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Unsteady Flow of Rotating Brinkman Type Fluid in Moving Disk

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In this paper, the exact solution for unsteady flow of rotating Brinkman type fluid over a moving disk is investigated. The momentum governing equation is modelled in the form of Partial Differential Equation together with the initial and boundary conditions. Using the suitable non-dimensional variables, the momentum governing equation as well as initial and boundary conditions are reduced to non-dimensional equations. The expressions of velocity and skin friction are obtained by using the Laplace transform method. Then, they are plotted graphically and discussed for different values of parameters such as Brinkman type fluid, rotation, and time. The obtained solution is satisfied for both initial and boundary conditions and skin friction shows the opposite behaviour to the velocity of the fluid.

Keywords: Brinkman type fluid, rotating, moving disk, Laplace transform

I. INTRODUCTION

Brinkman model is widely used as the basis for non-Newtonian fluid flow study, over a wide range of applications such as in chemical engineering, pharmaceutical and cosmetics (Aliseda *et. al.,* 2008; Lissant, 1977). The model was proposed by Brinkman (Brinkman, 1949a, Brinkman; 1949b) from the basis of Darcy's law which described viscous fluid through a porous medium. This model has a special term of viscosity and is applicable for the fluid flow past a high porous surface. In these papers, a viscous fluid flow is defined as a fluid through a cloud of spherical particles whose size is smaller than the characteristic length scale of the flow, and it occupies a negligible volume. Therefore, the viscous fluid flow in a porous medium can be accurately described by the Brinkman model for incompressible flow.

Numerous investigations have been reported in the literature to describe the fluid flow problem in various specific configurations, either by using analytical or numerical method. The study of the flow of viscous incompressible fluid flow through a porous channel using Brinkman model was presented in (Varma & Babu, 1985). Two cases have been considered in their study; (1) both channel boundaries are considered as a porous medium, and (2) only one side of the boundary is porous, while the other side is rigid wall. For the Stokes problem involving the Brinkman type fluid, its continuum solutions have been provided in (Fetecau *et. al.,* 2011). By means of Fourier sine transforms, the solutions have been presented using suitable forms in terms of the classical solution of the first Stokes problem for Newtonian fluid. Another study on the Brinkman type fluid was presented by (Ali *et. al.,* 2012), focusing on the new exact solutions for some unsteady motions of viscous fluid. By using the Laplace transform technique, the solutions for the problem were presented in

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simple forms, and the solutions can immediately be reduced to the solutions of inviscid fluid when the initial and boundary conditions were gratified.

The rotating flow near a vertical plate in viscous fluid conducted with stretching surface has been solved numerically by using Keller-box method in (Nazar *et. al.,* 2004). In the study, a comparison has been made between the numerical and analytical solutions. By using the Laplace transform method, the problem involving fluid flow past an infinite flat plate in incompressible viscous fluid was solved in (Manna *et. al.,* 2007). By focusing on the flow of rotating fluid, they considered that both plate and fluid rotate in uniform angular velocity about an axis normal to the plate. While for the rotating viscous fluid problem, the effects of magnetic field in oscillating plate filled by a saturated porous medium were studied in (Khan *et. al.,* 2013). By solving two types of boundary conditions (cosine oscillating and sine oscillating) using both methods (Laplace and Fourier Sine transforms), they summarized that the solutions satisfy the governing equations and imposed initial and boundary conditions.

The Brinkman fluid flow under rotation motion with various boundary conditions is also important in engineering application. However, this area of research is not as much studied as flow over a moving plate, oscillating plate, flow over stretching sheet and many more. Most probably, it is due to complex structure of this non-Newtonian fluid. Most of the studies focus on the other type of fluid flow. For example, the moving flow boundary condition of rotating second grade fluid in porous medium has been studied (Ismail *et. al.*, 2015). They found that the fluid velocity profiles were enhanced by the speed of rotation. In another example, (Mohamad *et. al.,* 2017) investigated the incompressible second grade fluid in oscillating plate wedge through a porous medium with the ramped wall temperature effect. From this study, they found that the ramped wall temperature in the oscillation motion is always lower compared to constant wall temperature.

This paper emphasizes on this research matter. Therefore, the aim of this paper is to study the unsteady fluid flow of rotating Brinkman type fluid in moving disc. Exact solution is obtained by using Laplace transform method.

II. MATHEMATICAL FORMULATION

Consider the unsteady flow of a rotating Brinkman type fluid passing through a moving disk. The axis of rotation is assumed to be in plane $x' = 0$. The y' axis is taken normal to the disk. Initially, both of the fluid and disk are at rest. At time $t' > 0$, the disk starts to move with constant velocity U_0 and the fluid starts solid body rotation with constant angular velocity Ω parallel to y' axis. Therefore, the appropriate governing equation is given as

$$\frac{\partial f'}{\partial t'} - 2i\Omega f' + \beta^* f' = \upsilon \frac{\partial^2 f'}{\partial y'^2} \tag{1.1}$$

together with initial and boundary conditions

$$\begin{aligned} &f'(y',0) = 0; \quad y' \geq 0, \\ &f'(0,t') = U_0; \quad t' > 0, \\ &f'(\infty,t') = 0; \quad t' > 0, \end{aligned} \tag{1.2}$$

where $f=f'(y',t')=u'(y',t')+iw'(y',t')$ is a complex velocity, $u'(y',t')$ is a primary velocity, i is unit vector for the flow, β^* is defined as $\beta^* = \alpha/\rho$ where α is drag coefficient in positive value, ρ is density of the fluid, υ is kinematic viscosity of the fluid. In order to simplify equation (1.1) and conditions (1.2) into non-dimensional form, the non-dimensional variables are introduced as

$$y = \frac{U_0}{\upsilon} y', \quad t = \frac{U_0^2}{\upsilon} t', \quad f = \frac{f'}{U_0}. \tag{1.3}$$

By using the non-dimensional variables (1.3), equation (1.1) together with conditions (1.2) are expressed as

$$\frac{\partial f}{\partial t} - a_1 f = \upsilon \frac{\partial^2 f}{\partial y^2} \tag{1.4}$$

and

$$\begin{aligned} &f(y,0) = 0; \quad y \geq 0, \\ &f(0,t) = 1; \quad t > 0, \\ &f(\infty,t) = 0; \quad t > 0, \end{aligned} \tag{1.5}$$

where $a_1 = 2ik - \beta_1$. Here, k is rotation parameter and β_1 is Brinkman fluid parameter. Our main aim is to find an exact solution of (1.4). By taking Laplace transform to equation (1.4) with respect to initial conditions (1.5), we obtained

$$\frac{d^2 \bar{F}}{dy^2} - (q - a_1)\bar{F} = 0 \tag{1.6}$$

and boundary conditions

$$\bar{F}(0,q)=\frac{1}{q},\qquad \bar{F}(\infty,t)=0. \tag{1.7}$$

By using the characteristic equation, (1.6) can be solved and we obtained

$$\bar{F}(y,q)=A_1\exp\left(-y\sqrt{q-a_1}\right)+A_2\exp\left(y\sqrt{q-a_1}\right). \tag{1.8}$$

We now need to find the values of A_1 and A_2 in equation (1.8). Imposing boundary conditions (1.7), we yield

$$\bar{F}(y,q)=\frac{1}{q}\exp\left(-y\sqrt{q-a_1}\right). \tag{1.9}$$

Then, the inverse Laplace transform of equation (1.9) is

$$f(y,t)=\frac{1}{2}\exp\left(-y\sqrt{-a_1}\right)erfc\left(\frac{y}{2\sqrt{t}}-\sqrt{-a_1t}\right)+\frac{1}{2}\exp\left(y\sqrt{-a_1}\right)erfc\left(\frac{y}{2\sqrt{t}}+\sqrt{-a_1t}\right), \tag{1.10}$$

and it is an exact solution for unsteady flow of rotating Brinkman type fluid over a moving disk. From this exact solution, the expression of skin friction τ can be obtained by using the non-dimensional form

$$\tau=-\left.\frac{\partial f}{\partial y}\right|_{y=0}. \tag{1.11}$$

By using this definition, we obtained the expression for the skin friction τ as

$$\tau=\frac{1}{2}\left[\sqrt{-a_1}\,erfc\left(\sqrt{-a_1t}\right)-\sqrt{-a_1}\,erfc\left(-\sqrt{-a_1t}\right)-2\frac{\exp(a_1t)}{\sqrt{\pi t}}\right]. \tag{1.12}$$

This skin friction expression defined the shear stress at the boundary layer of the rotating Brinkman type fluid in the moving disk.

III. RESULTS AND DISCUSSION

This section discusses the effect of embedded flow parameters on the fluid velocity. In order to illustrate such variations, the behaviour of velocity profiles has been plotted in Figure 1 until Figure 3. It is worth to note that, the panels **(a)** and **(b)** in each figure show the behaviour of the primary and secondary velocities, respectively. The effect of Brinkman type fluid β_1 is displayed in Figure 1 for both velocities. It is observed that, the primary (panel **(a)**) and secondary (panel **(b)**) velocities decrease on increasing the value of β_1. This is because, when the value of $\beta_1$increase, the viscous force will increase, and this will retard the movement of velocity profile.

The influence of rotation parameter k on velocity profile is graphically plotted in Figure 2. For larger values of rotation parameter, the fluid velocity decreases for primary velocity (panel **(a)**) whereas increases for secondary velocity (panel **(b)**). This is due to the Coriolis effect. In physics, the Coriolis effect is a deflection of moving objects in the frame rotating in the opposite direction. Therefore, primary velocity lost energy to deflect the movement of fluid to create the secondary velocity.

From the Figure 3, it shows that the velocity profiles increase in time t for both velocities (primary (panel **(a)**) secondary (panel **(b)**) velocities). Here, increasing in time will increase the movement of the disc and enhance the rotation speed of the fluid flow.

The values of skin friction are calculated in Table1. Its show that by increasing the values of β, k and t, the skin friction increases for β, k (primary velocity) and decreases for t, k (secondary velocity).

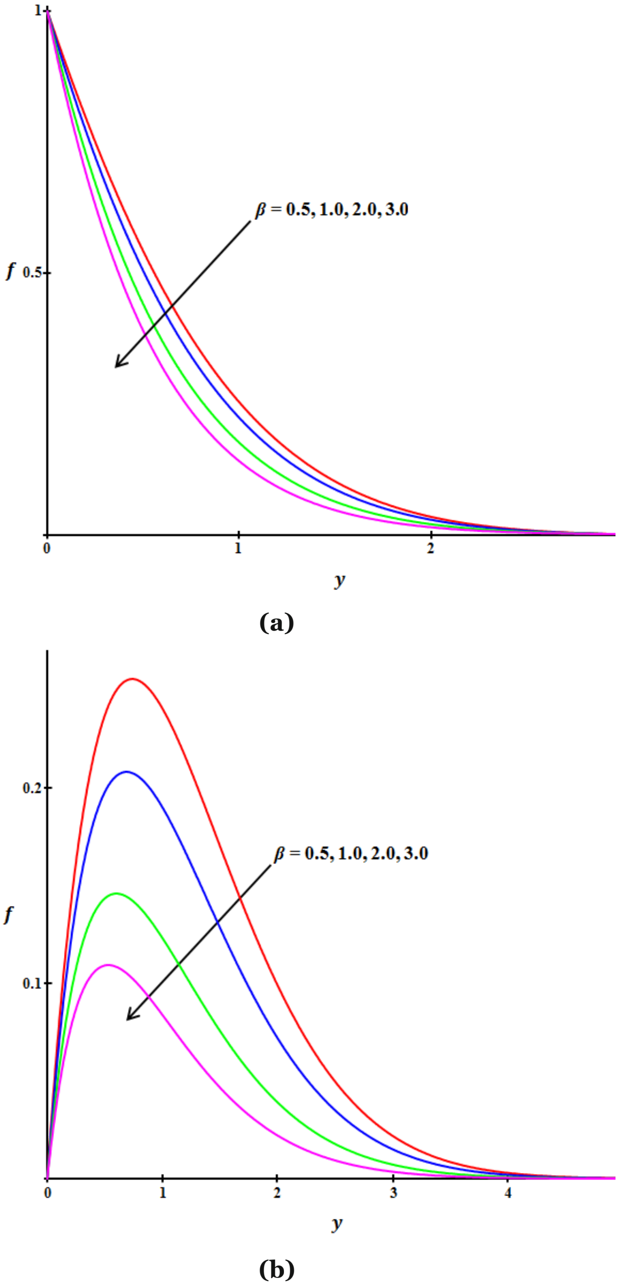


Figure 1. Velocity profiles for different values of $\beta_1$with k = 1.0 and t = 1.0 where **(a)** primary and **(b)** secondary velocities.

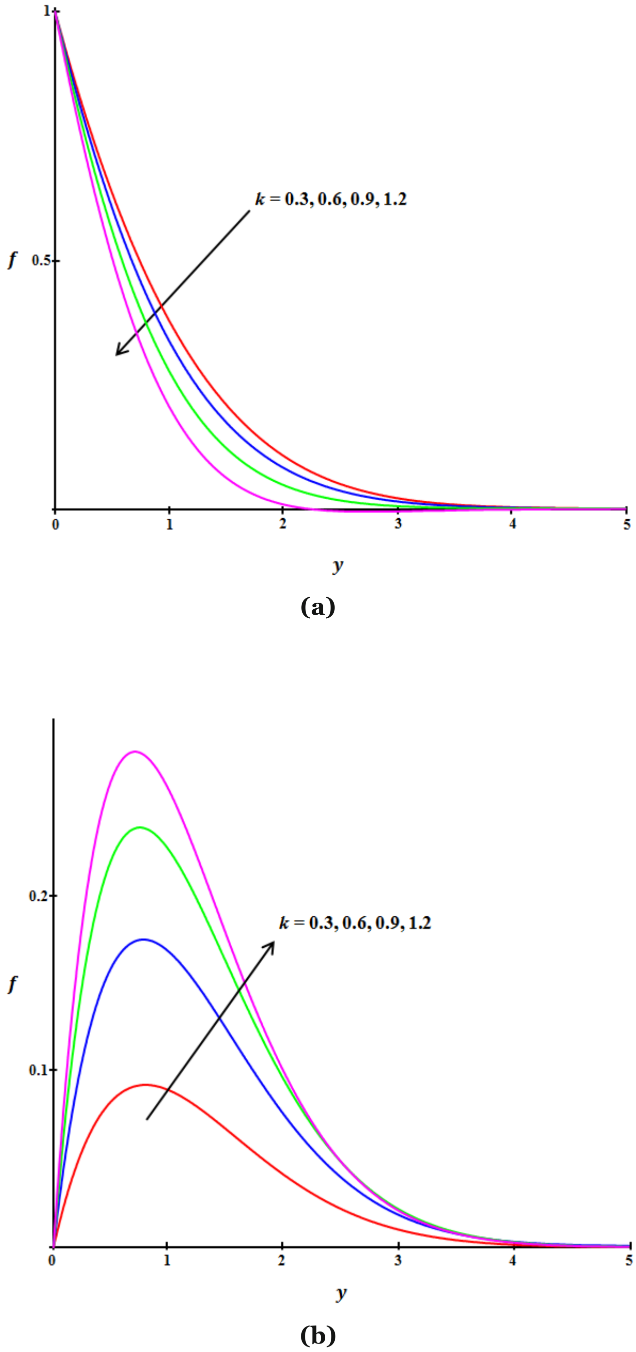
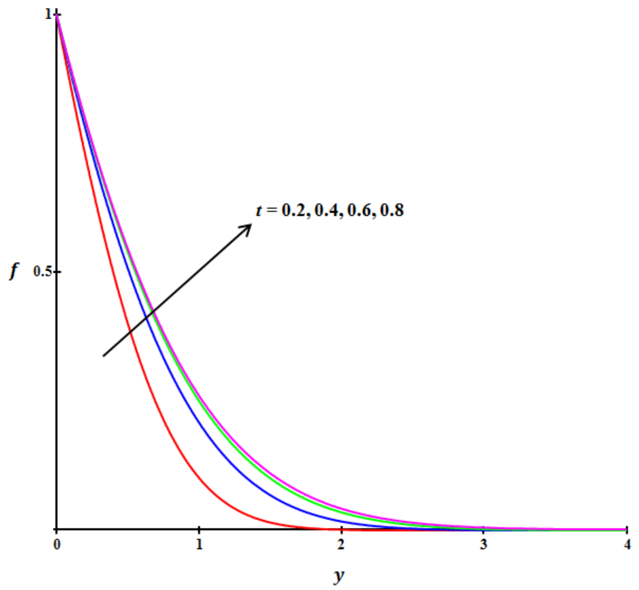
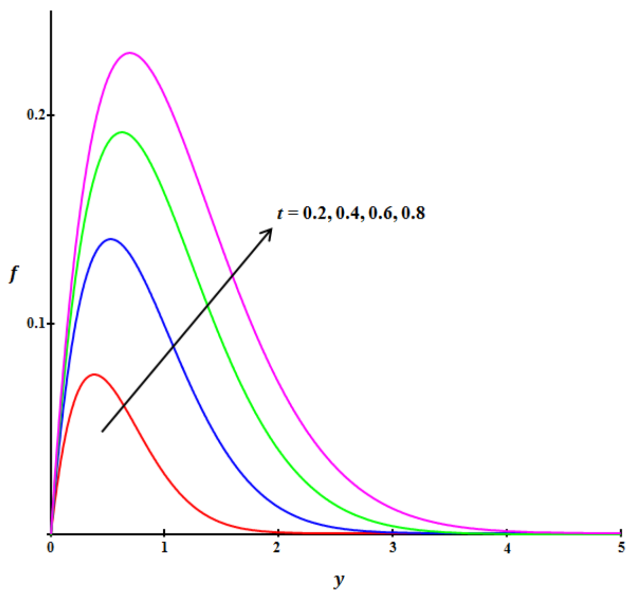


Figure 2. Velocity profiles for different values of k withβ_1 = 0.5 and t = 1.0 where **(a)** primary and **(b)** secondary velocities.



(a)



(b)

Figure 3. Velocity profiles for different values of t with β_1 = 0.5 and k= 1.0 where **(a)** primary and **(b)** secondary velocities.

Table 1. Variation of skin friction for all flow parameters

β	k	t	τ Primary	τ Secondary
0.500	**0.300**	**0.200**	**1.389**	**-0.146**
3.000			1.953	-0.126
	1.200		1.431	-0.582
		0.800	0.887	-0.265

IV. SUMMARY

An exact solution for unsteady flow of rotating Brinkman type fluid over a moving disk is obtained by using the Laplace transform method. The influence of flow parameters β, k and t on velocity and skin friction are plotted and calculated in graphs and table. The obtained solution is satisfied for both initial and boundary conditions

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