ASM Sci. J., Special Issue 6, 2019 for SKSM26, 53-59

Application of Graph Theory and Matrix Approach as Decision Analysis Tool for Smartphone Selection

Sumarni Abu Bakar*, Noor Ainy Harish, Kahartini Abdul Rahman, Mohd Agos Salim Nasir, Herniza Md Tahir, Elsie Janisip

Faculty of Computer and Mathematical Sciences
Universiti Teknologi MARA, 40450 Shah Alam, Selangor, Malaysia

Nowadays, there are various types of smartphones in the market offered to the customers with advanced applications, various types of design and features. All of these criteria (attributes) are assumed to be independence to each other in most studies of smartphone selection. The aim of this study is to propose graph theory and matrix approach as all of the criteria are possibly depend to each other. This method can be used as a decision analysis tool in smartphone selection in respect to the customer's preferences order. A case study of smartphone selection in Malaysia is used to illustrate the effectiveness of the proposed method.

Keywords: smartphone selection; smartphone criteria; graph theory; matrix approach

I. INTRODUCTION

Smartphones refer to mobile phones with advanced mobile applications like convenient and easy access to the internet that very popular in the new generations. It provides many functions to the users such as e-mail, web browsing, mobile video, app center, audiovisual amusement and Global Positioning System (GPS). According to Hu *et al.* (2014), smartphones allow mobile convenience, such as online payments, broadband internet access, communication performance and high computing, and multimedia platforms. Lane *et al.* (2010) and Hsiao and Chen (2015) additionally stated that myspace, friendster, facebook, twitter and instagram are type of social media application from the smartphone that allow people to share their routine life with others without connecting to the internet. With all of these advance applications, people are easily connected to each other and for a new generation having ones is a must.

In Malaysia, 35% of smartphone penetration was reported that resulting to more than 10 million smartphone users (Belkhamza *et al.*, 2016). This may due to rapid advancement in mobile marketing (Watson *et al.*, 2013), which cause in a decreasing of smartphone price and large selection of available smartphone model to people to choose. However, the selection of the most appropriate phone is a very difficult decision since it involves several perspectives (Büyüközkan & Güleryüz, 2013). Existing selection method which used Multi Criteria Decision Making (MCDM) approaches such as AHP, TOPSIS, ELECTRE I (Işıklar & Büyüközkan, 2007), Belbag *et al.* (2016) and hybrid techniques such as ANP & GCI, AHP & TOPSIS (Işıklar & Büyüközkan, 2007), Yildiz & Ergül (2015) are only applied with assumption that the attributes are not depend on the other attributes. As a result, it fails to capture information of interrelationship between attributes Rao & Padmanabhan (2006). Thus, this study proposed Graph Theory and Matrix Approach (GTMA) since GTMA has no such limitations (Agrawal *et al.*, 2016).

*Corresponding author's e-mail: sumarni@tmsk.uitm.edu.my

II. LITERATURE REVIEW

Many researchers have studied on smartphone selection using different MCDM methods in order to choose the most appropriate ones. MCDM approach used by Işıklar and Büyüközkanas stated in (Işıklar & Büyüközkan, 2007) used evaluation procedure namely analytical hierarchy process (AHP) is investigated in determining the relative importance of evaluation criteria and Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) to ranking the mobile phone alternatives. The most desirable features (attributes) influencing the choice of a smartphone are identified through a survey conducted among the telecommunication sector experts. However, the method is applied only if the attributes are independent. Akyene (2012) proposed Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) and Entropy in the mobile phone evaluation. Here the weight of the criteria is analysed by Entropy and the ranking of the alternatives determined by Similarity to Ideal Solution (TOPSIS). Yildiz & Ergül(2015) applied MCDM approach by combining Analytic Network Process (ANP) and the Generalised Choquet Integral (GCI) methods for the selection of smartphones. In the study, the best smartphone among 28 smartphone alternatives are selected by using three main criteria (attribute) and 17 sub-criteria. Still, the method does not represent the hierarchical interrelationship among the attributes. In addition, Büyüközkan and Güleryüz (2013) study on MCDM approach for smartphone selection using Intuitionistic Fuzzy TOPSIS (IF-TOPSIS) to better represent decision makers' preferences and to remove uncertainty. Belbag *et al.* (2016) use Fuzzy Elimination and Choice Expressing Reality (ELECTRE I) Method in the evaluation of smartphone brand choice involving 250 students at Gazi University, Ankara as their main sample. Here, seven criteria and five smartphone brands are investigated in their studies. Unfortunately, the methods do not provide a visualization of interrelationship between attributes.

Therefore, GTMA is utilized in this study because it does not have such disadvantages and could maintain the hierarchical structure and provide the information of interrelationship between attributes. Also, GTMA provide a visualisation of interrelationship between attributes based on the digraph representation. Thus, GTMA is a logical and systematic decision-making approach (Malik *et al.*, 2015; Geetha & Sekar, 2016).

III. PROPOSED METHOD

Graph theory and matrix approach is a new technique of decision-making (Malik *et al.*, 2015) which is reasonable and systematic (Geetha & Sekar, 2016), Rao (2007). The matrix is useful in analysing digraph models in easy way which explains the system and problems in numerous science and technology Rao (2007), Fathi *et al.* (2013). This approach consists of a digraph, its associated matrix and permanent function representation. The digraph representation consists of a number of nodes and directed edges while matrix representation of the graph represents a model which then analysed using permanent function to provide the information of decision making. The step by step explanation of the methodology is as follows:

***Step 1*:** List all potential attributes and alternatives for the smartphone selection.

In this step, all the criteria involved and smartphone brand that are available in the market are identified through literature or from the decision maker itself.

***Step 2*:** Develop a directed graph representation of interrelationship among the attributes. The diagraph consists of a set of nodes $V=\{v_i\}$ for i=1,2,3,...,m and set of directed edges $E=\{e_{ij}\}$ for i,j=1,2,3,...,m. The numbers of nodes m are equal to the number of smartphone attributes and directed edges e_{ij} represent therelative importance among attribute i to attribute j. The edge which is directed from node "i" to node "j" is simply because attribute "i" is more important than attribute "j". If the edge is directed from node "j" to node "i" then attribute $j$is more important than the attribute i. The interrelationship among attributes is illustrated in Figure 1.

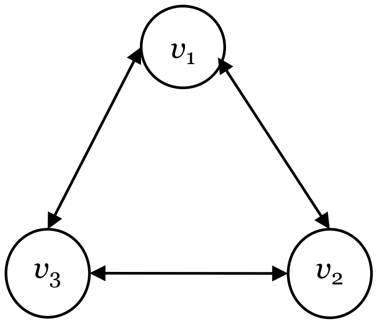

Figure 1. Digraph Representation of Interrelationship among Attributes

***Step 3*:** Obtain the relative importance x_{ij} of smartphone attributes on a suitable scale.

Here, the value of relative importance of smartphone attributes x_{ij} for attribute i to attribute j is collected through questionnaire using relative importance attributes scale as in Table 1.

Step 4: Develop the relative importance matrix [A] for the graph.

A matrix [A] is a square matrix whereby its element is composed into two parts, namely the off-diagonal and diagonal elements. The matrix [A] is written as:

$$[A]=\begin{bmatrix} y_1 & x_{12} & - & - & x_{1M} \\ x_{21} & y_2 & - & - & x_{2M} \\ - & - & y_3 & - & - \\ - & - & - & y_4 & - \\ x_{M1} & x_{M2} & - & - & y_M \end{bmatrix} \quad (1)$$

The off-diagonal elements of the matrix are represented as x_{ij} for $i \neq j$ and is calculated using arithmetic mean formula as follows:

$$x_{ij}= \frac{1}{n}\sum_{k=1}^{n} x_{ij}^{k} \quad \text{for } i \neq j \quad (2)$$

where x_{ij} = off-diagonal element of [A].

n = the number of decision maker.

x_{ij}^{k} = the relative importance value given by decision maker k which based on scale in Table 1.

Agrawal *et al.* in (Agrawal *et al.*, 2016) used highest mod value in determining the off-diagonal element of matrix [A]. However, the method is not suitable for a small number and even number of experts.

Meanwhile the diagonal elements of matrix [A] is represented as y_i for i= 1, 2,...n in which it constitutes the value of importance of the attributes for each alternative and is assigned based on the following scale in Table 2.

Table 1. Relative Importance Attributes Scale (Agrawal *et al.*, 2016)

Description	x_{ij}	1-x_{ij}
Two attributes are equally important	0.5	0.5
One attribute (i) is slightly more important over the other (j)	0.6	0.4
One attribute (i) is strongly more important over the other (j)	0.7	0.3
One attribute (i) is very strongly more important over the other (j)	0.8	0.2
One attribute (i) is extremely more important over the other (j)	0.9	0.1
One attribute (i) is exceptionally more important over the other (j)	1.0	0.0

The value of importance of attributes y_i for each alternative i is then calculated using arithmetic mean formula. Here, the off-diagonal elements of matrix [A] are equal for all smartphone alternatives but the diagonal elements value may be differing for each smartphone alternative.

Step 5: Develop the permanent function for the matrix. The permanent function is a standard matrix function and is used in combinatorial mathematics (Geetha & Sekar, 2016). This standard form of matrix function is calculated rather than determinant function because the negative sign does not appear in the permanent function of the matrix and hence no information will be lost (Roa, 2007; Fathi, 2013; Lanjewar *et al.*, 2015). In this study, the permanent function per (A) is adopted from Rao in Yildiz & Ergül (2015) and Agrawal *et al.* in (Agrawal *et al.*, 2016). The permanent function Per (A) as in (Rao (2007) and (Agrawal *et al.*, 2016), has (M+1) group for M×M matrix which represent the measure of attributes and the relative importance loops. Here, the effect of all attributes and the relative importance of attributes are characterized by the permanent function. Subsequently a preference index is calculated which is known as numerical value of the permanent function Lanjewar *et al.* (2015). In this study, the preference index is calculated with the help of MATLAB software.

Table 2. The Importance of Attributes Scale for each Smartphone Alternative (Agrawal *et al.*, 2016)

Qualitative measure of attributes	Assigned value of y_i
Exceptionally low (E)	0.0
Extremely low (EL)	0.1
Very low (VL)	0.2
Low (L)	0.3
Below average (BA)	0.4
Average (A)	0.5
Above average (AA)	0.6
High (H)	0.7
Very high (VH)	0.8
Extremely high (EH)	0.9
Exceptionally high (EPH)	1.0

IV. CASE STUDY

In this section, the illustration of the method discussed earlier is implemented for the selection of smartphone in Malaysia. The detailed of each step is explained below:

Step 1: Seven articles including Hsiao & Chen (2015), (Büyüközkan & Güleryüz, 2013), (Işıklar & Büyüközkan, 2007), (Belbag *et al.*, 2016), Yildiz & Ergül (2015), Lomonaco (nd) & Uddin *et al.* (2014) are analysed in order to select the attributes for evaluating the smartphone. Based on the literature, attribute such as dimensions (DI), memory capacity (MC), camera specifications (CS), brand choice (BC) and Price (PR) are selected in this study since they are among the most frequent attributes appeared and analysed in the previous study. Meanwhile, based on the telecommunication sector experts in Malaysia, smartphone brand namely Apple, Samsung, Oppo, Vivo, Asus and Lenovo which known to have high demand are selected in this study.

Step 2: Here, the attributes represent a set of nodes V such that $V=\{v_1, v_2, v_3, v_4, v_5\}$ where v_1 = Dimensions (DI), v_2 = Memory capacity (MC), v_3 = Camera specifications (CS), v_4 = Brand choice (BC) and v_5 = Price (PR). Next, the interrelationship between attributes which represented as directed edge e_{ij} for i, j=1,...,5 is developed. If there is a relationship between attributes i to attribute j then the edge is directed from attribute i to j. For this study, an assumption is made whereby there exists a relationship between every attribute to another. The edges are represented as set E such that $E=\{e_{12}, e_{13}, e_{14}, e_{15}, e_{21}, e_{23}, e_{24}, e_{25}, e_{31},. e_{32}, e_{34}, e_{35}, e_{41}, e_{42}, e_{43}, e_{45}, e_{51}, e_{52}, e_{53}, e_{54}\}$. Therefore, the graph can be obtained as follows.

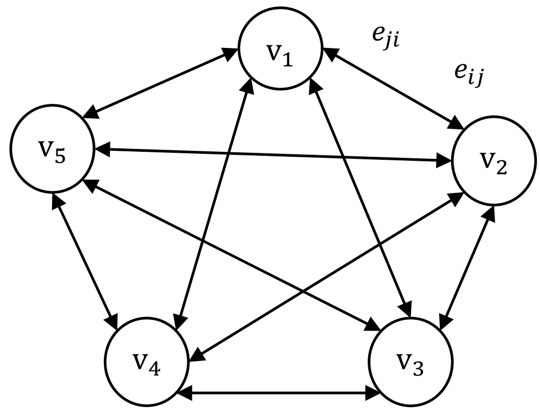

Figure 2. Digraph Representation of Interrelationship between Attributes

Step 3 and Step 4: Here, the value of relative importance of smartphone attributes x_{ij} for attribute i to attribute j is collected through questionnaire using relative importance attributes scale as in Table 1. The questionnaire is distributed to the four experts in the evaluation of smartphone brand based on the customer preference. The experts considered in the study are smartphone vendors who have a lot of experiences and knowledge dealing with smartphones for a long period of time. The relative importance value which is assigned by the four experts is shown in Table 3.

Table 3. Pairwise Comparison of Relative Importance among Attributes

Decision makers	attribute	DI	MC	CS	BC	PR
DM_1	DI	-	0.2	0.0	0.2	0.5
	MC	0.8	-	0.5	0.5	0.5
	CS	1.0	0.5	-	0.5	0.5
	BC	0.8	0.5	0.5	-	0.5
	PR	0.5	0.5	0.5	0.5	-
DM_2	DI	-	0.9	0.8	0.5	0.5
	MC	0.1	-	0.1	0.0	0.0
	CS	0.2	0.9	-	0.2	0.0
	BC	0.5	1.0	0.8	-	1.0
	PR	0.5	1.0	1.0	0.9	-
DM_3	DI	-	0.2	0.7	0.0	0.0
	MC	0.8	-	0.5	0.0	0.0
	CS	0.3	0.5	-	0.2	0.2
	BC	1.0	1.0	0.8	-	0.2
	PR	1.0	1.0	0.8	0.8	-
DM_4	DI	-	0.8	0.5	0.5	0.1
	MC	0.2	-	0.4	0.5	0.1
	CS	0.5	0.6	-	0.8	0.2
	BC	0.5	0.5	0.2	-	0.2
	PR	0.9	0.9	0.8	0.8	-

The pairwise comparisons for Table 3 are then examined and arithmetic mean for each attribute for every expert is then calculated in order to obtain the off-diagonal element of matrix [A] which is resulted as follows:

$$[A] = \begin{array}{c} \\ DI \\ MC \\ CS \\ BC \\ PR \end{array}\begin{array}{c} \begin{array}{ccccc} DI & MC & CS & BC & PR \end{array} \\ \begin{bmatrix} y_1 & 0.7 & 0.5 & 0.3 & 0.275 \\ 0.475 & y_2 & 0.375 & 0.25 & 0.15 \\ 0.5 & 0.625 & y_3 & 0.425 & 0.225 \\ 0.7 & 0.75 & 0.575 & y_4 & 0.475 \\ 0.725 & 0.85 & 0.775 & 0.75 & y_5 \end{bmatrix} \end{array}$$

Next, the five steps involved the calculation of the importance of attributes for each smartphone alternative y_i where the data is collected through questionnaire. The scale for the importance of attributes for each smartphone alternative is stated in Table 2. The experts are asked to select quantitative values of importance of the attributes for each alternative. The average value of the importance of the attributes with respect to each alternative is then calculated which then constitute the diagonal element of matrix [A]. The evaluation of attributes for each alternative is presented in Table 4.

Table 4. The Corresponding Value of Evaluation Result Based on the Expert

Alternative	**Attribute**	**DM$_1$**	**DM$_2$**	**DM$_3$**	**DM$_4$**
A_1	DI	0.8	0.8	0.8	0.5
	MC	1.0	1.0	0.8	0.9
	CS	1.0	0.9	0.6	1.0
	BC	1.0	0.9	0.7	0.9
	PR	1.0	1.0	0.7	0.9
A_2	DI	0.8	0.6	0.7	1.0
	MC	0.8	0.6	0.7	0.7
	CS	1.0	0.9	0.7	0.7
	BC	0.9	0.7	0.7	0.7
	PR	0.5	0.6	0.5	0.7
A_3	DI	0.3	0.4	0.3	0.2
	MC	0.3	0.7	0.5	0.5
	CS	0.3	0.9	0.6	0.5
	BC	0.3	0.4	0.6	0.4
	PR	0.3	0.3	0.5	0.5
A_4	DI	0.7	0.6	0.7	1.0
	MC	0.8	0.8	0.7	0.8
	CS	1.0	0.9	0.7	0.9
	BC	0.9	0.7	0.7	0.9
	PR	0.5	0.6	0.5	0.8
A_5	DI	0.7	0.5	0.5	0.6
	MC	0.7	0.7	0.5	0.6
	CS	0.8	0.5	0.6	0.6
	BC	0.7	0.2	0.6	0.6
	PR	0.5	0.4	0.5	0.6
A_6	DI	1.0	0.5	0.7	0.6
	MC	1.0	0.7	0.5	0.9
	CS	1.0	0.8	0.7	0.9
	BC	1.0	0.7	0.8	0.9
	PR	0.7	0.8	0.6	0.9

The value assigned by the experts for each attribute with respect to each alternative is then calculated using arithmetic mean. Subsequently the following diagonal element values for each smartphone alternative are obtained and shown in Table 5.

Table 5. Diagonal Element Values for Each Smartphone Alternatives

Attributes	A_1	A_2	A_3
DI	0.725	0.775	0.3
MC	0.925	0.7	0.5
CS	0.875	0.825	0.575
BC	0.875	0.75	0.425
PR	0.9	0.575	0.4
Attributes	A_4	A_5	A_6
DI	0.75	0.575	0.7
MC	0.775	0.625	0.775
CS	0.875	0.625	0.85
BC	0.8	0.525	0.85
PR	0.6	0.5	0.75

In the study, six relative importance matrices [A_i], i = 1,...,5 are obtained and the permanent function for each of the matrix is then calculated using permanent function adopted in Rao (2007) and (Agrawal *et al.*, 2016).

Step 5: Here, the permanent function Per (A), for each of the smartphone alternatives is obtained and has (5+1) groups which represent the existence of attributes and the relative importance loops where the number of attributes is five. Here, the permanent function for 5 attributes produces 5! = 120 terms that is arranged in (5+1) groups. The preference index value for the permanent function of matrix [A_1], [A_2], [A_3], [A_4], [A_5] and [A_6] are then calculated with the help of MATLAB-R2016a software. The preference index measures the degree of choice of smartphone alternative with respect to all attributes. The higher the index value, the preferable the smartphone is. The value of preference index for this study is presented in Table 6.

Table 6. Preference index and ranking for the smartphone brands

Alternative	**Preference index**	**Ranking**
Apple (A_1)	6.6962	1
Vivo (A_2)	4.9807	4
Lenovo (A_3)	2.7822	6
Oppo (A_4)	5.3666	3
Asus (A_5)	3.6414	5
Samsung (A_6)	5.6975	2

From Table 6, the highest preference index is **6.6962** representing Apple brand. This shows that Apple brand is the most preferred choice followed by Samsung brand. Oppo brand is rank in the third place followed by Vivo and Asus. The Lenovo brand which hold value of preference index **2.7822** is less preferred choice among the five smartphone alternatives. The sequences of ranking of smartphone alternatives using preference index is Apple (A_1) >Samsung (A_6) >Oppo (A_4) > Vivo (A_2) > Asus (A_5) > Lenovo (A_3).

V. CONCLUSION

In this study, Graph Theory and Matrix Approach (GTMA) is used as an evaluation method in smartphone selection process. The weighted directed graph provides a visualization of interrelationship among attributes related to the smartphone which does not provided by previous study. This study also provided useful information to the mobile manufacturing firms to make improvements on their smartphone products in order to satisfy customers' needs and provide a better selection of a smartphone brand to consumers in purchasing a smartphone.

VI. ACKNOWLEDGEMENT

We greatly acknowledge FSKM, Shah Alam for providing us the financial support and the authors are greatly appreciate reviewers for constructive comments.

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