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Free Convection Boundary Layer Flow of Viscoelastic Micropolar Fluid Past a Horizontal Circular Cylinder

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This study intends to observe the behaviour of the flow of viscoelastic micropolar fluid under boundary layer approximation moving over a horizontal circular cylinder at constant wall temperature. First, a set of governing equations is formulated before they are transformed into dimensionless form. Stream function is then applied to the obtained equations, producing sets of partial differential equations which are then solved numerically using a finite difference scheme, namely the Keller-box method. The skin friction and heat transfer characteristics of the flow are observed at multiple values of viscoelastic parameter, K, material parameter, K_1, and the magnetic parameter, M.

Keywords: viscoelastic micropolar; horizontal circular cylinder; aligned MHD

I. INTRODUCTION

Free or natural convection is a process where heat transfer causes fluid motion while energy is transferred from hotter to colder region that occur if temperature difference exists in the fluid. The process has been proven significant in both nature and engineering with extensive application in aeronautics, solar water heating system, cooling of electrical components and gas turbine blades as well as being a major factor in the design of chemical processing equipment.

Even though the study of free convection over horizontal circular cylinder has started a few decades back, the literature on this topic is still not as common as the case of mixed convection. Merkin (1976) was the first to work out the exact solution of the problem for Newtonian fluid. Motivated by the pioneer study, Ingham & Pop (1987) studied free convection about a heated horizontal circular cylinder in porous medium while Merkin & Pop (1988) considers the free convection boundary layer on horizontal circular cylinder in viscous fluid with constant heat flux. Since then, the problem of free convection over horizontal circular cylinder is then extended to various types of fluids, boundary conditions and effect in both theoretical and experimental study.

The previous studies have inspired Nazar *et al.* (2002) to extend the idea to micropolar fluid with constant heat flux problem before Salleh & Nazar (2010) adopted the idea for the case of Newtonian heating and Sarif *et al.* (2014) investigates the flow with convective boundary conditions in viscous fluid. While Mohd Kasim *et al.* (2012) considers the case when the circular cylinder is embedded in viscoelastic fluid, Sheikholeslami *et al.* (2012) is interested in the magnetic field effect on the flow of nanofluid around horizontal circular cylinder.

This study concentrate on free convection of boundary layer flow of viscoelastic micropolar fluid over horizontal circular cylinder are driven by the previous work (Aziz *et al.*, 2017; Kasim & Shafie, 2010) that investigates the same problem on mixed convection mode. To the best of author’s knowledge, the study of free convection of viscoelastic micropolar fluid moving along the circular cylinder geometry has not yet been considered in detail.

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II. MATHEMATICAL FORMULATION

A horizontal circular cylinder heated at constant temperature with radius a and embedded in viscoelastic micropolar fluid with temperature T_∞ is considered.

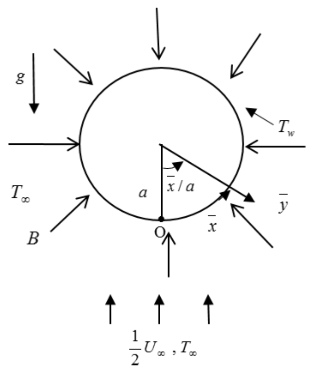

Figure 1. Physical model and coordinate system for free convection from a horizontal circular cylinder with constant surface temperature

In the derivation process of the equations in our model, the Boussinesq and boundary layer approximations are used.The Boussinesq approximation is applied in buoyancy-driven flow where density differences can be ignored except in momentum equation Gebhart *et al.*(1988) while according to the boundary layer approximation, $\frac{1}{\mathrm{Re}} \to 0$, hence such terms can be discarded from the equations. Assuming the validity of the approximations, the dimensional governing equation of the free convection boundary layer flow is given by equation (1) to (5).

Continuity equation:

$$\frac{\partial \bar{u}}{\partial \bar{x}} + \frac{\partial \bar{v}}{\partial \bar{y}} = 0 \tag{1}$$

Energy equation:

$$\bar{u}\frac{\partial T}{\partial \bar{x}} + \bar{v}\frac{\partial T}{\partial \bar{y}} = \alpha \frac{\partial^2 T}{\partial \bar{y}^2} \tag{2}$$

Micropolar equation:

$$\rho j\left(\bar{u}\frac{\partial \bar{H}}{\partial \bar{x}} + \bar{v}\frac{\partial \bar{H}}{\partial \bar{y}}\right) = -\kappa\left(2\bar{H} + \frac{\partial \bar{u}}{\partial \bar{y}}\right) + \gamma \frac{\partial^2 \bar{H}}{\partial \bar{y}^2} \tag{3}$$

Momentum Equation:

$$\begin{aligned} \bar{u}\frac{\partial \bar{u}}{\partial \bar{x}} + \bar{v}\frac{\partial \bar{u}}{\partial \bar{y}} &= \left(\frac{\mu+\kappa}{\rho}\right)\frac{\partial^2 \bar{u}}{\partial \bar{y}^2} + \\ &\frac{k_0}{\rho}\left[\frac{\partial}{\partial \bar{x}}\left(\bar{u}\frac{\partial^2 \bar{u}}{\partial \bar{y}^2}\right) + \bar{v}\frac{\partial^3 \bar{u}}{\partial \bar{y}^3} - \frac{\partial \bar{u}}{\partial \bar{y}}\frac{\partial^2 \bar{u}}{\partial \bar{x}\partial \bar{y}}\right] \\ &+ g\beta\left(T - T_\infty\right)\sin\left(\frac{\bar{x}}{a}\right) + \frac{\kappa}{\rho}\frac{\partial \bar{H}}{\partial \bar{y}} \\ &-\frac{\sigma}{\rho}\bar{u}B^2\sin^2\alpha \end{aligned} \tag{4}$$

subject to the boundary conditions

$$\begin{aligned} &\bar{u} = \bar{v} = 0,\ T = T_w,\ \bar{H} = -n\frac{\partial \bar{u}}{\partial \bar{y}} \text{ on } \bar{y} = 0, \\ &\bar{u} \to 0,\ \bar{v} \to 0,\ T \to T_\infty, \bar{H} \to 0 \text{ as } \bar{y} \to \infty \end{aligned} \tag{5}$$

The following variables are introduced to convert the above equations into non-dimensionless form.

$$\begin{aligned} &x = \frac{\bar{x}}{a},\ y = \frac{Gr^{1/4}\bar{y}}{a},\ u = \frac{aGr^{-1/2}\bar{u}}{\nu}, \\ &v = \frac{aGr^{-1/4}\bar{v}}{\nu},\ H = \frac{a^2Gr^{-3/4}\bar{H}}{\nu},\ \theta = \frac{T - T_\infty}{T_w - T_\infty} \end{aligned} \tag{6}$$

Equations (7) to (11) are the outcomes of the non-dimensionalisation process.

Continuity equation:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{7}$$

Micropolar equation:

$$\begin{aligned} &u\frac{\partial H}{\partial x} + v\frac{\partial H}{\partial y} = \\ &-K_1\left(2H + \frac{\partial u}{\partial y}\right) + \left(1 + \frac{K_1}{2}\right)\frac{\partial^2 H}{\partial y^2} \end{aligned} \tag{8}$$

Energy equation:

$$u\frac{\partial \theta}{\partial x} + v\frac{\partial \theta}{\partial y} = \frac{1}{\mathrm{Pr}}\frac{\partial^2 \theta}{\partial y^2} \tag{9}$$

Momentum equation:

$$\begin{aligned} &u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \left(1 + K_1\right)\frac{\partial^2 u}{\partial y^2} \\ &+K\left[\frac{\partial}{\partial x}\left(u\frac{\partial^2 u}{\partial y^2}\right) + v\frac{\partial^3 u}{\partial y^3} - \frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial x\partial y}\right] \\ &+\theta\sin\left(x\right) + K_1\frac{\partial H}{\partial y} - Mu\sin^2\alpha \end{aligned} \tag{10}$$

subject to the boundary conditions

$$u = v = 0,\ \theta = 1,\ H = -n\frac{\partial u}{\partial y} \text{ on } y = 0 \tag{11}$$
$$u \to 0,\ v \to 0,\ \theta \to 0,\ H \to 0 \text{ as } y \to \infty$$

where M, K and K_1 is the magnetic parameter, viscoelastic parameter and material parameter ,respectively, defined by

$$j = a^2 Gr^{-1/2},\quad K = \frac{k_0 Gr^{1/2}}{a^2\rho},\quad K_1 = \frac{\kappa}{\mu}, \tag{12}$$
$$\mathrm{Gr} = \frac{g\beta(T_w - T_\infty)a^3}{\nu^2},\qquad M = \frac{\sigma B_0^{\,2} a^2}{\rho Gr^{1/2}\nu}$$

In order to reduce the complexity of equation (7-11) and boundary conditions (12), the following variables are assumed.

$$\psi = xf(x,y),\ \theta = \theta(x,y),\ H = xG(x,y) \tag{13}$$

where ψ is the non-dimensional stream function defined as,

$$u = \frac{\partial\psi}{\partial y},\ v = -\frac{\partial\psi}{\partial x} \tag{14}$$

producing the following equations as a result.

Micropolar equation:

$$\left(1 + \frac{K_1}{2}\right)\frac{\partial^2 G}{\partial y^2} + f\frac{\partial G}{\partial y} - \frac{\partial f}{\partial y}G - K_1\left(2G + \frac{\partial^2 f}{\partial y^2}\right) = x\left(\frac{\partial f}{\partial y}\frac{\partial G}{\partial x} - \frac{\partial f}{\partial x}\frac{\partial G}{\partial y}\right) \tag{15}$$

Energy equation:

$$\frac{1}{\Pr}\frac{\partial^2\theta}{\partial y^2} + f\frac{\partial\theta}{\partial y} = x\left(\frac{\partial f}{\partial y}\frac{\partial\theta}{\partial x} - \frac{\partial f}{\partial x}\frac{\partial\theta}{\partial y}\right) \tag{16}$$

Momentum equation:

$$\begin{aligned}&(1+K_1)\frac{\partial^3 f}{\partial y^3} + f\frac{\partial^2 f}{\partial y^2} - \left(\frac{\partial f}{\partial y}\right)^2 + \frac{\sin x}{x}\theta \\ &+ K_1\frac{\partial G}{\partial y} - M\frac{\partial f}{\partial y}\sin^2\alpha \\ &+K\left\{2\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3} - f\frac{\partial^4 f}{\partial y^4} - \left(\frac{\partial^2 f}{\partial y^2}\right)^2\right. \\ &\left.+x\left(\frac{\partial^2 f}{\partial x\partial y}\frac{\partial^3 f}{\partial y^3} - \frac{\partial f}{\partial x}\frac{\partial^4 f}{\partial y^4} + \frac{\partial f}{\partial y}\frac{\partial^4 f}{\partial x\partial y^3} - \frac{\partial^2 f}{\partial y^2}\frac{\partial^3 f}{\partial x\partial y^2}\right)\right\} \\ &= x\left(\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial x\partial y} - \frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial y^2}\right)\end{aligned} \tag{17}$$

subject to the boundary conditions

$$f = \frac{\partial f}{\partial y} = 0,\ \ \theta = 1,\ G = -n\frac{\partial^2 f}{\partial y^2} \text{ at } y = 0 \tag{18}$$
$$\frac{\partial f}{\partial y} \to 0, \frac{\partial^2 f}{\partial y^2} \to 0, \theta \to 0, G \to 0 \text{ as } y \to \infty$$

where $n$is the ratio of microrotation vector component and the skin friction at the wall with a range of values $0 \le n \le 1$. A model with $n = 1$ represents turbulent flow inside the boundary layer while when $n = 0$, the flow is concentrated with high density of microelements thatcauses the microelements near the wall to lose their rotating ability(Abdel-Rahman, 2009; Jena & Mathur, 1981). For this study, our interest is on the case when $n = 0.5$, that represents weak concentration of microelements. At this n value, the rotation of the fine particles and fluid vorticity are equal.

Near the lower stagnation point of the cylinder, where $x \approx 0$, Equation (15) to (17) are reduced to a set of ordinary differential equations as follows.

$$\left(1 + \frac{K_1}{2}\right)G'' + f\,G' - f'G - K_1\left(2G + f''\right) = 0 \tag{19}$$

$$\begin{aligned}&(1+K_1)f''' + f\,f'' - f'^2 + \theta + K_1 G' \\ &- Mf'\sin^2\alpha \\ &+ K\left(2f' f''' - f\,f^{iv} - f''^2\right) = 0\end{aligned} \tag{20}$$

$$\frac{1}{\Pr}\theta'' + f\,\theta' = 0 \qquad (21)$$

subject to the boundary conditions

$$f(0) = f'(0) = 0,\ \theta(0) = 1,\ G(0) = -\eta f''(0)$$
$$f' \to 0,\ f'' \to 0,\ \theta \to 0,\ G \to 0 \text{ as } y \to \infty \qquad (22)$$

The physical quantities of interest are the reduced skin friction coefficient, $C_f Gr^{1/4}$ and the reduced Nusselt number, $Nu_x Gr^{-1/4}$ which are given by

$$C_f Gr^{1/4} = \left(x\frac{\partial^2 f}{dy^2}\right)_{\bar{y}=0},\quad Nu_x Gr^{-1/4} = -\left(\frac{\partial\theta}{\partial y}\right)_{\bar{y}=0}$$

Due to the non-linearity of the equations, exact solutions are non-existent. Therefore, numerical solutions are obtained using the Keller-box method with the assistance of Fortran program.

III. RESULTS AND DISCUSSION

For validation purpose, the magnetic parameter, viscoelastic parameter and micropolar parameter are all set to 0 (M=K=K_1=0) and the reduced skin friction coefficients obtained from this model are compared to the published studies by Molla (2006),Mohamed *et al.* (2016) and Zokri *et al.* (2018). Thefiguresin Table 1show strong agreement between the previous and current resultsthat lead us to believe that the numerical solutions are reliable.

Table 1. Values of $C_f Gr^{1/4}$ at different values of $x$when Pr=1.

x	[12] (2005)	[13] (2016)	[14] (2018)	Present
0	0.000	0.0000	0.000	0.000
$\pi/6$	0.4145	0.4121	0.4120	0.4150
$\pi/3$	0.7539	0.7538	0.7507	0.7556
$\pi/2$	0.9541	0.9563	0.9554	0.9580
$2\pi/3$	0.9696	0.9743	0.9728	0.9760
$5\pi/6$	0.7739	0.7813	0.7761	0.7832

Figure 2, 4 and 6 illustrate the variations of reduced skin friction coefficient against x for different values of material parameter, viscoelastic parameter and magnetic parameter, respectively. The figure shows that the reduced skin friction coefficient becomes higher with the increase of the material parameter, K_1. This outcome is predictable since larger material parameterimplies higher vortex viscosity of fluid. It is also evident that the value of the skin friction coefficient peaked at the top surface of the cylinder before it starts decreasing to a finite value.

As for the viscoelastic and magnetic parameter, adverse effects are observed. The increase of the parameter values cause the skin friction coefficient to decrease. Higher magnetic parameter indicates the existence of stronger Laurent force in the fluid which acts on the opposite direction of the flow causing the velocity to decrease. Hence, the skin friction coefficient becomes smaller.

From Figure 3, 5 and 7, we can observe that heat transfer becomes more efficient as the material parameter increases but depleted as the viscoelastic and magnetic parameter becomes larger. The jagged lines observed in the figures when x is small could have been caused by the flow instability as it touches the cylinder surface at the stagnation point.

IV. SUMMARY

In this paper, the free convection boundary layer flow on a horizontal circular cylinder embedded in viscoelastic micropolar fluid with MHD effect is investigated. From the study, it can be concluded that the reduced skin friction coefficient and heat transfer have a positive relationship with material parameter while viscoelastic parameter and magnetic parameter react oppositely with the physical quantities. From the result, viscoelastic micropolar fluid with high material parameter might be a potential booster of heat transfer in boundary layer flow.

V. ACKNOWLEDGEMENT

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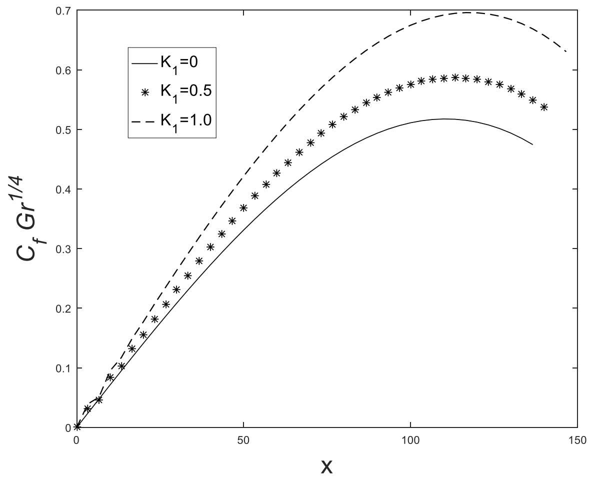

Figure 2. Variation of reduced skin friction coefficient against *x* for various K_1

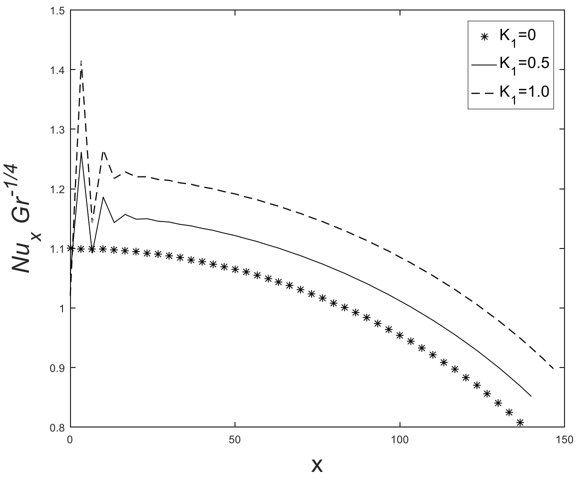

Figure 3. Variation of reduced Nusselt number against *x* for various K_1

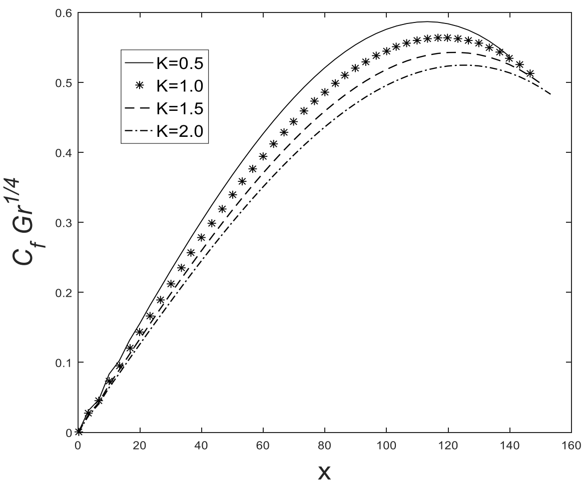

Figure 4. Variation of reduced skin friction coefficient against *x* for various *K*

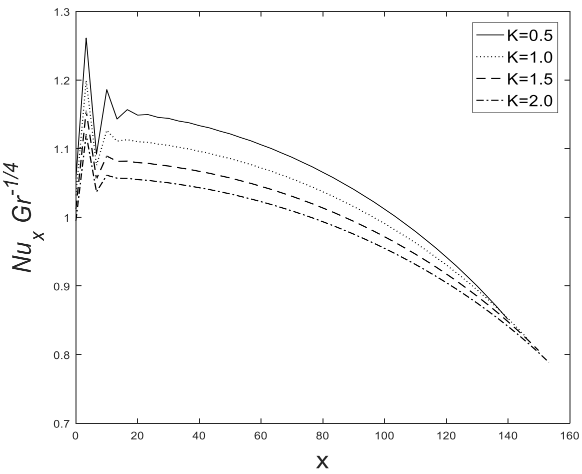

Figure 5. Variation of reduced Nusselt number against *x* for various *K*

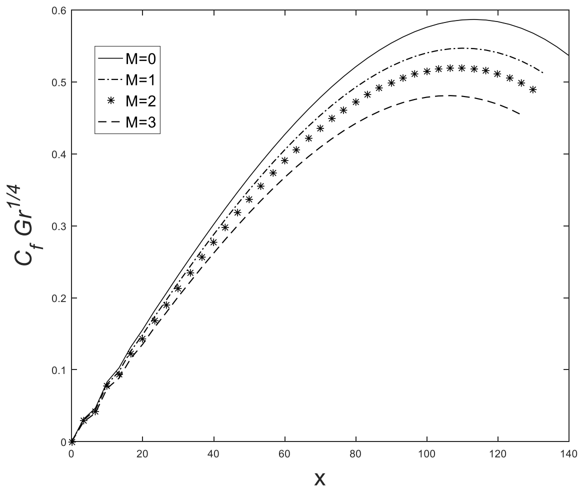

Figure 6. Variation of reduced skin friction coefficient against *x* for various *M*

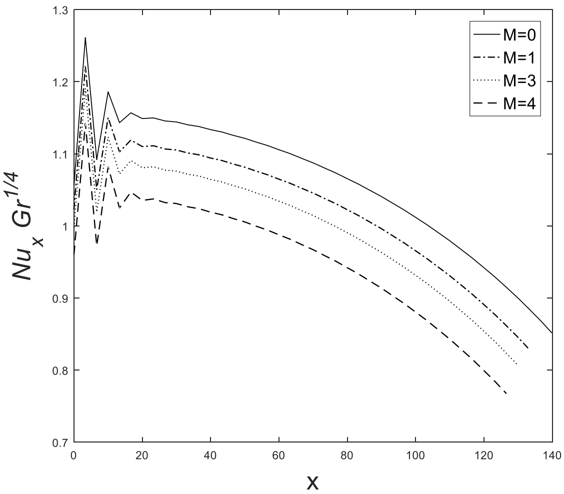

Figure 7. Variation of reduced Nusselt number against x for various *M*

VI. REFERENCES

Abdel-Rahman, G.M.: Studying effect of MHD on thin films of a micropolar fluid. Physica B: Condensed Matter **404**(21), 3859-3866 (2009).doi:https://doi.org/10.1016/j.physb.2009.07.112.

Aziz, L.A., Kasim, A.R.M., Salleh, M.Z., Shafie, S., Yusoff, W.N.S.W.: Boundary layer flow of mixed convection viscoelastic micropolar fluid over a horizontal circular cylinder with aligned magnetohydrodynamic effect. Malaysian Journal of Fundamental and Applied Sciences **13**(4), 567-571 (2017).

Gebhart, B., Jaluria, Y., L. Mahajan, R., Sammakia, B.: Buoyancy-Induced Flows and Transport, vol. 111. (1988).

Ingham, D., Pop, I.: Natural convection about a heated horizontal cylinder in a porous medium. Journal of fluid Mechanics **184**, 157-181 (1987).

Jena, S.K., Mathur, M.N.: Similarity solutions for laminar free convection flow of a thermomicropolar fluid past a non-isothermal vertical flat plate. International Journal of Engineering Science **19**(11), 1431-1439 (1981). doi:https://doi.org/10.1016/0020-7225(81)90040-9.

Kasim, A.R.M. and Shafie. S.:Mixed convection boundary layer of a viscoelastic fluid past a circular cylinder with constant heat flux.Proceeding on Regional Conference on Applied and Engineering Mathematics. Vol. 1, pp. 124-129, (2010).

Merkin, J.: Free convection boundary layer on an isothermal horizontal cylinder. In: American Society of Mechanical Engineers and American Institute of Chemical Engineers, Heat Transfer Conference, St. Louis, Mo., Aug. 9-11, 1976, ASME 5 p. 1976.

Merkin, J., Pop, I.: A note on the free convection boundary layer on a horizontal circular cylinder with constant heat flux. Heat and Mass Transfer 22(1), 79-81 (1988).

Mohd Kasim, A.R., Mohammad, N.F., Shafie, S.: Effect of heat generation on free convection boundary layer flow of a viscoelastic fluid past a horizontal circular cylinder with constant surface heat flux. In: AIP Conference Proceedings 2012, vol. 1, pp. 286-292. AIP.

Mohamed, M.K.A., Noar, N., Salleh, M.Z., Ishak, A.: Free convection boundary layer flow on a horizontal circular cylinder in a nanofluid with viscous dissipation. Sains Malaysiana 45(2), 289-296 (2016).

Molla, M.M., Hossain, M.A., Paul, M.C.: Natural convection flow from an isothermal horizontal circular cylinder in presence of heat generation. International Journal of Engineering Science 44(13-14), 949-958 (2006).

Nazar, R., Amin, N., Pop, I.: Free convection boundary layer on an isothermal horizontal circular cylinder in a micropolar fluid. Heat Transfer 2, 525-530 (2002).

Salleh, M.Z., Nazar, R.: Free convection boundary layer flow over a horizontal circular cylinder with Newtonian heating. Sains Malaysiana 39(4), 671-676 (2010).

Sarif, N.M., Salleh, M.Z., Tahar, R.M., Nazar, R.: Numerical solution of the free convection boundary layer flow over a horizontal circular cylinder with convective boundary conditions. In: AIP Conference Proceedings 2014, vol. 1, pp. 179-185. AIP.

Sheikholeslami, M., Gorji-Bandpay, M., Ganji, D.D.: Magnetic field effects on natural convection around a horizontal circular cylinder inside a square enclosure filled with nanofluid. International Communications in Heat and Mass Transfer 39(7), 978-986 (2012).

Zokri, S.M., Arifin, N.S., Mohamed, M.K.A., Kasim, A.R.M., Mohammad, N.F., Salleh, M.Z.: Influence of viscous dissipation on the flow and heat transfer of a Jeffrey fluid towards horizontal circular cylinder with free convection: A numerical study. Malaysian Journal of Fundamental and Applied Sciences 14(1), 40-47 (2018).