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Fuzzy B-spline Curved Surface Modeling

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This paper will discuss on constructing fuzzy B-spline curve model in surface form. This model is constructed based on B-spline curved surface model and the fuzzy set theory. For fuzzy set theory, this theory is being used to define the uncertainty data which are in interval forms and later, these data are modeled through B-spline curved surface model. The process of defining the uncertainty data in interval forms is done by using the concepts of fuzzy number in trapezoidal form. After that, through this model, the fuzzification and defuzzification processes are being discussed. The preferred numerical example is also given after the constructed of fuzzy B-spline curved surface to increase the understanding of this model.

Keywords: B-spline curve; B-spline surface; trapezoidal fuzzy number; fuzzification; defuzzification

I. INTRODUCTION

The construction curve model in order to represent a set of data become a method of conducting research, analysis and making conclusions. Each developed curve model will face the blending process between the data sets and the designing curve model. There is various type of curve functions which has been constructed along together with the requirements of visualization and representing data sets according to the suitability of the curve function. For each curve, they also have the advantages and disadvantages of modeling data sets by following the distribution and the positioning of the data. The mentioned curve functions such as spline, Bezier, and B-spline curve functions (Rogers, 2001; Farin, 2002; Salomon, 2006; Yamaguchi, 1988).

The extension of curve development is certainly a surface which becomes a way of data representation which more effective. In designing a surface, the surface function is required in addition with a set of data points in 3-dimensional form. These kinds of functions can be formed based on the existing curve functions (Rogers, 2001; Farin, 2002).

For a set of data points which contains the uncertainty attributes or perhaps data uncertainty, then this issue can be solved by defining the uncertainty data points by using fuzzy set theory and fuzzy number concept which will be become fuzzy data points. Later, these fuzzy data points also being modeled by blending them with curves and surfaces in order to generate a fuzzy curve and surface. The construction of fuzzy curve and surface models used the curve and surface functions such as spline, Bezier and B-spline can be referred through (Zakaria and Wahab, 2014; Zakaria *et al.*, 2014, Zakaria and Wahab, 2012; Wahab and Zakaria, 2015; Zakaria *et al.*, 2016).

However, if the curve is constructed within the shape of the surface which is modeled the uncertainty set of data points, then it has become a technique in designing curve and surface. consequently, this study discussed the construction fuzzy curve surfaced which can be modeled via both B-spline curve and surface models. Then, for fuzzy data points, it is described in trapezoidal shape based on trapezoidal fuzzy number concepts when there exists uncertainty on an interval from uncertainty data. After the

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uncertainty interval of uncertainty data points is defined and being modeled through a curved surface by using B-spline function, the next applied process is the fuzzification process which used the alpha-cut operation of trapezoidal fuzzy number concepts. Then, the defuzzification process next is applied to obtain a crisp fuzzy interval. An example of application also mentioned by specifying the border of hilly roads as the additional understanding of this constructed model.

A. Definitions and Basic Concepts

Definition 1. A fuzzy set in universal of discourse, U can be defined as a membership function, $\mu_A(x)$ which maps the values on interval $[0,1]$ such as $\mu_A : U \to [0,1]$ which can be presented as $A = \{(x, \mu_A(x)) \mid x \in U\}$.

Definition 2. A fuzzy number, $\vec{A} = (a,b,c,d)$ is the trapezoidal fuzzy number with $a \le b \le c \le d$, if the membership function can be given as follows.

$$\mu_{\vec{A}}(x) = \begin{cases} 0, & x < a \\ \dfrac{x-a}{b-a}, & a < x \le b \\ 1, & b < x < c \\ \dfrac{d-x}{d-c}, & c \le x < d \\ 0, & x > d \end{cases} \qquad (1)$$

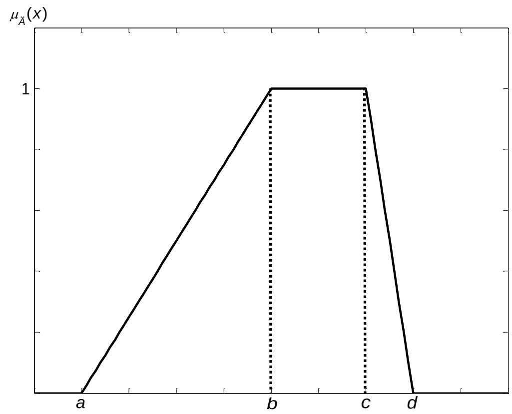


Figure 1. Trapezoidal fuzzy number of $\vec{A}$ with $\vec{A} = (a,b,c,d)$

In order to define the uncertainty intervals which consisting of uncertainty data in the real number forms, fuzzy relation will be used as defining the uncertainty data. Therefore, the definition of fuzzy relation and fuzzy interval can be given as follows (Zimmermann, 1985; Dubois and Prade, 1980; Klir and Yuan, 1995).

Definition 3. Let $X,Y \subseteq R$ be a universal set, then $\vec{R} = \{((x,y), \mu_R(x,y)) \mid (x,y) \subseteq X \times Y\}$ is called fuzzy relation on $X \times Y$ with $\times$ represented as multiplication operation.

Definition 4. Let $X,Y \subseteq R$ and $\vec{A} = \{(x, \mu_A(x)) \mid x \in X\}$ and $\vec{B} = \{(y, \mu_B(y)) \mid y \in Y\}$ are two fuzzy sets. Then, $\vec{R} = \{[(x,y), \mu_R(x,y)], (x,y) \in X \times Y\}$ is a fuzzy relation on $\vec{A}$ and $\vec{B}$ if

$\mu_R(x,y) \le \mu_A(x), \quad \forall (x,y) \in X \times Y$ and

$\mu_R(x,y) \le \mu_B(y), \quad \forall (x,y) \in X \times Y$.

Definition 5. Let $X,Y \subseteq R$ and $\vec{M} = \{(x, \mu_M(x)) \mid x \in X\}$ and $\vec{N} = \{(y, \mu_N(y)) \mid y \in Y\}$ are two fuzzy points. Then, fuzzy relation for both fuzzy point can be defined as $\vec{p} = \{[(x,y), \mu_p(x,y)], (x,y) \in X \times Y\}$.

Definition6. Let $\vec{p} = \{(x,y),\ x \in X,\ y \in Y \mid x \text{ and } y \text{ are fuzzy points}\}$ and $\vec{P} = \{P_i \mid P \text{ is data points}\}$ are set of fuzzy data points which is $P_i \in \vec{p} \subset X \times Y \subseteq R$ with R is universal set and $[\mu_P(P_i)] : P \to [0,1]$ is membership function where it can be defined as $[\mu_P(P_i)] = 1$ and then can be summarized as $[\vec{P}] = \{(P_i, [\mu_P(P_i)]) \mid P_i \in R\}$. Then,

$$\mu_P(P_i)=\begin{cases} 0 & \text{if and only if } P_i \notin R \\ c\in(0,1) & \text{if and only if } P_i \,\tilde{\in}\, R \\ 1 & \text{if and only if } P_i \in R \end{cases} \quad (2)$$

with $\mu_P(P_i)=\left\langle \mu_P(\vec{P}_i^{\leftarrow}),[\mu_P(P_i)],\mu_P(\vec{P}_i^{\rightarrow})\right\rangle$ where $\mu_P(\vec{P}_i^{\leftarrow})$ and $\mu_P(\vec{P}_i^{\rightarrow})$ are left and right membership functions respectively. And for $[\mu_P(P_i)]=[\mu_P(P_i)^{\leftarrow},\mu_P(P_i)^{\rightarrow}]$ is the crisp interval data points with $\mu_P(P_i)^{\leftarrow}$ dan $\mu_P(P_i)^{\rightarrow}$ are left and right crisp interval data. This can be defined as

$$[\vec{P}]=\left\{[\vec{P}_i]=(x_i,y_i)\,|\,i=0,1,..,n\right\} \quad (3)$$

for all *i*, $[\vec{P}_i]=\left\langle \vec{P}_i^{\leftarrow},[P_i]=[P_i^{\leftarrow},P_i^{\rightarrow}],\vec{P}_i^{\rightarrow}\right\rangle$ with $\vec{P}_i^{\leftarrow}$, $\vec{P}_i^{\rightarrow}$ and $[P_i]=[P_i^{\leftarrow},P_i^{\rightarrow}]$ are left and right fuzzy data points and crisp interval data respectively. The crisp interval data consisted by left, $P_i^{\leftarrow}$ and right, $P_i^{\rightarrow}$ crisp interval data points. This definition also can be extended to 3-dimension forms. The definition of fuzzy interval data points can be illustrated as follows.

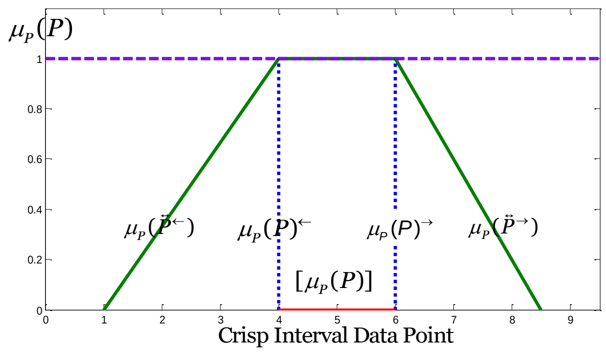


Figure 2. The illustration of fuzzy interval data point

Figure 2 shows that the illustration for fuzzy interval data point which the crisp interval data bounded by left and right fuzzy data points. The properties of this fuzzy interval data point also is same with the properties of trapezoidal fuzzy number. This was due to the fuzzy interval data point defined in accordance with the use of trapezoidal fuzzy number. Through the definition of fuzzy interval data point, the process of fuzzification and defuzzification can be defined with each of them use alpha-cut and mean operations.

Definition 7. Let α be an alpha-cut towards every fuzzy interval data points, $[\vec{P}_i]$ with $[\vec{P}_i]\in[\vec{P}]$. Then, the alpha-cut process of $[\vec{P}_i]$ also can be called as fuzzification process which can be defined through the following equation.

$$[\vec{P}_{i_\alpha}]=\left\langle \vec{P}_{i_\alpha}^{\leftarrow},[P_i]=[P_i^{\leftarrow},P_i^{\rightarrow}],\vec{P}_{i_\alpha}^{\rightarrow}\right\rangle \quad (4)$$

The alpha-cut operation also can be represented as

$$[\vec{P}_{i_\alpha}]=\Big\langle \vec{P}_{i_\alpha}^{\leftarrow}=\left(P_i^{\leftarrow}-\vec{P}_i^{\leftarrow}\right)\alpha+\vec{P}_i^{\leftarrow},[P_i], \\ P_{i_\alpha}^{\rightarrow}=-\left(\vec{P}_i^{\rightarrow}-P_i^{\rightarrow}\right)\alpha+\vec{P}_i^{\rightarrow}\Big\rangle \quad (5)$$

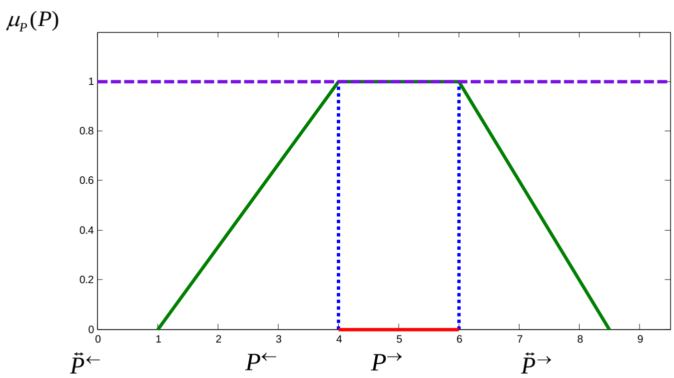
Definition 8. Let $[\vec{P}_{i_\alpha}]$ be a fuzzification of fuzzy interval data points. Then, $[\bar{P}_{i_\alpha}]$ is a set of defuzzify interval data points for $[\vec{P}_{i_\alpha}]$ if every each $[\vec{P}_{i_\alpha}]\subset[\vec{P}_i]\in[\vec{P}]$ can be stated as

$$[\bar{P}]=\left\{[\bar{P}_i]\right\} \text{ for each } i=0,1,...,n \quad (6)$$

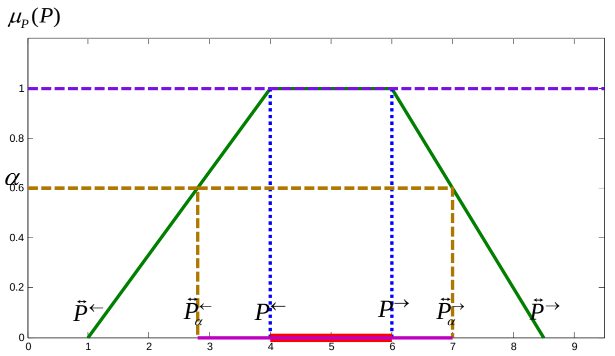
where every each

$$[\bar{P}_{i_\alpha}]=\sum_{i=0}\left[\left(\frac{1}{2}\vec{P}_{i_\alpha}^{\leftarrow}+P_i^{\leftarrow}\right),\left(\frac{1}{2}P_i^{\rightarrow}+\vec{P}_{i_\alpha}^{\rightarrow}\right)\right].$$

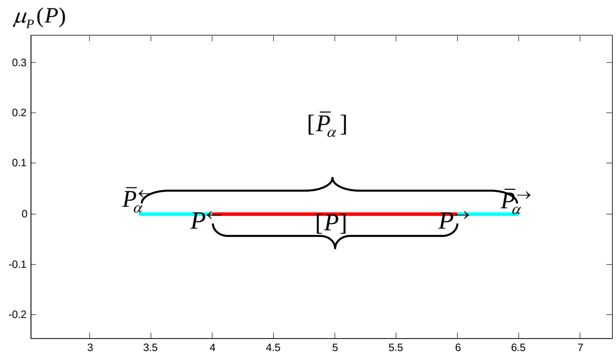
The defuzzification process for each fuzzy interval data points can be illustrated through Figure 3 as follows.



(a) Fuzzy Data Point Model



(b) Fuzzification Fuzzy Interval Data Point Model



(c) Defuzzification Fuzzy Interval Data Point Model

Figure 3. Modeling process of (a) fuzzy interval data point; (b) fuzzification fuzzy interval data point and (c) defuzzification fuzzy interval data point

II. FUZZY B-SPLINE CURVED SURFACE MODEL

The next step is modeling the fuzzy interval data points through fuzzy curved surface which used B-spline curve and surface functions. The definitions of B-spline curve and surface model can be given as follows.

Definition 9. A fuzzy B-spline curve is a function, $\overleftrightarrow{Bs}(t)$ which represent a curve for a set of real fuzzy number and can be defined as

$$\overleftrightarrow{Bs}(t)=\sum_{i=1}^{n}[\vec{P}_i]B_{i,h}(t), \quad (7)$$

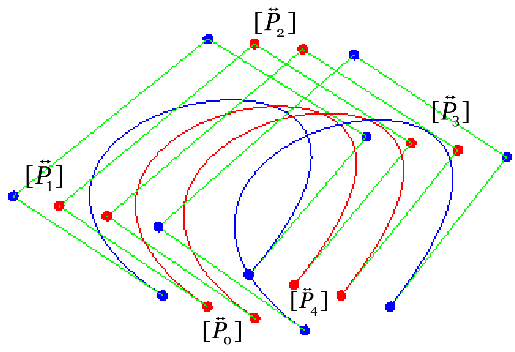
with $[\vec{P}_i]$ are fuzzy interval control points in 2- or 3-dimension which also known as fuzzy interval data points and $B_{i,h}(t)$ are B-spline basis functions together with crisp knot series, $t_1,t_2,\ldots,t_{m=h+n+1}$ where h represent as degree of B-spline function and n represent as sum of fuzzy control points.

Definition 10. A fuzzy B-spline surface can be defined by the equation as follows.

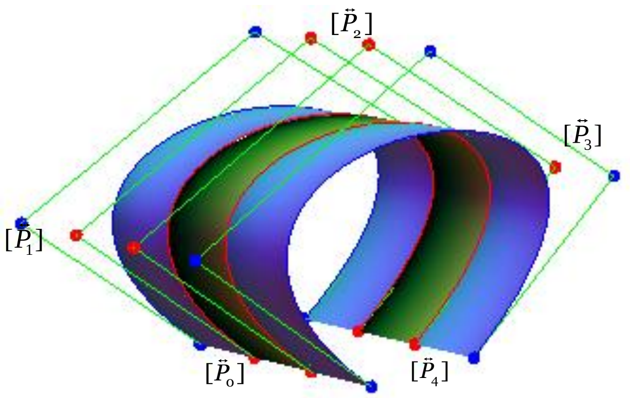
$$\overleftrightarrow{BsS}(s,t)=\sum_{i=0}^{m}\sum_{j=0}^{n}[\vec{P}_{(i,j)}]N_{i,p}(s)N_{j,q}(t), \quad (8)$$

where (i) $N_{i,p}(s)$ and $N_{j,q}(t)$ are basis functions with degree p and q along with crisps parameter s and t in $[0,1]$; (ii) every vector knot must fulfill the property as $t=m+p+1$ and $s=n+q+1$; (iii) $[\vec{P}_{(i,j)}]=\left\langle \vec{P}_{(i,j)}^{\leftarrow},[P_{(i,j)}],\vec{P}_{(i,j)}^{\rightarrow}\right\rangle$ are fuzzy interval control points at rows, i and columns, j with $[P_{(i,j)}]=[P_{(i,j)}^{\leftarrow},P_{(i,j)}^{\rightarrow}]$.

The illustration for both Definition 9 and 10 can be given by Figure 4 and Figure 5 respectively.



Crisp interval control points and B-spline curve;
Fuzzy left and right control points and B-spline curve



Crisp B-spline curved surface;
Fuzzy left and right B-spline curved surface

Figure 4. Fuzzy B-spline curved surface modeling

The next processes are the fuzzification and defuzzification for fuzzy B-spline curved surface which can be illustrated through Figure 5 as follows.

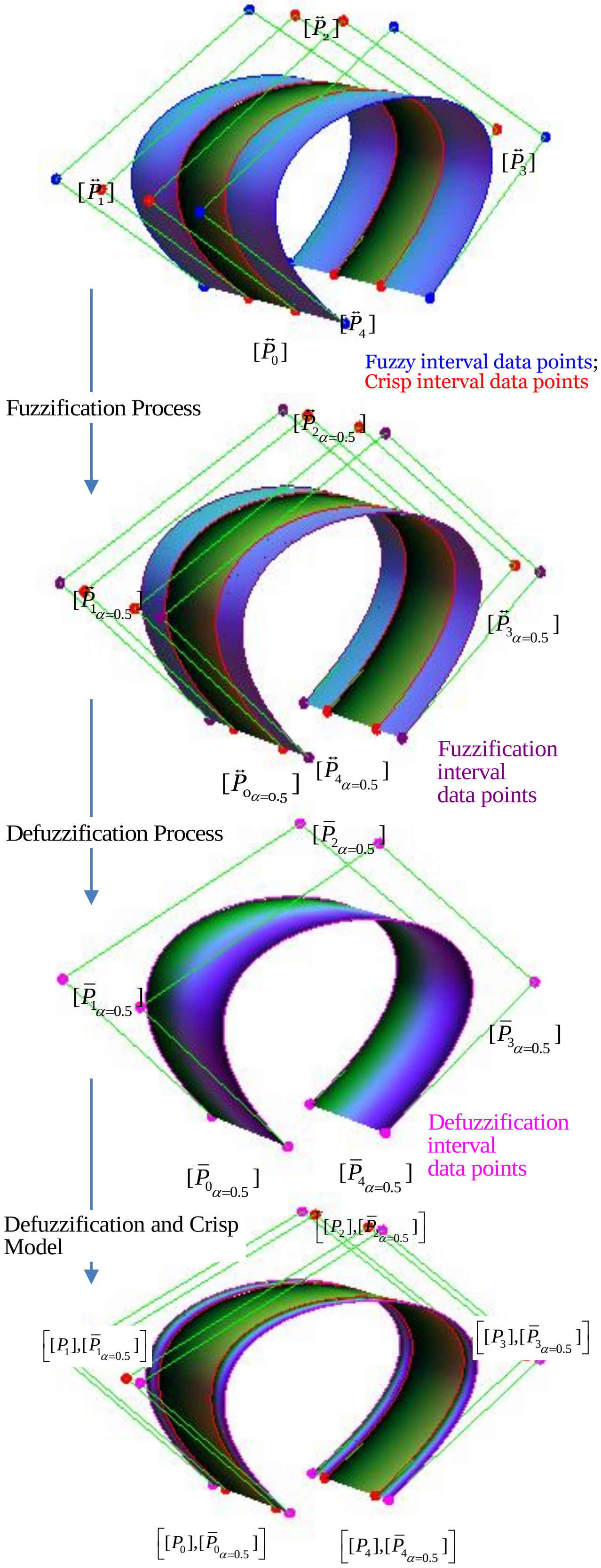


Figure 5. Fuzzy B-spline curved surface model which is being through fuzzification and defuzzification processes

Figure 5 shows that the fuzzy curved surface model has been modeled through defining, fuzzificationand defuzzification processes. Based on the Figure 5, the set of fuzzy interval data points can be defined as fuzzy interval control points.

III. EXAMPLE OF APPLICATION

In this section, the example of application for this developed model which modeling the fuzzy interval data points will be discussed. This application is based on the curved surface of unpaved roads on hilly areas. This road modeling is done to determine the boundary between the road and the shoulder which is unknown clearly with some data had been taken.

Therefore, the road modeling through fuzzy curved surface model can be accomplished by using the previous definitions. The processes such as fuzzification and defuzzification processes are also applied to obtain crisp fuzzy roads model. Thus, the illustrations for each definition as well as the processes used can be given through the following figures.

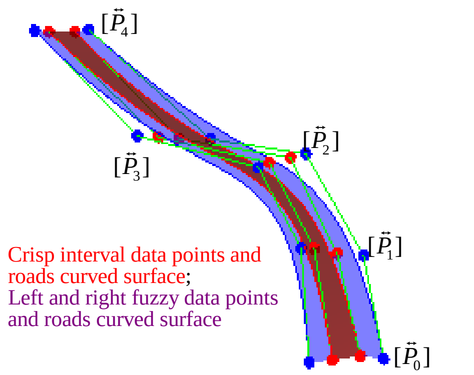


Figure 6. Fuzzy curved surface roads and boundary modeling with fuzzy interval data points

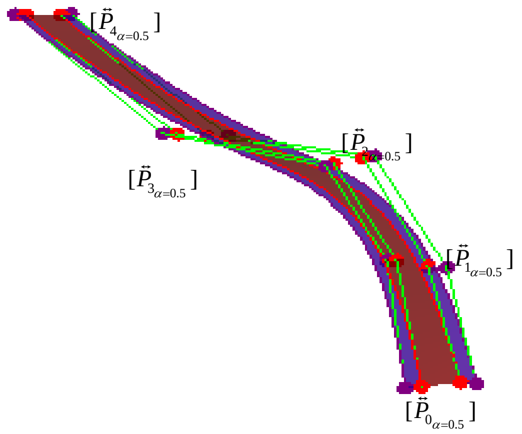


Figure 7. Fuzzification fuzzy curved surface roads model along with fuzzificationinterval roads data points

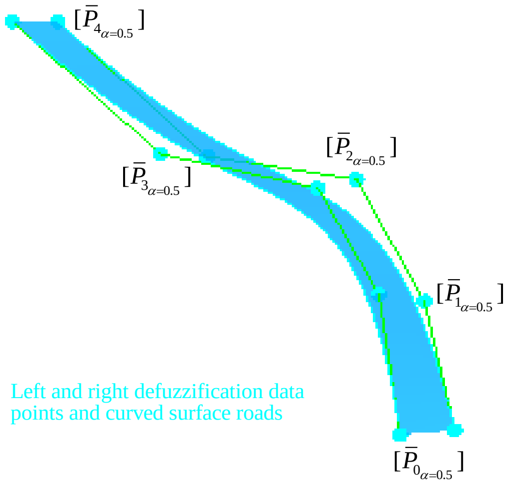


Figure 8. Defuzzfication curved surface roads with defuzzify interval roads data points

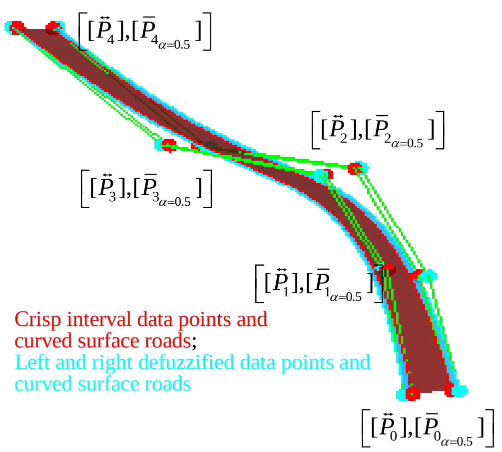


Figure 9. Crisp and defuzzified curved surface roads model with crisp and defuzzified interval roads data points

Figure 6 shows the modeling of fuzzy surface road which are comprised by the road surface and the boundary of the road. This constructed model uses fuzzy curved surface B-spline model which is formed based on the definition of fuzzy interval road data points comprising of the left and right fuzzy interval data points and the crisp interval road data points.The next process involves the process of fuzzification and defuzzification that is applied in Figures 7 based on Definition 7 and Definition 8 with the fuzzification process using trapezoidal alpha cut operation and meanwhile the defuzzifying process uses the min method. Both results of the process can be described respectively in Figure 8 and Figure 9. Next, both crisp and defuzzifiedcurved surface models are modeled by Figure 10 to find out the difference between the two models through graphic illustration.

IV. SUMMARY

In this paper, the fuzzy B-spline curved surfacemodel has been developed which can be represented by the interval of uncertainty data point set before being defined as a fuzzy interval data point set. Then, to obtain a crisp fuzzy curved surface B-spline, then this model needs to apply the fuzzification and defuzzification process against fuzzy interval data points. Fuzzification process used alpha-cut operation in the trapezoidal form with alpha value can be determined, i.e. $\alpha \in (0,1]$. The next process is a defuzzification process that gives a crisp fuzzy curved surface model.

In addition, the application for thismodel can also be given, i.e. the modeling of road surfaces and hilly road boundary with a set of data points whichcannot be determined. The fuzzy curved surface model becomes an alternative method in modeling the set of uncertainty interval data point. This model can also be extended to an interpolation model that provides perfection in modeling the curved surface in further studies.

V. ACKNOWLEDGEMENT

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