate is reduced. The oscillatory thermal Rayleigh number decreases with an increase in Λ_1. The effect of Λ_1 is to advance the onset of convection in a viscoelastic nanofluid layer and the system is unstable.

Figure 2 shows the neutral stability curves for the variations of thermal Rayleigh number, Ra as a function of the wave number α for various values of Λ_2. It is observed that, an increase in the value of Λ_2 increase the oscillatory thermal Rayleigh number. Λ_2 delays the onset of convection in a viscoelastic nanofluid layer thus stabilize the system. Figure 3 shows the thermal Rayleigh number increases for both stationary and oscillatory convection as the value of Q increases. Increased magnetic parameter signifies the strength of Lorentz force since the magnetic field is the measure of the relative importance of Lorentz force to the viscous hydrodynamic force. Hence, the influence of magnetic field delays the onset of oscillatory and stationary convection thus stabilizes the system.

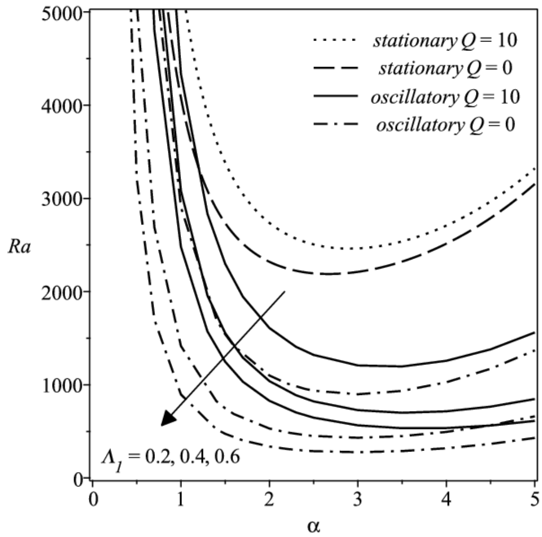

Figure 1. Neutral stability curves of the stationary and oscillatory convection for various values of Λ_1 when $\Lambda_2 = 0.1, P_r = 5, Le = 100, N_A = 5$ and $Rn = -10$

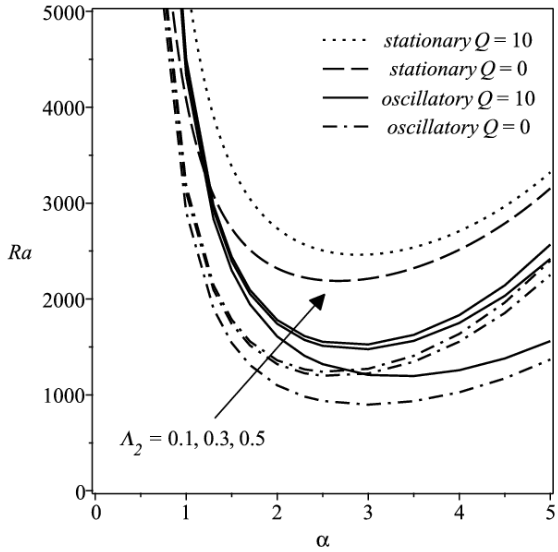

Figure 2. Neutral stability curves of the stationary and oscillatory convection for various values of Λ_2 when $\Lambda_1 = 1.0, P_r = 5, Le = 100, N_A = 5$ and $Rn = -10$

The critical thermal Rayleigh number is increasing when the value of Chandrasekhar number increases is shown in Figure 4. The critical thermal Rayleigh number remains stable for stationary convection while the critical thermal Rayleigh number decreasing as Λ_1 increases for oscillatory convection. The critical thermal Rayleigh number for the stationary and oscillatory convection as functions of the Λ_2 for several values of Q is shown in Figure 5. As the value of Q and Λ_2 increase, the critical thermal Rayleigh number increase. The critical thermal Rayleigh number remains stable for stationary convection while the critical thermal Rayleigh number increasing as Λ_2 increases for oscillatory convection.

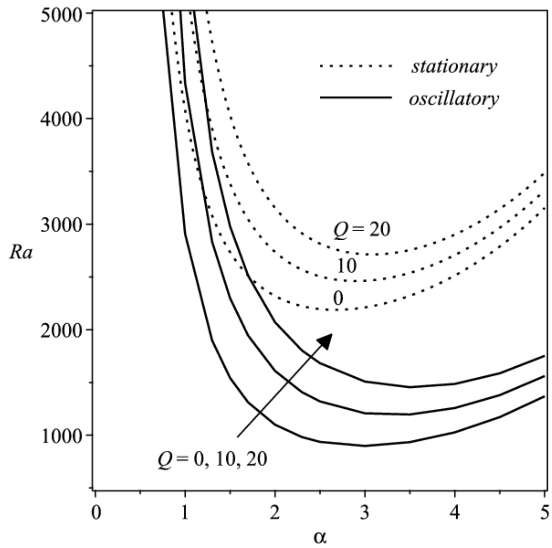

Figure 3. Neutral stability curves of the stationary and oscillatory convection for various values of Q when $\Lambda_1 = 1.0, \Lambda_2 = 0.5, P_r = 5, Le = 100, N_A = 5$ and $Rn = -10$

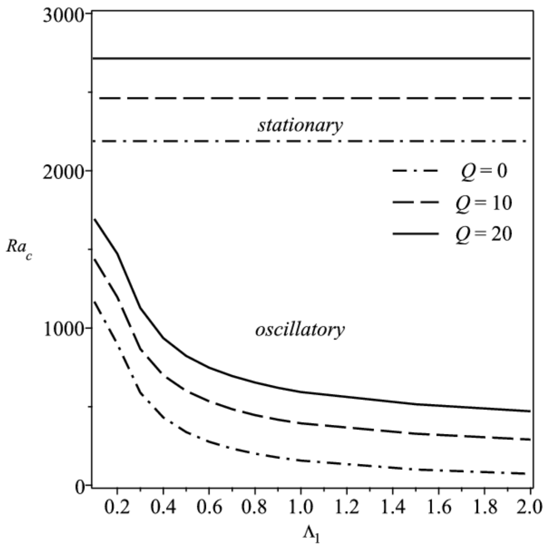

Figure 4. The critical thermal Rayleigh number as function of Λ_1 for various values of Q when $\Lambda_2 = 0.1, P_r = 5, Le = 100, N_A = 5$ and $Rn = -10$

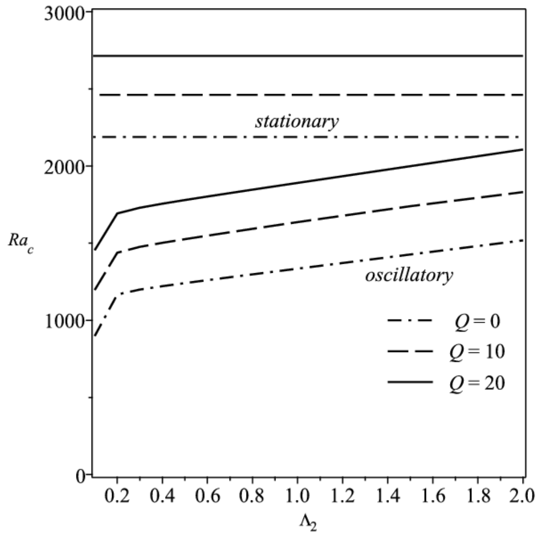

Figure 5. The critical thermal Rayleigh number as function of Λ_2 for various values of Q when $\Lambda_1 = 1.0, P_r = 5, Le = 100, N_A = 5$ and $Rn = -10$

V. CONCLUSIONS

The influences of stress relaxation, strain retardation and magnetic field on the convective instability in a deep viscoelastic nanofluid layer were studied analytically. The system of conservation equations was solved using a linear stability analysis. The effect of stress relaxation is to advance the onset of oscillatory convection while the effect of strain retardation is to delay the onset of oscillatory convection. The magnetic field delays both stationary and oscillatory convection.

VI. ACKNOWLEDGEMENT

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