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Marangoni Convection in a Double Diffusive Binary Fluid with Temperature Dependent Viscosity, Coriolis Force and Internal Heat Generation

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A steady Marangoni convection in a horizontal double diffusive binary fluid is considered. Present study investigated the effects of temperature dependent viscosity, Coriolis force and internal heat generation to the onset of convection. The bottom boundary was set to be insulating or conducting to temperature. A detailed numerical calculation of the marginal stability curves were performed by using the Galerkin method and it is shown that temperature dependent viscosity, internal heat generation and Soret number destabilize the binary fluid layer system while Taylor number and Dufour number act oppositely to the system. **Keywords:** binary fluid, double diffusive, variable viscosity, coriolis force, heat generation.

I. Introduction

The theory of double diffusive convection has been studied by researchers as it has many important applications in astrophysics, engineering and geology. One of the most related phenomena is oceanography where Stommel et al. (1956) start to study on their curiosity in this field. However, Huppert and Turner (1981) explain it thoroughly based on theoretically, experimentally and in sea-going oceanographers. Double diffusive is a problem when there exist two competing diffusive elements which are the temperature and salinity gradients. Effects that exist in a double diffusive binary fluid convection are called the Soret effect (thermo-diffusion) and the Dufour effect (diffusion-thermo). Bergman (1986) studied double diffusive in a Marangoni convection where the results show that convection may occur even when the Marangoni number is zero. Other researchers also studied the inclusion of other effects in a binary fluid (Bergeon et al. (1998); Slavtchev et al. (1999); Saravanan and Sivakumar (2009); Abidin et al. (2017); Abidin et al. (2017)). The first linearly stability analysis was started by Nield and Kuznetsov (2011) where they studied the effect of thermosolutal in both stationary and oscillatory mode in a nanofluid.

In earlier research, fluid is assumed to have a constant viscosity whereby we knew that viscosity may vary depending on other factors such as the temperature. Ramírez and Sáez (1990) stated that temperature dependent viscosity should be taken into account for every case studied since the effect has a huge impact on the instability of a convection. Griffiths (1986) mentioned that viscosity decreasing exponentially with temperature in some oils. Booker (1976) had done an experiment to study large temperature dependent viscosity where he states that mean temperature on the upper rigid and lower rigid boundaries removes the effect of a strong temperature dependent but the results are significant as it follows previous studies on this effect. Trompert and Hansen (1998) also analyzed a fluid with a strongly temperature dependent viscosity but the study was done numerically. White (1988) studied the planform of a convection with this effect theoretically and experimentally. Manga et al. (2001) investigated the effect experimentally on Benard convection. Sparrow et al. (1964) and Roberts (1967) analyze the nonlinear temperature distribution that is caused by the internal heat generation in a horizontal fluid layer. Gasser and Kazimi (1976) and Kaviany (1984) include the internal heat generation effects in a porous medium.

McDonald (1952) state that Coriolis effect is perpendicular to the rotating object's axis and the speed of the rotating obejct will define the magnitude. The importance of Coriolis effects in a Marangoni convection was presented numerically by Vidal and Acrivos (1966) where they found that the system stability is caused by the uniform rotation. Meanwhile, the study also shows the same correlation with previous experimental studies where the direction of the flow is downward along the cell boundaries and upward in the core in Marangoni convection. In 2014, Yad (2014) have carried out a study on the coupled effect of rotation (Coriolis) and magnetoconvection in a nanofluid. The study shows that rotation stabilizes the system. However, for the slip conditions, free-free boundary enhanced rapidly compare to rigid-rigid boundaries. Mcconaghy and Finlayson (1969) studied the Coriolis effect in a Marangoni oscillatory instability using smaller Prandtl number and shows similar results as Vidal and Acrivos (1966) and Yad (2014).

In this research paper, we are interesting to study the effect of temperature dependent viscosity in a double diffusive binary fluid layer together with the coupled effects of Coriolis force and internal heat generation. We have not come across any literature that combines all the effects in one dynamical system. The Soret and Dufour effects are taken into account as these effects were often being ignored in previous research problem due to their small magnitude. This model aims to be beneficial for problems in oceanography or in other geophysics areas.

The upper boundary were set to be free and insulating. Meanwhile, the lower boundary is set to be rigid. However, the temperatre conditions were set to be insulating or conducting. We assume that the upper surface to be non-deformable and employed the stability analysis theory. The resulting eigenvalue problem is solved numerically using Galerkin method.

II. Mathematical Formulation

A system where a horizontal Boussinesq binary fluid with a depth d with a temperature difference of $\triangle T$ between the lower and upper boundary is considered. The layer rotates about the vertical axis with angular velocity and is heated from below. The physical properties of the fluid are assumed constant except the surface tension, density and kinematic viscosity. The surface tension at the upper-free undeformable surface is assumed to vary linearly with temperature and solute concentration gradient in the form

$$\sigma = \sigma_0 + \sigma_t(T - T_0) + \sigma_c(C - C_0) \quad (1)$$

where σ_0 is the unperturbed value, T_0 is the reference value of temperature, C_0 is the reference value of concentration, σ_t is the rate of change of surface tension with temperature and σ_c is the rate of change of surface tension with concentration. The fluid density, ρ takes the form

$$\rho = \rho_0[1 - \alpha(T - T_0) + \alpha_c(C - C_0)] \quad (2)$$

where ρ_0 is the reference value of density at $T = T_0$, α and α_c are the coefficients of thermal and solute expansion respectively. The viscosity, μ of the binary fluid vary exponentially with temperature and solute concentration gradients in the form

$$\mu = \mu_0 \exp\left[\mu_t(T - T_0) + \mu_c(C - C_0)\right] \quad (3)$$

where μ_0 is the reference value at the reference temperature T_0. μ_t and μ_c are both positive constant. With this assumption and by following Nield and Kuznetsov (2011), Mokhtar et al. (2017), Nan (2013) and Yadav et al. (2016), the governing equations of mass, momentum and energy used for Marangoni convection is as below

$$\nabla \cdot \overrightarrow{v} = 0 \quad (4)$$

$$\rho[\frac{\partial \overrightarrow{v}}{\partial t} + (\overrightarrow{v} \cdot \nabla)\overrightarrow{v}] + 2\Omega v = -\nabla p + \mu \nabla^2 \overrightarrow{v} + \rho \overrightarrow{g} \quad (5)$$

$$\rho c[\frac{\partial T}{\partial t} + (\overrightarrow{v} \cdot \nabla)T] = \kappa \nabla^2 T + q + \rho c D_{TC} \nabla^2 C \quad (6)$$

$$\frac{\partial C}{\partial t} + (\overrightarrow{v} \cdot \nabla)C = D_s \nabla^2 C + D_{CT} \nabla^2 T \quad (7)$$

where $\overrightarrow{v}$ is the velocity, ρ is the density, Ω is the Coriolis, p is the pressure, $\overrightarrow{g}$ is the gravity, α is the thermal volumetric coefficient, α_c is the solutal volumetric coefficient, κ is the thermal diffusivity, D_{TC} is the Dufour diffusivity, q is the uniformly distributed volumetric internal heat generation, D_s is the solutal diffusivity and D_{CT} is the Soret diffusivity.

In order to study the system stability using linear stability theory, equations (4)-(7) are non-dimensionalize using the following definitions:

$$(x, y, z) = \frac{(x', y', z')}{d}, (u, v, w) = \frac{d(u', v', w')}{\kappa},$$
$$t = \frac{t'\kappa}{d^2}, p = \frac{p'd^2}{\mu\kappa}, C = \frac{C' - C'_0}{\Delta C'}, \overline{f} = \frac{\mu(z)}{\mu_0},$$
$$\Psi = d^2 \frac{\Psi'(z)}{\kappa} \quad (8)$$

where t is time, $\overline{f}$ is the z component of viscosity due to the temperature dependent viscosity and Ψ is the z component of vorticity due to the rotation.

The quiescent basic state of the fluid is given by

$$(u, v, w) = (0, 0, 0), \rho = \rho_b(z), C = C_b(z),$$
$$\Psi = \Psi_b(z),$$
$$p = p_b(z) = p_0 - \rho_0 g z - \frac{1}{2}\rho_0 g \beta z(z - d),$$
$$T = T_b(z) = -\frac{q}{2\kappa}z^2 + (\frac{qd}{2\kappa} - \frac{\Delta T}{d})z + T_0$$
$$= \frac{T_l + T_u}{2} - \beta(z - \frac{d}{2}) \quad (9)$$

where $\beta = \frac{\Delta T}{d}$ is the temperature gradient, T_l is the lower temperature, T_u is the upper temperature and subscript b denotes the basic state.

The superpose perturbations on the basic solution is in the following form

$$(u, v, w, p, \rho, C, T, \Psi) =$$
$$(0 + u', 0 + v', 0 + w', p_b(z) + p', \rho_b(z) + \rho',$$
$$C_b(z) + C', T_b(z) + T', \Psi_b(z) + \Psi') \quad (10)$$

The perturbed non-dimensional governing equations can be obtained as

$$\nabla \cdot \overrightarrow{v}' = 0 \quad (11)$$

$$\frac{1}{Pr}\frac{\partial \overrightarrow{v}'}{\partial t} = -\nabla p' + \nabla^2 \overrightarrow{v}' + RaT'\hat{E} + RsLeC'\hat{E} + \sqrt{Ta}(\overrightarrow{v}' \times \hat{E}) \quad (12)$$

$$\frac{\partial T'}{\partial t} - [Q(1 - 2z) - 1]w' = \nabla^2 T' + Df \nabla^2 C' \quad (13)$$

$$\frac{\partial C'}{\partial t} - w' = Le \nabla^2 C' + Sr \nabla^2 T' \quad (14)$$

where $Ra = \frac{\alpha g d^3 \Delta T}{\mu\kappa}$ (Rayleigh number), $Rs = \frac{\alpha_c g d^3 \Delta C}{\mu D_s}$ (Solutal Rayleigh number), $Le = \frac{D_s}{\kappa}$ (Lewis number), $Sr = \frac{D_{CT}\Delta T}{\kappa \Delta C}$ (Soret parameter), $Df = \frac{D_{TC}\Delta S}{\kappa \Delta T}$ (Dufour parameter),

$Pr = \frac{\mu}{\rho\kappa}$ (Prandtl number) , $Ta = \frac{d^4 4\Omega^2}{\mu^2}$ (Taylor number) and $Q = \frac{qd^2}{2\kappa\Delta T}$ is the dimensionless heat source strength (Sparrow et al., 1964).

Applying the curl operator twice on equation (12) and using equation (11), we obtained equation (15)

$$[\nabla^4 - \frac{1}{Pr}\frac{\partial}{\partial t}\nabla^2]w' + RsLeC'\nabla_d^2 + RaT'\nabla_d^2 - \sqrt{Ta}\frac{\partial \Psi'}{\partial z} = 0 \quad (15)$$

and the z-component of equation (15) is represented by equation (16).

$$[\nabla^2 - \frac{1}{Pr}\frac{\partial}{\partial t}]\Psi'_z + \sqrt{Ta}\frac{\partial w'}{\partial z} = 0 \quad (16)$$

The perturbation quantities in a normal mode are in the form

$$(w', T', C', \Psi') = [W(z), \Theta(z), \Phi(z), \Gamma(z)]e^{i(a_x x + a_y y)} \quad (17)$$

Equation (17) is being substituted into equations (13)-(16), to obtain the linearized form

$$\overline{f}(D^2 - a^2)^2 W + D^2\overline{f}(D^2 - a^2)W + 2D\overline{f}(D^2 - a^2)DW - a^2 Ra\Theta - Lea^2 Rs\Phi - \sqrt{Ta}D\Gamma = 0 \quad (18)$$

$$(D^2 - a^2)\Theta + [1 - Q(1 - 2z)]W + Df(D^2 - a^2)\Phi = 0 \quad (19)$$

$$W + Sr(D^2 - a^2)\Theta + Le(D^2 - a^2)\Phi = 0 \quad (20)$$

$$(D^2 - a^2)\Gamma + \sqrt{Ta}DW = 0 \quad (21)$$

where $a = \sqrt{a_x^2 + a_y^2}$, $D = \frac{d}{dz}$, $\overline{f}(z) = exp[B(z - \frac{1}{2}) + \frac{(T_0 - T_s)}{\beta d}]$ where $B = (\frac{\mu_t + \mu_c}{2})\beta d$ is the dimensionless viscosity parameter and $T_s = T_l - \beta d$.

The boundary conditions at the upper free surface ($z = 1$) are

$$W = D^2 W = \Phi = \Gamma = 0 \quad (22)$$

$$D\Theta = 0 \quad (23)$$

$$(D^2 + a^2)W + Ma\ a^2\Theta = 0 \quad (24)$$

The boundary condition at the lower surface ($z = 0$) are

$$W = DW = \Phi = \Gamma = 0 \quad (25)$$

and the temperature is set to be conducting

$$\Theta = 0 \quad (26)$$

or insulating

$$D\Theta = 0 \quad (27)$$

Equations (18)-(21) are solved depending on the boundary conditions (22)-(27) to obtain the eigenvalue Ma.

III. Methodology

Governing equations (18)-(21) together with the boundary conditions (22)-(27) constitute a linear eigenvalue problem of the system and being solved by using the Galerkin-type weighted residuals method. A trial based function was composed based on the boundary conditions.

$$W = \sum_{n=1}^{N} A_n W_n, \Theta = \sum_{n=1}^{N} B_n \Theta_n, \Phi = \sum_{n=1}^{N} C_n \Phi_n, \Gamma = \sum_{n=1}^{N} D_n \Gamma_n \quad (28)$$

where A_n, B_n, C_n and D_n are unknown coefficients. To approximate the solutions, W_n, Θ_n, Φ_n and Γ_n are chosen generally based on the boundaries conditions. Using expression for W, Θ, Φ and Γ in the linearized equations as well as multiplying all equations with the base functions respectively and integrating the functions, a system of 4×4 linear algebraic equations in 4 unknowns A_n, B_n, C_n and $D_n, n = 1, 2, 3, ..., N$, where N is the natural number is obtained. Marangoni number, Ma act as the eigenvalue when the determinant of the coefficient matrix is vanished to obtain a system with a non-trivial solution.

IV. Results and Discussion

The Rayleigh number, Ra was set to zero ($Ra = 0$) since in this paper, only the steady Marangoni convection is considered. By using the Galerkin method, we present the marginal stability parameters, Ma and the corresponding critical wave number, a numerically. Two cases were studied where the lower boundary is set to be conducting or insulating. Ignoring all the effects, we recover the classical Marangoni problems by Pearson (1958) where the critical Marangoni number is 79 for conducting case, and 48 for the insulating case.

Figure 1 shows the marginal curves of Marangoni number, Ma as a function of wavenumber, a for several values of Taylors number, Ta. Ta represents the Coriolis effect on the system. The critical Marangoni number which is the minimum value of the Marangoni number increases as Ta increases. When $Ta = 100$, we can see that the critical Marangoni number is higher compared to $Ta = 1$ and $Ta = 10$. This indicate that Coriolis force is a stabilizing factor where it surpress the convection. This condition is due to the force effect in the vertical movement and hence, restrict the movement of thermal convection to the horizontal plane (Yadav et al. (2016)). In the insulating case, since both boundaries are insulated to temperature, we can found that the critical Marangoni number is at $a = 0$. Figure 2 is a plot of Ma versus a for different values of temperature dependent viscosity, B. It shows that for both cases, the marginal stability curve decreases as the temperature dependent viscosity number, B increases. In the insulating case, the result shows a good agreement with previous findings (Kalitzova-Kurteva et al. (1996); Abidin et al. (2018)). Kalitzova-Kurteva et al. (1996) also study the temperature dependent viscosity effect in a conducting case where they state that temperature dependent viscosity has a stabilizing effect for a moderate viscous fluids and a destabilizing effect for a strong viscous fluid. However, their finding was done in a regular liquid layer instead of a binary fluid layer where commonly binary fluids are more stable compared to a regular fluid. The same destabilizing effect was presented by Abidin et al. (2017) in a binary fluid.

Figure 3 represent Ma against a with values of internal heat generation $Q = 1$, 5 and 10. It is found that the marginal stability curves shift downwards as the internal heat generation, Q increase. Results are similar with Char and Chiang (1994) where the authors present their finding for $Q = 0$, 1 and 5 in a regular fluid. We obtain the same critical values and same significant influence on the system. It shows that internal heat generation is a destabilizing factor to make the system more unstable. The thermal mode being transformed into the surface tensile mode as Q increases making the system destabilized. Figure 4 and Figure 5 shows the trends of stability for two parameter that exists due to the double diffusive problem which are the Soret number, Sr and Dufour number, Df. Soret analyzes the thermodiffusion and Dufour analyzes the diffusion thermos effect on the flow. As seen clearly in the figures, the critical Marangoni number decreases as Soret number increases. The temperature flux increase when the system is heated from below and this contributes to the initiation of natural convection in a binary fluid. Meanwhile, the Dufour number shows the opposite reaction where the critical Marangoni number decreases as Dufour number increases. The energy flux from lower and higher solute concentration is driven by the mass gradient in the binary system.

Figure 6 and 7 represent the variation of critical Marangoni number, Mc for different values of B and Ta for the conducting case. The plots in Figure 6 represent the values of temperature dependent viscosity, $B = 1$, 2, 3 on the critical Marangoni number, Mc against the Solutal Rayleigh number, Rs. As Rs increases, the Mc also increases. In Figure 7 shows the coupled effect of Df and Ta. When Df increase, the Mc will also increase. However, the stability effect is more obvious for a higher Ta value compared to a lower Ta.

V. Conclusion

The stability analysis of the Marangoni convection in a double diffusive binary fluid with Coriolis force, temperature dependent viscosity and internal heat generation has been studied theoretically. The Coriolis force and the Dufour number are clearly a stabilizing factor to make the system more stable. Meanwhile, temperature dependent viscosity, internal heat generation and Soret number destabilize the system where the marginal shift downwards as we increase the effects.

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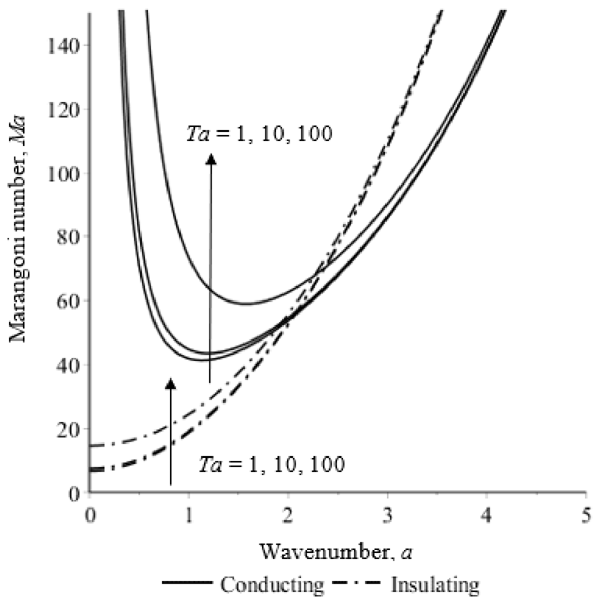


Figure 1: Variation of Ma for Ta

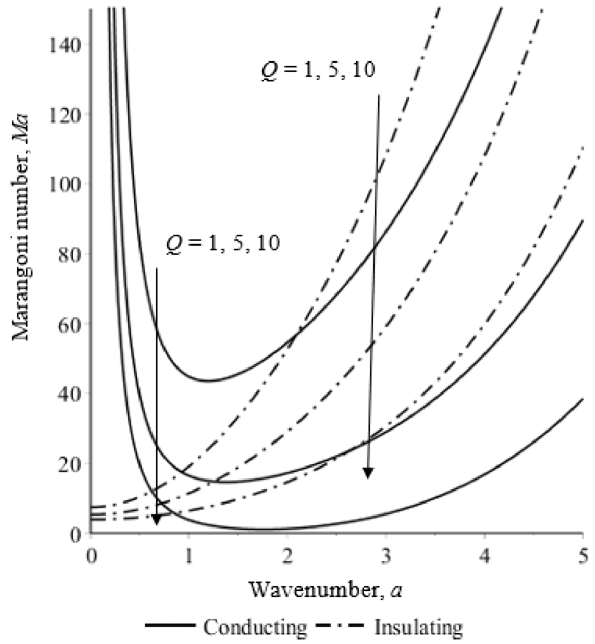


Figure 3: Variation of Ma for Q

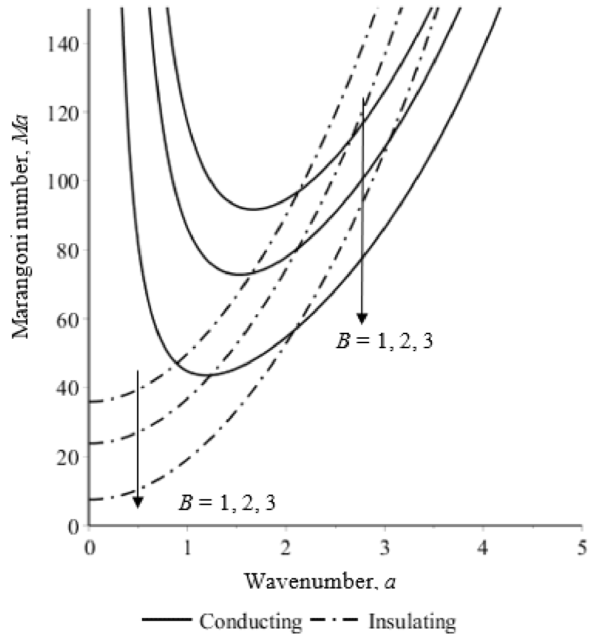
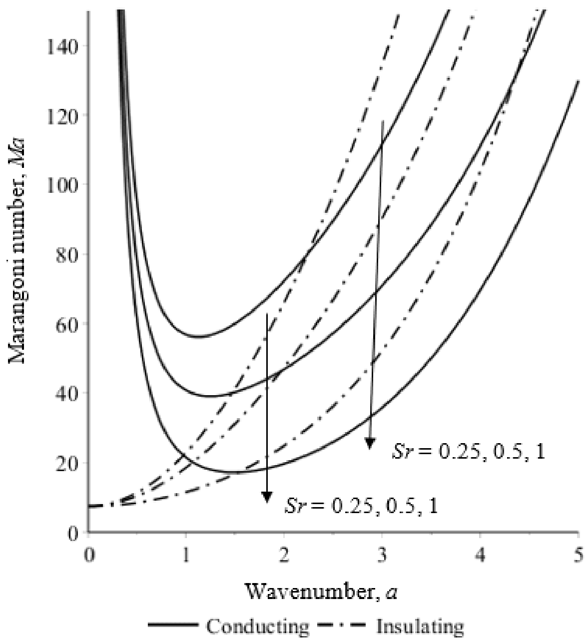


Figure 2: Variation of Ma for B



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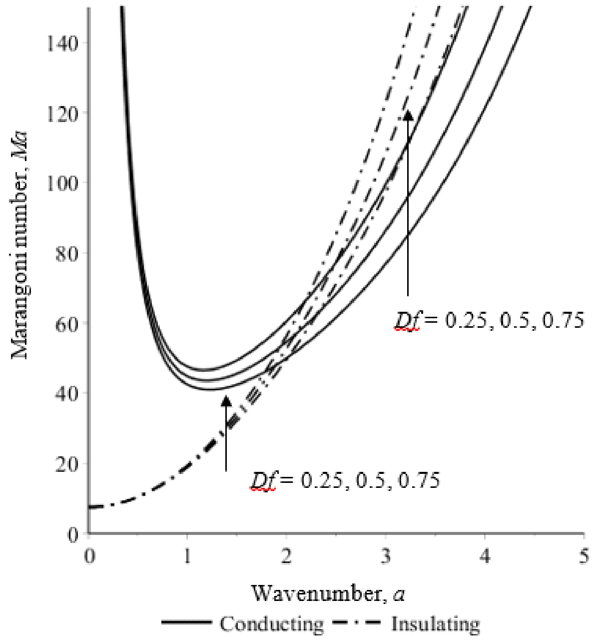

Figure 5: Variation of Ma for Df

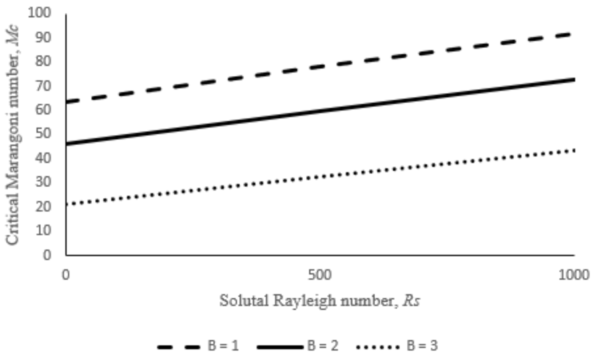

Figure 6: M_c vs. Rs for various values of B

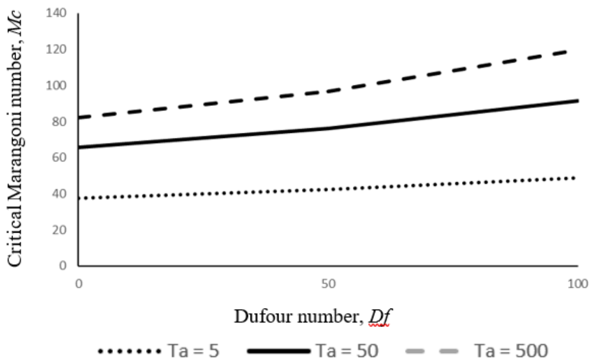

Figure 7: M_c vs. Df for various values of Ta