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Sensitivity Analysis for Fuzzy Similarity Measures

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Fuzzy similarity measure (FSM) is the method used to calculate similarity between fuzzy sets. Various techniques have been formulated, but it can be observed that there is a lack of formal method to determine which method is better or suitable to be used for certain applications. Many researchers make comparisons between methods based on selected cases only, which is, in essence not enough to conclude which method is better. This study proposes sensitivity analysis where parameter adjustment will be made relative to the height, distance, area, or perimeter so that the similarity values obtained can be compared and analysed. The result shows that the parameter adjustment will result in changes of similarity values. Therefore, the sensitivity analysis should be regarded as very important when comparing between two fuzzy numbers. Hence, the proposed method is expected to be very useful in assisting researchers to determine the behaviour of the selected fuzzy similarity measure. This finding suggests that the current practice by comparing fuzzy numbers based on selected sample are not enough to provide conclusive result. The concept of analysis introduced in this study can be a starting point for more systematic analysis on fuzzy similarity measures. This hopefully will open for broader implementations of fuzzy similarity measures in real world decision-making.

Keywords: Fuzzy similarity measure; Sensitivity analysis; Generalised fuzzy numbers; Decision-making

I. INTRODUCTION

Similarity measure is essentially known as a crucial concept to be used for comparison of various types of objects, such as image and geometry (Baccour *et al.*, 2014). Basically, it is a function of a real-valued that measures the similarity between two objects. Similarity measure is widely used whether in fuzzy concept or non-fuzzy concept. The implementation of similarity measures can be found in various real-world applications such as market prediction, pattern recognition, machine learning, engineering, business management, social science, psychology, and others.

Many researchers have reviewed and analysed the previous Fuzzy Similarity Measure (FSM) and made further explorations in order to introduce new approaches to FSM. Since there are many different FSM currently available, one could find that there is a lack of guidelines on which method is better or suitable to be used according to the problem under study. In particular, there is a lack of formal analysis on the behaviour of FSM. Previous studies determine the effectiveness of the method by exploiting the result of similarity measures between the pattern set of generalised trapezoidal fuzzy numbers with other available methods. But it has a major limitation since some results of the comparison are expected to be varied if certain parameters related to the formulation are adjusted. Sensitivity analysis (SA) is an important tool for examining issues that relate with uncertainties in the structure of the model or the parameter or the input value (Micovic *et al.*, 2017). Eventually, it is an approach that explores the uncertainty of output of the model if different parameters are used. SA is expected to be used to

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provide some insight on which FSM method is suitable based on the comparison made between the selected methods. In other words, it could help users in deciding or estimating the outcome of a decision if a situation turns out to be different from the initial prediction. This study proposes sensitivity analysis to show the different ways to examine the efficiency of the method. In this research, the main assumption used to differentiate fuzzy similarity methods is that changes in parameters will subsequently change the similarity value between fuzzy sets. A series of experiments will be conducted by changing the parameters related to the formulation of FSM such as distance, area, perimeter, and height. Graphical analysis will be used to observe the changes of the similarity values against parameter adjustment to gain better understanding on the behaviour of FSM under study. It is also may be used to distinguish between methods so that better inference could be made in the decision-making process.

II. CONCEPTS USED IN FUZZY SIMILARITY MEASURES

Fuzzy Similarity Measure (FSM) can be categorised according to the main concept that is used to distinguish between fuzzy sets such as the membership function values, distance between fuzzy sets, comparison of height, area, and perimeter of the fuzzy sets or even combination of several different concepts. Ideally, the selection of each concept used is based on the domain of the problems, the processed information of the research and also the suitable properties of a particular measure (Cross & Sudkamp, 2002) .

Based on the existing categorisation, most of FSMs are based on fuzzy set theory, which is formulated based on fuzzy membership function concept. Fuzzy set theory method is the most useful in fuzzy measure and has been broadly used in frequent application (Zheng, 2012). According to Zadeh (1965), fuzzy set theory is a concept to symbolise how humans perceive the information. The similarity based on fuzzy set theory can be subdivided into two groups, which are measures based on fuzzy logic or crisp logic (Beg & Ashraf, 2009). Moreover, Beg and Ashraf (2009) also determine that the focus on the concept that is based on set theory would be on the degree of similarity that considers both the amount of overlap between the given sets and symmetric difference. There are many FSMs based on fuzzy set theory developed in the literature. For example in Pappis and Karacapilidis (1993), comparative assessments of three newly proposed FSMs are presented. Whereas in Candan *et al.* (2000), fuzzy set theory is used to develop the similarity measure to be used for query processing. Shahari and Rasmani (2020) compared and used four different types of fuzzy similarity measure to investigate the consistency of the decision outcomes on job satisfaction evaluation.

The second most useful concept used in FSM is the similarity based on distance. In a research conducted by Allahviranloo *et al.* (2012), it was argued that distance measure and similarity measure are related concepts since it is possible to express the measures between fuzzy numbers by a functional relationship. Furthermore, according to Allahviranloo *et al.* (2012), distance measure between fuzzy numbers is one of theoretical assumptions that describe the similarity measure that is inversely related to distance measure.

Shape is another concept that has been used in the formulation of FSM. In the research conducted by Gadi *et al.* (1999), Fourier Descriptors are used to determine the fuzzy similarity measure based on shape. The process of similarity measures is determined by comparing the features of image with image in the database to identify which images are similar with the given features.

Some enhancements also have been made by researchers to obtain a better interpretation of similarity measures by combining several different concepts. For example, the method proposed in Xu *et al.* (2010) is based on a combination of two factors which are distance and height whereas the research by Patra and Mondal (2015) is based on distance, height and area. Meanwhile research reported in Khorshidi and Nikfalazar (2017) and Mat Saffie *et al.* (2017), the similarity degree is based on combination of four factors which are distance, height, area and perimeter

III. PROPOSED SENSITIVITY FOR FUZZY SIMILARITY MEASURE

In this study, sensitivity analysis is proposed to be conducted to determine the behaviour of fuzzy similarity methods. The benefit of sensitivity analysis is it allows the user to be flexible to test the sensitivity of the dependent variables to the independent variables. The sensitivity analysis can be done through several steps. Firstly, a series of parameters adjustment needs to be determined according to the types or concepts of fuzzy similarity measure. For example, if the method is based on distance, then adjustment on distance should be prepared. This will allow different results based on a chosen set of fuzzy numbers to be compared. The next step is to calculate the similarity value by using selected FSM formulation. After the similarity measure is obtained, the similarity values can be plotted against the parameter adjustment. The higher the value of parameter adjustment indicates that the higher number of different pairs of fuzzy sets were compared. The proposed method is based on the assumption that the larger the parameter values, the two compared objects may become more similar or in contrast, become more dissimilar. Hence, the similarity value should become smaller or larger. One simple example is when two objects are taken apart from each other, based on the concept of similarity by distance, the similarity between the objects should become smaller or both objects become dissimilar if the distance becomes larger. This can be illustrated through the graphical analysis as shown in Figure 1. Let say there are two different methods being compared namely Method 1 and Method 2. Thus, the comparison of the behaviour between the two methods can be achieved. In the graph, S_1 indicates the initial similarity measure, while S_2 and S_3 indicate the similarity values after a series of parameter adjustments were made. Therefore, the plotted graphs show some patterns which indicate the behaviour of each method that can be properly interpreted by the user in order to decide which method is better or shall be used.

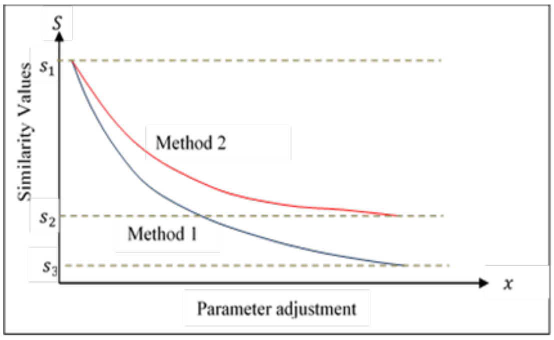

Figure 1. Graphical Analysis between Fuzzy Similarity Methods

IV. SELECTED METHOD FOR THE EXPERIMENTATION

Although numerous fuzzy similarity methods have been proposed, in this study, the method proposed by Xu *et al.* (2010) will be used for comparison purposes to determine the behaviour of the selected methods. Note that comparison between methods can only be made if both methods use the same concept or the same input. Therefore, the selection was made purposely so that calculation of similarity value can be conducted. Let M and N are two generalised fuzzy numbers to be compared, denote as $M = (m_1, m_2, m_3, m_4; w_M)$ and $N = (n_1, n_2, n_3, n_4; w_N)$. The descriptions of the four selected FSM are given as follows:

a) Method 1: The degree of similarity proposed in Xu *et al.* (2010) is using the new arithmetic operators of the linguistic values of trapezoidal fuzzy numbers. The distance Centre of Gravity (COG) of M and N, $d(M, N)$ are computed as $d(M,N) = \frac{\sqrt{(x_M - x_N)^2 + (y_M - y_N)^2}}{\sqrt{1.25}}$. Note that the calculation for y_N and x_N is using the same condition as y_M and x_M. The degree of similarity is computed as:

$$S(M,N) = 1 - \frac{\sum_{i=1}^{4} |m_i - n_i|}{8} - \frac{d(M,N)}{2} \quad (1)$$

b) Method 2: The technique proposed by Patra and Mondal (2015) used the concept of geometric distance, area and height of a generalised trapezoidal fuzzy number. The area of the fuzzy number denoted by $Ar(M)$ is defined by $Ar(M) = \frac{(m_4 + m_3 - m_2 - m_1) \times w_M}{2}$. The similarity measure is computed as:

$$S(M,N) = \left(1 - \frac{1}{4}\sum_{i=1}^{4} |m_i - n_i|\right) \times \left(1 - \frac{1}{2}\{|Ar(M) - Ar(N)| + |w_M - w_N|\}\right) \quad (2)$$

c) Method 3: Fuzzy similarity measure that was proposed by Khorshidi and Nikfalazar (2017) contains geometric distance, centre of gravity (COG), area, perimeter, and height. The COG, $d(M, N)$ is calculated using the equation proposed in Xu *et al.* (2010) and the area, $Ar(M)$ is based on a method proposed by Patra and Mondal (2015). Whereas the perimeter is computed as $P(M) = \sqrt{(m_1 - m_2)^2 + w_M^2} + \sqrt{(m_3 - m_4)^2 + w_M^2} + (m_3 - m_2) + (m_4 - m_1)$. Then the degree of similarity between these two fuzzy numbers, denoted by S (M, N), is computed as:

$$S(M,N) = \left(1 - \frac{\sum_{i=1}^{4}|m_i - n_i|}{4} \times d(M,N)\right) \times \left(1 - \frac{|Ar(M) - Ar(N)| + |w_M - w_N| + \frac{|P(M) - P(N)|}{\max(P(M), P(N))}}{3}\right) \quad (3)$$

d) Method 4: In Mat Saffie *et al.* (2017), in order to calculate the degree of similarity between two generalised trapezoidal fuzzy numbers (GTFNs), M and N, the fuzzy similarity measure based on geometric distance, centre of gravity (COG) points (x^*, y^*), height (w), area (Ar), and perimeter (P) has been introduced. The COG, area and perimeter are based on the previous 3 methods described in a), b), and c) From the equation, If $S_M + S_N > 0$, the value of $B(S_M, S_N) = 1$, while when $S_M + S_N = 0$, the value of $B(S_M, S_N) = 0$ such that $S_M = m_4 - m_1$ and $S_N = n_4 - n_1$. The similarity measure is calculated as:

$$S(M,N) = \left(1 - \frac{\sum_{i=1}^{4}|m_i - n_i|}{4}\right) \times \left(1 - |x_M^* - x_N^*|\right)^{B(S_M,S_N)} \times \left(1 - \frac{|w_M - w_N| + \frac{|y_M^* - y_N^*|}{\max(y_M^*, y_N^*)} + |Ar(M) - Ar(N)| + \frac{|P(M) - P(N)|}{\max(P(M), P(N))}}{4}\right) \quad (4)$$

The four selected methods will be tested with different pairs of fuzzy numbers, and the parameter (distance, height, perimeter, and area) will be adjusted in order to obtain a series of results (similarity values) so that graphical analysis can be made to compare the behaviour of methods being investigated.

V. EXPERIMENTATION RESULTS

Experiments involving four selected FSM with different similarity concepts and twenty parameter adjustments were prepared. The main purpose of this study is to introduce the significance of the proposed sensitivity analysis rather than to choose which method is better. Hence, the four selected existing FSM will just be referred to as Method 1, Method 2, Method 3 and Method 4. Note that the selected FSM used in the experimentations are based on a combination of various concepts in which the changes in certain parameters might also change the other parameters as well.

Table 1. Summary of Six Experiments Conducted using the Four Selected Fuzzy Similarity Measure (FSM)

Exp	Geometric Shape	Parameter Adjustment	Set of Generalised Fuzzy Numbers (Patra & Mondal, 2015)
1	Trapezoidal and triangular	Height (Triangular)	$M = \{0.1, 0.2, 0.3, 0.4; 1\}$ $N = \{0.1, 0.25, 0.25, 0.4; 1\}$
2	Trapezoidal and triangular	Height (Trapezoidal)	$M = \{0.1, 0.2, 0.3, 0.4; 1\}$ $N = \{0.1, 0.25, 0.25, 0.4; 1\}$
3	Trapezoidal and trapezoidal	Distance	$M = \{0.1, 0.2, 0.3, 0.4; 1\}$ $N = \{0.1, 0.2, 0.3, 0.4; 1\}$

4	Trapezoidal and trapezoidal	Distance (Non-normal; Different height)	$M = \{0.1, 0.2, 0.3, 0.4; 0.8\}$ $N = \{0.2, 0.3, 0.4, 0.5; 0.4\}$
5	Trapezoidal and trapezoidal	Area	$M = \{0.1, 0.2, 0.3, 0.4; 1\}$ $N = \{0.3, 0.4, 0.5, 0.6; 1\}$
6	Triangular and triangular	Perimeter	$M = \{0.1, 0.25, 0.25, 0.4; 0.8\}$ $N = \{0.1, 0.25, 0.25, 0.4; 0.6\}$

The six experiments have been prepared using six pairs of generalised fuzzy numbers previously used and discussed in Patra and Mondal (2015). The series of experiments are summarised and tabulated in Table 1.

The first experiment is the comparison of similarity measures between a pair of generalised fuzzy numbers which are a trapezoidal fuzzy number and a triangular fuzzy number. Figure 2 illustrates the geometrical representation of the two generalised fuzzy numbers where the trapezoidal fuzzy number is $M = \{0.1, 0.2, 0.3, 0.4; 1\}$ and the triangular fuzzy number is $N = \{0.1, 0.25, 0.25, 0.4; 1\}$. In order to analyse the behaviour of the selected FSM, twenty parameter adjustments were made by reducing the height of the triangular fuzzy number by 0.005 units each time an adjustment is made. While for the second experiment, it is done by using the same pair of fuzzy numbers but with the adjustment of height of the trapezoidal fuzzy number. Similar to the previous experiment, the height of the trapezoidal fuzzy number was reduced by 0.005 units. Analysis of the similarity values versus parameter adjustment for both experiments are done as illustrated in Figure 3 and Figure 4, respectively.

From Figure 3, it can be seen that as the height of the triangular fuzzy number being adjusted or in other word, reduced, the similarity values obtained using all selected FSM are also decreasing. This is reasonable because the height of the triangular fuzzy number becomes lower or more dissimilar from the trapezoidal fuzzy number. Figure 3 also shows that the values of similarity between Method 2 and Method 3 became closer to each other as the adjustments were made.

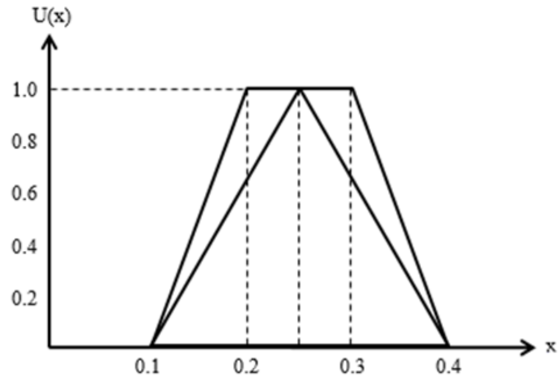

Figure 2. Geometrical Representation of a Trapezoidal and a Triangular Fuzzy Numbers
(Source: Patra & Mondal, 2015)

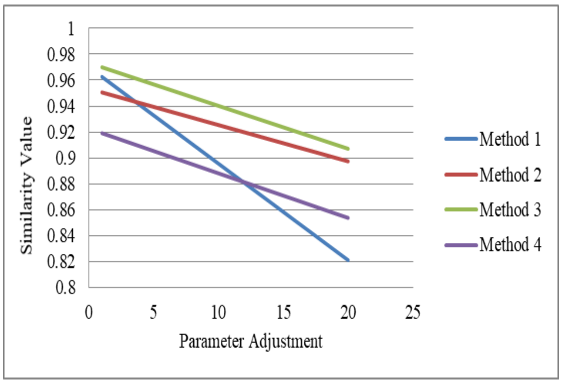

Figure 3. Sensitivity Analysis of Selected FSM using Trapezoidal and Triangular Fuzzy Numbers based on Height Adjustment of Triangular Fuzzy Number

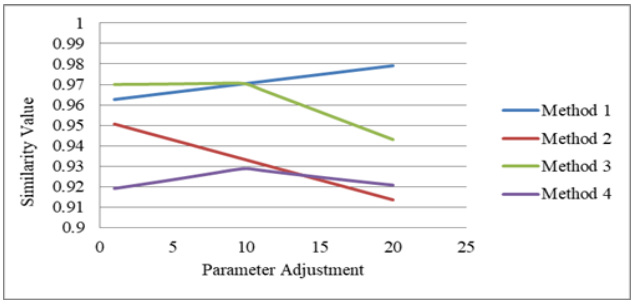

Figure 4. Sensitivity Analysis of Selected FSM using Trapezoidal and Triangular Fuzzy Numbers based on Height Adjustment of Trapezoidal Fuzzy Number

This means for this case; these two methods will not give much difference in similarity value. Also, at the early stage of the experiment, the similarity values obtained using Method 1 seem to be quite similar with Method 2 and Method 3. However, as the adjustments of the parameter were further made, it can be said that the similarity values obtained using Method 1 varied greatly as compared to the two methods. It also can be seen that at some point the line graphs of Method 1 and Method 4 are intersecting which means both methods produce an identical similarity value at a certain point, even so, most of the time there are obvious differences in terms of the similarity value. This shows that when making a comparison between methods using only one similarity value for each pair of fuzzy numbers, it can cause misinterpretation since the behaviour of each method is not fully analysed.

Based on Figure 4, it can be seen that all of the selected methods have different behaviours. Method 1 and Method 2 seem to be contrasting where the similarity values obtained using Method 1 become larger as the adjustments were made, while Method 2 is the opposite. From this analysis, it can be said that Method 1 probably did not give reasonable similarity values. Also, similarity values obtained using Method 3 did not give much difference although height adjustments have been made, until it reaches a certain point where it is suddenly decreasing. This shows how uncertain the values obtained using this method can be for this pair of generalised fuzzy numbers. Figure 4 also shows that at an early stage of the experimentation, the similarity values obtained using Method 1, Method 2 and Method 3 are almost the same. However, as the parameter adjustments were made, the similarity values between these methods seem to be in a great difference. Moreover, the similarity values obtained using Method 2 and Method 4 varied greatly at first, then as the experimentation continues, the values seem to be closed to each other. The same goes with Method 1 and Method 3. This shows how important sensitivity analysis is, which directly indicates that making comparisons between methods using only one parameter for each pair of fuzzy numbers will not give an accurate overview of the FSM behaviour. From Experiment 1 and Experiment 2, it can be concluded that although the same pair of generalised fuzzy numbers were used, changes in shape on either one of the fuzzy numbers might not produce the same results.

For the next experiment, a pair of two identical trapezoidal fuzzy numbers were used where $M = \{0.1,0.2,0.3,0.4; 1\}$ and $N = \{0.1,0.2,0.3,0.4; 1\}$. Figure 5 illustrates the comparison of the two trapezoidal fuzzy numbers which are exactly the same, while Figure 6 shows the similarity values obtained using the selected FSM based on distance adjustment. The distance adjustment was made such that one of the two fuzzy numbers was moved to the right by 0.005 units each time an adjustment was made.

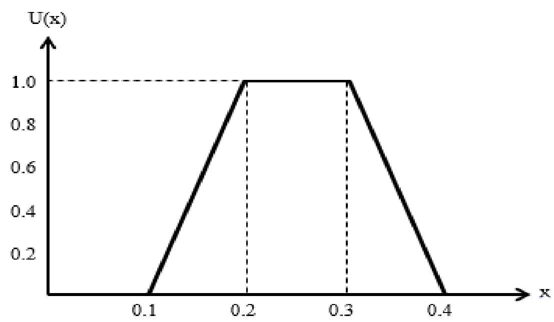

Figure 5. Geometrical Representation of a Pair of Two Identical Trapezoidal Fuzzy Numbers
(Source: Patra & Mondal, 2015)

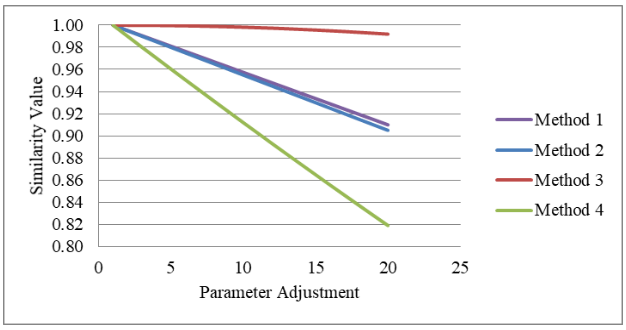

Figure 6. Sensitivity Analysis of Selected FSM using a Pair of Two Identical Trapezoidal Fuzzy Numbers based on Distance Adjustment

It is assumed that as two fuzzy numbers become further from each other, the similarity value will also decrease, and vice versa. From the results obtained, it can be seen that as the generalised fuzzy number N becomes further from generalised fuzzy number M, the similarity values obtained using all selected FSM are also decreasing which fall under the assumption mentioned above. Also, it can be seen in Figure 6 that all of the selected FSM were giving out the same similarity value at first which was equivalent to 1 and started to differ as the adjustments of distance on the trapezoidal fuzzy number N were made. This is because the pair of fuzzy numbers used are identical at first and started to differ when the adjustments were made. Throughout the experimentation, the similarity values obtained using Method 3 are almost the same although the adjustments have been made. This shows for this type of generalised fuzzy numbers, Method 3 is the least sensitive as compared to the other methods. As for Method 1 and Method 2, both have identical or almost identical similarity values although distance adjustments have been made. This means for this case, choosing either Method 2 or Method 3 would not give much difference. Among all of the selected FSM, Method 4 might be the most sensitive for this pair of fuzzy numbers.

For the fourth experiment, a pair of two distinct non-normal trapezoidal fuzzy numbers where $M = \{0.1, 0.2, 0.3, 0.4; 0.8\}$ and $N = \{0.2, 0.3, 0.4, 0.5; 0.4\}$ are used. The fuzzy number N is adjusted to the left in terms of distance such that the fuzzy number was reduced by 0.01 units, each time an adjustment was made.

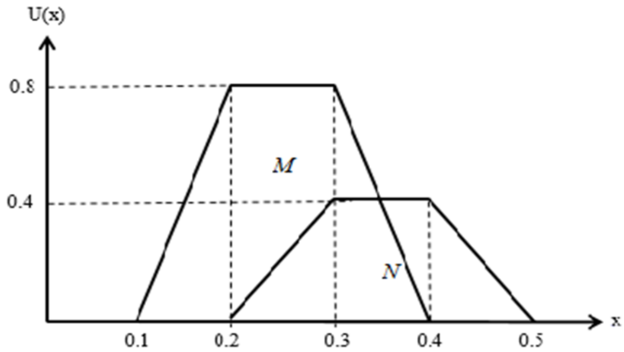

Figure 7. Geometrical Representation of a Pair of Two Distinct Non-Normal Trapezoidal Fuzzy Numbers (Source: Patra & Mondal, 2015)

Figure 7 shows the illustration of the two distinct non-normal trapezoidal fuzzy numbers and Figure 8 shows the similarity values obtained using the four selected FSM based on the distance adjustment. Based on Figure 8, since the two fuzzy numbers are initially located at different points and both are not identical, then it is understandable that the initial similarity values obtained using all selected FSM are different unlike the initial similarity values obtained in Experiment 3 where they all start with same similarity values which is equivalent to 1.

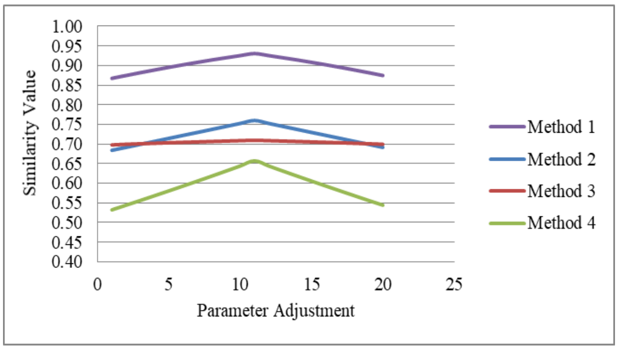

Figure 8. Sensitivity Analysis of Selected FSM using a Pair of Two Distinct Non-Normal Trapezoidal Fuzzy Numbers based on Distance Adjustment

Further than all selected FSM, Method 3 seems to be the least sensitive method as it does not give much difference in terms of similarity value as the parameter adjustments were made. Also, Method 1, Method 2 and Method 3 seem to have the same behaviours where the similarity values obtained are increasing at the early adjustments and then, when they reach a certain point, the values start to decrease. This seems reasonable enough since when the fuzzy number N was moved by 0.01 units to the left, the two fuzzy numbers used are getting close to each other, but as further adjustments were made, the two fuzzy numbers will become more dissimilar. Therefore, the similarity values obtained using the three methods fall under the assumption as mentioned before. However, among the three methods, Method 2 and Method 4 probably are the best techniques to be used for this type of fuzzy numbers. Even so, it is all up to researchers themselves to choose which method is the best one for their application, with the help of this sensitivity analysis.

For the next experiment, another pair of trapezoidal fuzzy numbers are used where $M = \{0.1,0.2,0.3,0.4; 1\}$ and $N = \{0.3,0.4,0.5,0.6; 1\}$as illustrated in Figure 9, but this time both are normal fuzzy numbers. The adjustments were made based on the area where the height of one of the fuzzy numbers which is *N,* was reduced by 0.005 units. The graphical representation of the similarity values obtained using the selected FSM is shown in Figure 10. From the experimental results presented in Figure 10, the similarity values evaluated using all the selected FSM were decreasing throughout the experimentation which is reasonable since one of the two fuzzy numbers was getting smaller from the other one. Also, Figure 10 shows the similarity values obtained using Method 1 and Method 2 are almost the same, and to compare these two methods with the other two FSM which are Method 3 and Method 4, it can be said that they consistently differ from the start until the last parameter adjustment. Although similarity values obtained using Method 1 and Method 2 are quite similar, as the adjustments were further made, the values obtained using both methods started to differ even more. Method 1 seems to be the least sensitive as compared with the other three selected methods which means the method does not give much difference in terms of similarity value although twenty parameter adjustments have been made. Figure 10 also shows that the similarity values obtained using Method 2, Method 3 and Method 4 have the same pattern for this type of fuzzy numbers. Therefore, by considering this analysis on the behaviour of each selected FSM and other research on aspects of similarity measure, researchers are expected to be able to choose the most suitable technique to be used in their studies or applications.

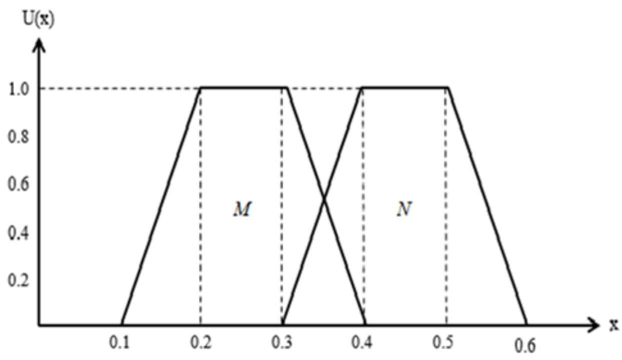

Figure 9. Geometrical Representation of a Pair of Normal Trapezoidal Fuzzy Numbers
(Source:(Patra & Mondal, 2015))

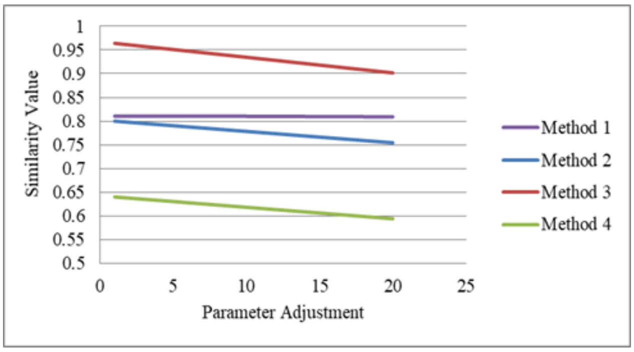

Figure 10. Sensitivity Analysis of Selected FSM using a Pair of Normal Trapezoidal Fuzzy Numbers based on Area

As for the last sensitivity analysis, a pair of non-normal triangular fuzzy numbers are used where $M = \{0.1,0.25,0.25,0.4; 0.8\}$ and $N = \{0.1,0.25,0.25,0.4; 0.6\}$.

For experimental purposes, the fuzzy number *N* is adjusted in terms of perimeter where the right side of the fuzzy number was moved to the right by 0.005 units, while the left side was moved to the left also by 0.005 units. This means the perimeter of the triangular fuzzy number *N* becomes larger as the parameter adjustments were made. Figure 11 illustrates the two non-normal triangular fuzzy numbers with different height, while Figure 12 shows the similarity values obtained using the selected FSM for the sensitivity analysis purpose Figure 12 shows that all the selected FSM have diverse behaviours. From the results obtained, among all of the selected FSM, Method 2 and Method 4 did not seem to make much difference as the twenty adjustments were made. This shows how insensitive the methods are towards the change in parameter. However, for Method 2, there is a point where it is drastically decreasing which means this method has uncertain similarity values. This will make it harder for researchers to interpret or decide without the help of sensitivity analysis. The same goes with Method 3 where the similarity values obtained using the method are increasing at the early stage of the experimentation, and as the parameter adjustments were made, the slope seems to be flattened. Among all the selected FSM, Method 3 is probably not suitable to be used for this case as it seems to produce unreasonable similarity values as compared to the other methods. Figure 12 also shows that Method 2 and Method 3 are intersecting at certain points which means they will give out the same similarity value. However, if this sensitivity analysis is not conducted, the researchers will not be able to see that the two methods are only giving the same similarity value at a certain point and not at all times. Moreover, it can be seen in Figure 12 that Method 2 and Method 3 are contrasting. Therefore, this can cause misinterpretation when making a comparison between methods if this sensitivity analysis is not conducted together with the methods comparison. Throughout the experimentation, Method 1 gives smooth similarity values as compared to the other techniques. Nevertheless, these values might be inaccurate because the values are not consistent with the values obtained using the other three methods. So, it is depending on the researchers to decide which method is the best one after considering this analysis which is expected to be able to assist them in making the right decision.

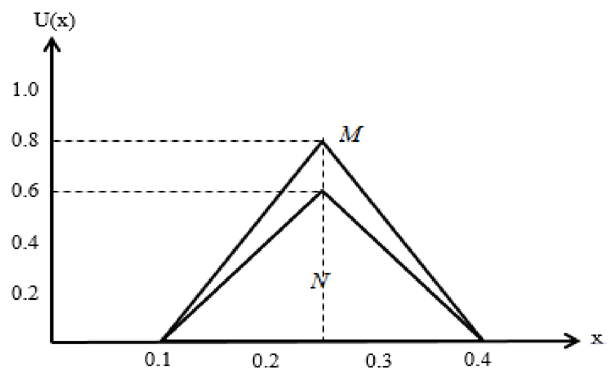

Figure 11. Geometrical Representation of a Pair of Non-Normal Triangular Fuzzy Numbers
(Source: Patra & Mondal, 2015)

Results from the six experiments presented in this section indicate that the sensitivity analysis can be very helpful in assisting researchers in many ways. The sensitivity analysis can be done by analysing the graphical representation of the similarity values obtained using the selected FSM against parameter adjustment. By doing the proposed analysis, the behaviour of each FSM can be investigated and at the same time it provides comparison between methods. Also, the results from the sensitivity analysis can be used in determining the degree of sensitivity of each method towards parameter adjustment.

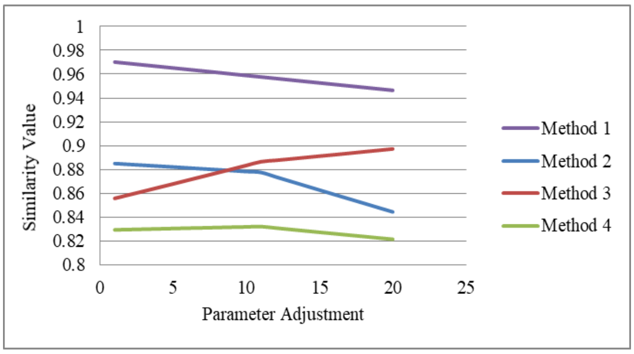

Figure 12. Sensitivity Analysis of Selected FSM using a Pair of Non-Normal Triangular Fuzzy Numbers based on Perimeter Adjustment

It is expected that the sensitivity analysis for FSM is very useful to assist researchers in choosing the right type of fuzzy numbers or fuzzy membership functions to be compared. Additionally, it is also can be said that the results obtained from this sensitivity analysis are more informative as compared to the comparison made without any parameter adjustment.

VI. CONCLUSION

This study proposed sensitivity analysis by using graphical representation to observe the changes of the fuzzy similarity values against parameter adjustment. Six experimentations have been conducted using examples taken from a previous research report. Four selected FSM which are based on a combination of different concepts used to formulate the methods are employed in the experimentations. The findings show that the parameter adjustment will result in changes of similarity values. Therefore, by doing the sensitivity analysis, the behaviour of each selected FSM can be analysed. Also, this study has proven that comparisons based on selected cases only that have been done by many researchers are not enough to jump into conclusion on which method is the best one. Hence, it can be concluded that the proposed sensitivity analysis can be very useful in assisting researchers in analysing the behaviour of FSM and making the right decision in order to choose the most suitable FSM to be used. Potential future work would include the visualisation tool can be used to investigate intermediate results. This visualisation tools also allow the user to map the changes in the output distribution back into the input space, to gain more insights about the model behaviour. Further research can be continued to investigate the proposed sensitivity analysis in FSM that can be applied in different case studies for handling uncertainty or inconsistency and checking the robustness of the conclusions.

VII. REFERENCES

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