

A Non-Linear Stochastic Consols Yield Model of Malaysia

Nur Syahirah Abd Munil¹, Norizarina Ishak^{2*}, Ahmad Fadly Nurullah Rasadee³ and Abu Bakar Hasan⁴

^{1,2}*Faculty of Science and Technology, Universiti Sains Islam Malaysia, Bandar Baru Nilai, 71800 Nilai, Negeri Sembilan, Malaysia*

³*Faculty of Economics and Muamalat, Universiti Sains Islam Malaysia, Bandar Baru Nilai, 71800 Nilai, Negeri Sembilan, Malaysia*

⁴*Faculty of Engineering and Built Environment, Universiti Sains Islam Malaysia, Bandar Baru Nilai, 71800 Nilai, Negeri Sembilan, Malaysia*

This study examines the non-linear stochastic Consols yield model for Malaysia inspired by Whitten and Thomas model in year 1999. Whitten and Thomas model consist six subs-models which are price inflation model, wage inflation model, share dividends, share yields, Consols yield and base rate. We only considered the Consols yield model in this study. Aim of this study is to apply the Consols yield inspired Whitten and Thomas model to Malaysian data by using Threshold Autoregressive (TAR) method for data estimation. From that, we able to determine the most suitable model for Consols yield in Malaysia. Hence, we compare our model with the inspired Wilkie model for Malaysia developed by Ishak (2015). The data that we used in this study is a 10-Year Malaysian Government Securities (MGS) yield. The data is based on monthly data from year 2006-2016. Descriptive statistics is used to test the statistical behavior of the data. This study shows a positive kurtosis and skewness which indicates that the distribution has heavier tail than the normal distribution. This study also shows that there is threshold effect on Malaysian Consols yield which is below 3.71% and the most suitable TAR model for Malaysian Consols yield is TAR (3).

Keywords: Whitten and Thomas model; consols yield; mgs yield; threshold autoregressive (tar)

I. INTRODUCTION

Stochastic investment model quite popular and has attracts the actuaries and financial experts around the world in recent years. Wilkie is the first person built a new model named as Wilkie model in his first research about investment model (Wilkie, 1984). Wilkie has used the methodology for the linear time series models based on Box Jenkins method that developed by Box and Jenkins in year 1978. This model is widely use among the actuaries in actuarial work and become the benchmark for future development. The two popular model which are autoregressive moving average (ARMA) model for stationary time series and autoregressive integrated moving average (ARIMA) model for non-stationary time series have been used for his study. All the actuaries and

financial experts from different countries around the world have shown the great concern towards the stochastic investment model in recent years. This model has frequently used by the experts at this moment to model the investment returns to see how the behavior of the investment returns in the future using forecasting in the long run. The data from the past can give everybody know the information of overall economy in term of volatility, the variation of economic factors such as the inflation can give big impact towards economy and market shocks happened in years. All this information encouraged Wilkie to build this kind of model. He wanted to consider all the multifactor of investment in stochastic aspect.

*Corresponding author's e-mail: norizarina@usim.edu.my

A. The Wilkie Model and Previous Refinement

David Wilkie has developed Wilkie Model in 1984 and presented a paper to Faculty of Actuaries with title “A stochastic investment model for actuarial use” (Wilkie, 1984). The original Wilkie Model is a linear stochastic model and consists of four interconnected models:

- A price inflation model (retail prices index)
- A share dividend index model
- A share dividend yield model
- A Consols yield model (long term interest rate model)

The Wilkie model used a cascade structure where the retail price influenced other asset returns. In Figure 1, the arrows have represented the direction of influence between the four variables.

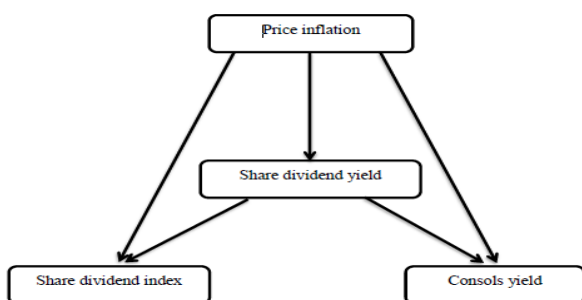


Figure 1. Direction of influence in Wilkie Model (Wilkie, 1984)

In 1995, he updated his model when he presented to the Institute of Actuaries with title “More on a stochastic asset model for actuarial use” Wilkie (1995). He included an alternative Autoregressive Conditional Heteroscedastic (ARCH). The new models are:

- A wage inflation model
- A short-term interest rate model (base rate)
- A property yield model
- An index- linked yield model.
- An exchange rate model

In Figure 2 illustrates the 1995 Wilkie model, he also use the cascade structure which included new variables such as wages index and base rates.

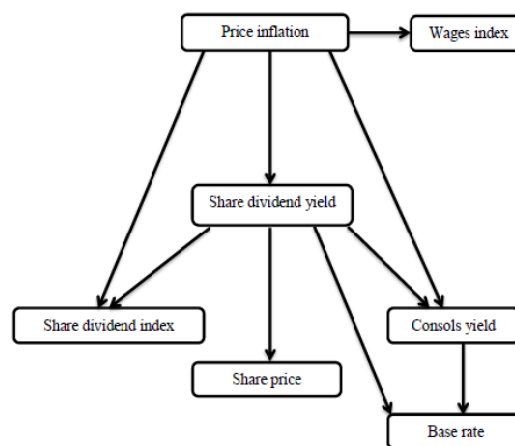


Figure 2. The direction of influence in Wilkie model (Wilkie, 1995)

However, Whitten & Thomas (1999) showed a slight variation in the Wilkie model by introducing the threshold autoregressive (TAR) model. Whitten & Thomas (1999) considered a non-linear stochastic asset model. The models are including:

- A price inflation model
- A wage inflation model
- Share dividends
- Share yields
- Consols yield (long term interest rate)
- Base rate (short term interest rate)

The structure of Wilkie model improved by Whitten & Thomas (1999) is shown in Figure 3.

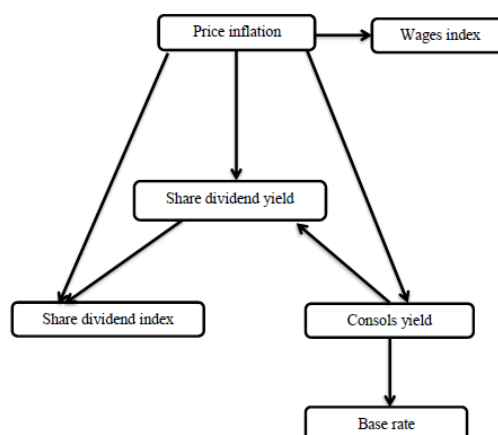


Figure 3. The direction of influence in Whitten and Thomas model Whitten & Thomas (1999)

Whitten & Thomas (1999) have found something difference from the structure updated Wilkie model in 1995.

They re-examine the correlation between the share dividend yield and the Consols yields. The result is there is positively correlation between the log differences of share dividend yield and Consols yield. From the result, they are strongly believed that the share yield is depending upon the Consols yield (Whitten & Thomas, 1999). So, the structure became as shown in Figure 1.3.

II. MATERIALS AND METHODS

This study used the secondary data that can be obtained from 10-Year Malaysian Government Securities (MGS) yield. We updated the data based on monthly data from January 2006 to December 2016. This data represents the bond yield indicates the long-term interest-bearing securities for Malaysia.

B. Descriptive Statistics

Descriptive statistics can be used to analyze the basic features of the data in a study. It summarizes the given data about the sample and the measures. The measures are including mean, mode, median, standard deviation, skewness and kurtosis. Mean is an average of the sample data. Mode is the most occurring value of the data. The median is the exact value in the middle of the data. Standard deviation is expressing as how much the data differ from its mean. The skewness is a measure of the lack of symmetry and the meaning of kurtosis is a measure whether the data are heavy-tailed or light-tailed relative to normal distribution.

C. Unit Root Test

Before we used the data for regression, we must test the data first to ensure the data is stationary or not stationary. So, we use Augmented Dickey Fuller Test (ADF) to test the stationary of the data. Augmented Dickey Fuller is the inspiration from the name of American statisticians, David Dickey and Wayne Fuller who has developed this test in 1979. This test is appropriate for trending time series. ADF test is the simple method to test the unit root but for the economic and financial time series which have more

complex system are better to use the simple autoregressive model. The unit root test can be tested by using E-views software.

No Intercept and No Trend:

$$\Delta y_t = \gamma y_{t-1} + \sum_{i=1}^p \beta \Delta y_{t-i} + \mu_t \quad (2.1)$$

Intercept:

$$\Delta y_t = \alpha_0 + \gamma y_{t-1} + \sum_{i=1}^p \beta \Delta y_{t-i} + \mu_t \quad (2.2)$$

Intercept and Trend:

$$\Delta y_t = \alpha_0 + \gamma y_{t-1} + \alpha_2 t + \sum_{i=1}^p \beta \Delta y_{t-i} + \mu_t \quad (2.3)$$

Test for unit root – using Augmented Dickey Fuller (ADF) Test. These three possible forms are given by the following equations:

where,

y_t : Consols yield

H_0 : Consols yield has a unit root (non-stationary)

H_A : Consols yield does not has unit root (stationary)

α : constant

β : the coefficient on a time trend

p ; the lag order of autoregressive process

μ_t : error term.

If the data is not stationary, it contains a unit root. After we test using ADF and the results are stationary it means the data has no unit root. The null hypothesis will be rejected if there are strong evidences that show the data was significant. This ADF technique basically involves Ordinary Least Squares (OLS) method to estimate the coefficient parameters of the model. To find whether the data was significant or not we must compare the t-statistics with critical value. If the t-statistics value less than the critical value, so we will reject the null hypothesis and conclude that the dependent variable, y_t will be significant.

D. Wilkie Consols Yield Model

The model used by Ishak (2015) was following the Wilkie model (Wilkie, 1984) and the model is,

$$\ln CNt = \ln CMU + CA \cdot (\ln CNt-1 - \ln CMU) + CY \cdot YSD \cdot YZt + CSD \cdot CZt \quad (2.4)$$

where CMU is the mean, the CSD is the standard deviation, the CA is the AR parameter and CZt is the sequence of independent identically distributed normal random variables, i.i.d. This model is AR (1) model (Wilkie, 1995).

E. Whitten and Thomas Consols Yield

Model

The model was developed by (Whitten & Thomas, 1999) based on Threshold modelling. The structure of the equation is similar to the Wilkie model, but the equation is divided into two regimes which upper and lower indicates high and lower effects of threshold for Consols yield. The equation of Consols yield using TAR model can be estimated as the equation below:

$$\begin{aligned} \ln(CRt) = \{ & \ln(CMU1) + CA1 \cdot (\ln(CRt-1) - \ln(CMU1)) + \\ & (CSD1 \cdot CZt); It-1 \leq QR \\ \ln(CMU2) + CA2 \cdot & (\ln(CRt-1) - \ln(CMU2)) + (CSD2 \cdot CZt); \\ It-1 > QR \} & \quad (2.5) \end{aligned}$$

where $CZt \sim iid(0,1)$.

This model consists of $CMU1$ and $CMU2$ are the constants mean of lower and upper regimes, $CA1$, $CA2$, $CSD1$, $CSD2$ are the autoregressive parameter and standard deviation for the lower and upper regimes. QR is the threshold and $It-1$ is the threshold variable that dictates which regime the process is in at time t and CZ as the sequence of independent identically distributed normal random variables, i.i.d.

F. Threshold Regression Model

The threshold regression model described simply form of non-linear regression with piecewise linear specifications and regime switching occurs when the observed variable has crossed unknown threshold. The applications of TR models such multiple equilibria, models for sample splitting, self-exciting Threshold Autoregression (SETAR)

and the very popular model that we also use in this paper is Threshold Autoregressive (TAR). TAR specifications are quite famous where they are easy to use for estimation and interpret and able to produce interesting nonlinearities results. The basic equation for TAR is the observations in regime when $j = 0, 1, \dots, m$. The linear regression specification

$$y_t = X_t\beta + Z_t\delta_t + \epsilon_t \quad (2.6)$$

The regressor has divided into two groups. First, the X variables are those parameters do not across regimes while the other variable which is Z variables has coefficients that are regime-specific.

For an observable threshold variable, q_t and increasing threshold values ($\gamma_1 < \gamma_2 < \dots < \gamma_m$) such that if regime j

$$\gamma_j \leq q_t < \gamma_{j+1}$$

where we set $\gamma_0 = -\infty$ and $\gamma_{m+1} = \infty$. Thus, for example for single threshold basically has two regime models the equation become

$$\begin{aligned} y_t = X_t\beta + Z_t\delta_t + \epsilon_t \text{ if } -\infty \leq q_t < \gamma_1, \\ y_t = X_t\beta + Z_t\delta_t + \epsilon_t \text{ if } \gamma_1 < q_t < \infty. \end{aligned} \quad (2.7)$$

III. RESULTS AND DISCUSSIONS

G. Descriptive Statistics Results

We performed descriptive statistics to this series using E-views to present quantitative descriptions in manageable form. The results of descriptive statistics for the MGS yield or known as Consols yield are shown in Table 1.

The results of descriptive statistics for the total number of observations for Consols yield are 132 ($n=132$) had an average mean from all the observations are 3.93 percent in between January 2006 to December 2016 with 0.348 percent away from the average which is the standard deviation. Since the standard deviation is smaller it is more precise to estimate the population mean. The maximum of the Consols yield data at 5.053 percent on June 2006 and the minimum is 3.093 percent on January 2009. The value of kurtosis which equal to zero indicates that the following

data will follow the normal distribution perfectly. In this case, the data has shown the kurtosis value is 3.67. It means that the value of kurtosis is positive value. The value of skewness is at 0.578 points showing that the distribution of this series positively skewed. The result of probability for Consols yield as shown below is 0.007 which is less than 0.01. It means that the Consols yield data is statistically significant.

Table 1. Descriptive Statistics for monthly Consols yield (January 2006 – December 2016)

Descriptive Statistics	Consols Yield (%)
Mean	3.931
Standard Deviation	0.348
Maximum	5.053
Minimum	3.093
Skewness	0.578
Kurtosis	3.670
Probability	0.007

H. Unit Root Test Results

First and foremost, we test the time series data using ADF Test. We test the Consols yield data using E-views software and the result is stationary at level form. The results are shown in the table below.

Null Hypothesis, H_0 : Consols yield is not stationary and has a unit root

Alternative Hypothesis, H_A : Consols yield are stationary and not has unit root

Table 2. Unit Root Test Results

Variable	Level Form				
	Coefficient	T-Test	P-value	Result	Series $I(n)$
Consols Yield	-0.192226	-3.679484	0.0055	-Reject H_0 -stationary	$I(0)$ Stationary at level form

Sources: Unit Root Test in E-views Software

From the results, we can see the data for Consols yield from January 2006 to December 2016 is statistically significant at all significance level as the null hypothesis is rejected. The Consols yield data is already stationary at level form as the estimated coefficient is small with the negative value. The t-statistics and the probability value are very small compared with critical value at 10 percent, 5 percent and 1 percent.

I. Threshold Autoregressive Results

We estimate the Consols yield data using Threshold regression method in E-views with both dependent variable lag length i and threshold lag length d from 1 to 5. The threshold for Consols yield data are through 132 months starting from January 2006 to December 2016.

Each of the additional lags will decrease the size of sample and the number possible of threshold. This is single threshold with two regimes with an AR (5) in each regime and the model selection is between lags from 1 to 5. From the lags 1 until 5, the model selected is TAR (3) as the autoregressive of order lag threshold $d=3$ in the lower and upper regimes with the threshold value used are 3.714. The value of the coefficients and standard error for TAR (3) in lower and upper regimes as stated in Table 3.

Table 3. The coefficient and standard error for TAR (3)

Variable	Lower regime	Upper regime
Coefficient	-0.006161	0.235108
Standard error	0.151665	0.329827

* QR = 3.714 the best fitted model TAR (3)

One of the parameter coefficients is significant in this model indicates the decreasing in SSR values. The TAR (3) model can be estimated as equation:

$$Cnsls_3 = \begin{cases} -0.243510 Cnsls_2 - 0.006161 Cnsls_3 ; Cnsls_3 \leq 3.714 \\ 0.043126 Cnsls_2 + 0.23510 Cnsls_3 > 3.714 \end{cases} \quad (3.1)$$

The model is selected based on the value of the Sum of Squared Residual (SSR) where the best fitted model must

have the lowest value of SSR. It can be proved the TAR (3) has the lowest SSR value compared to the other model. The result for the Sum of Squared Residual (SSR) for each model and each regime can be seen in the Table 4.

Table 4. Results of Sum of Squared Residual of possible model

Threshold Variable	SSR	Regimes
TAR (3)	3.776	2
TAR (2)	3.893	2
TAR (1)	3.920	2
TAR (4)	4.025	2
TAR (5)	4.828	1

*SSR: Sum Squared Residual

The value of SSR for TAR (3) is the lowest followed by TAR(2), TAR(1), TAR(4) and TAR(5). So, this is the suitable model for long term interest rates for government bond yield in Malaysia using non-linear threshold modelling.

IV. SUMMARY

There are many financial experts and actuaries are interested do the research about Threshold modelling. But, the system of threshold can give some difficulties for the

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long-term economic time series for examples are asymmetry, heteroscedasticity and non-normality. This study is focus about the non-linear stochastic Consols yield model in Malaysia. Aim for our studies is to apply the Consols yield inspired Whitten and Thomas model to Malaysian data with parameter estimation by using Threshold regression and choose the suitable model for Consols yield for investment data in Malaysia. We also determined the behavior of Consols yield in Malaysia. The data that has been used is monthly data from January 2006 to December 2016. The TAR model is suitable to use for long term economic series. From the results, we found that there is threshold effect on Consols yield for Malaysia below 3.71%. The TAR model which is suitable for Malaysian investment data is TAR (3) after we estimates all the possible model by using E-views software. The other findings are the value of kurtosis showed the positive value and the skewness also showed the positive result as the distribution of this series positively skewed.

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