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Magnetohydrodynamics (MHD) Boundary Layer Flow and Heat Transfer over Shrinking Sheet with Suction and Stability Analysis

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This case study seeks to examine the fluid flow over shrinking sheet towards suction. This works also investigate the heat transfer in the present of magnetic parameter, heat generation and Lewis number. The basic governing partial differential equations are reduced to a set of ordinary differential equations by using appropriate similarity transformation. To obtain the numerical results, we used MATLAB software. We notice the dual similarity solutions are available in certain range of shrinking sheet parameter. Thus, this results make us continue further in perform the stability analysis by using bvp4c solver in MATLAB software. As expected, our study proved that the solution is stable only the first one and the second solution is not.

Keywords: boundary layer, dual solutions, MHD, stability analysis.

I. Introduction

The term of boundary layer was introduced by German engineer named Ludwig Prandtl in 1904. According to Prandtl theory, when a flow past an object, the flow region can be divided into two regions. The first region is a thin layer adjoining the solid boundary where the viscous force and rotation cannot be neglected while the second region is an outer region where the viscous force is very small and can be neglected. The analysis of boundary layer flow and heat transfer passing stretching/shrinking sheets has gained attention of many researchers in a few years ago due to the importance in engineering applications. (Sakiadis, 1961) was the first researcher that studied the problem of boundary layer flow on a surface of continuous. Then, (Crane, 1970) obtained an exact solution of the boundary layer flow of the Newtonian fluid caused by the stretching of an elastic sheet moving in its own plane linearly. Very recently, (Zaimi and Ishak, 2015) investigated the problem of boundary layer flow with convective boundary condition and they found that there exist dual solutions for both stretching and shrinking parameter.

Stability analysis is a method that evaluate the most stable solution among dual solutions. According to (Merkin, 1986), the solution with positive eigenvalues indicate stability and the solution with negative eigenvalues imply otherwise. (Junoh et al., 2018) investigated the problem of magnetohydrodynamic flow on nonlinear passing shrinking sheet in the present of radiation. They found that there exist dual solutions. Hence, they continued to perform stability analysis to identify which solution is stable and unstable. Finally, they found that the first solution is stable and physically realizable. The stability analysis is done here by the following works of (Sharma et al., 2014), (Hamid and Nazar, 2016), (Najib et al., 2017) and (Salleh et al., 2018). The aim of the current paper is to investigate numerically the problem of MHD boundary layer flow and heat transfer towards shrinking with stability analysis.

II. Mathematical Formulation

Let us consider an incompressible and two dimensional laminar boundary layer flow over a permeable shrinking sheet. The boundary layer approximations is employed and the equations of the governing problem are as follows:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2}-\frac{\sigma\beta_0^2}{\rho}u \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{\kappa}{\rho C_p}\frac{\partial^2 T}{\partial y^2}+\frac{Q}{\rho C_p}\left(T-T_\infty\right) \tag{3}$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D_B\frac{\partial^2 C}{\partial y^2} \tag{4}$$

where u and v are velocity component of the fluid along the x and y directions, respectively. $\nu=\mu/\rho$ is the kinematic viscosity where μ is the fluid viscosity and ρ is the fluid density. σ is the electrical conductivity while β_0 is the constant applied magnetic field. T represents temperature and C is the cocentration. Further, κ refer to thermal conductivity, C_p refer to specific heat of the fluid, Q refer to volumetric rate of heat generation and D_B refer to Brownian diffusion coefficient. And the appropriate boundary conditions are given by

$$\begin{aligned} u=U_w(x)=cx,\quad v=v_w,\\ T=T_w,\quad C=C_w \quad \text{at} \quad y=0\\ u\to 0,\quad T\longrightarrow T_\infty,\\ C\longrightarrow C_\infty \quad \text{at} \quad y\longrightarrow\infty \end{aligned} \tag{5}$$

Here, c denotes the stretching/shrinking rate where $c>0$ refers stretching plate while $c<0$ refer to shrinking plate. Mass tranfer velocity, $v=v_w$ where $v_w<0$ refers injection and $v_w>0$ refer suction and we assumed v_w as below:

$$v_w=-\sqrt{a\nu}S \tag{6}$$

Now, we introduced stream function which are defined as $u=\partial\psi/\partial y$ and $v=-\partial\psi/\partial x$. The continuity equation (1) is satisfied by stream function. Then, we assume the similarity transformations that defined as follow:

$$\begin{aligned} \eta=y\left(\frac{c}{\nu}\right)^{1/2},\quad \psi=\sqrt{a\nu}xf\left(\eta\right),\\ T=T_\infty+\left(T_w-T_\infty\right)\theta\left(\eta\right),\\ C=C_\infty+\left(C_w-C_\infty\right)\phi\left(\eta\right) \end{aligned} \tag{7}$$

Using equation (7), equations (2), (3) and (4) transformed into nonlinear ordinary differential equations as below:

$$f'''+ff''-f'^2-Mf' \tag{8}$$

$$\theta''+Pr\left(f\theta'+\Delta\theta\right)=0 \tag{9}$$

$$\phi''+Lef\phi'=0 \tag{10}$$

where prime indicates differentiation with respect to η. $M=\frac{\sigma\beta_0^2}{a\rho}$ refer to the magnetic parameter and $Pr=\frac{\mu C_p}{\kappa}$ denotes the Prandtl number. Then, $\Delta=\frac{Q}{\rho a C_p}$ is the heat source ($\Delta<0$) or sink ($\Delta>0$) parameter while $Le=\frac{\nu}{D_B}$ is Lewis number. The transformed boundary conditions are:

$$\begin{aligned} f\left(0\right)=S,\quad f'\left(0\right)=\lambda,\quad \theta\left(0\right)=1,\quad \phi\left(0\right)=1\\ \text{and}\quad f'\left(\infty\right)\to 0,\quad \theta\left(\infty\right)\to 0,\quad \phi\left(\infty\right)\to 0 \end{aligned} \tag{11}$$

where S is the constant mass transfer parameter with $S>0$ is for suction and $S<0$ is for injection. $\lambda=\frac{c}{a}$ is the velocity ratio parameter. The main physical quantities of interest are the value of $f''\left(0\right)$, being measure of the skin friction, the temperature gradient, $-\theta'\left(0\right)$ and the concentration gradient, $-\phi'\left(0\right)$.

III. Stability Analysis

In order to perform stability analysis, we need to consider the unsteady problem. Equation (1) holds, while equations (2)-(4) are replaced by

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2}-\frac{\sigma\beta_0^2}{\rho}u \tag{12}$$

$$\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{\kappa}{\rho C_p}\frac{\partial^2 T}{\partial y^2}+\frac{Q}{\rho C_p}\left(T-T_\infty\right) \tag{13}$$

$$\frac{\partial C}{\partial t}+u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D_B\frac{\partial^2 C}{\partial y^2} \tag{14}$$

where t denotes the time. Then, based on similarity transformations in (7), we introduced the new dimensionless variables as follow:

$$\eta=y\left(\frac{c}{\nu}\right)^{1/2},\quad \psi=\sqrt{a\nu}xf\left(\eta,\tau\right),$$
$$T=T_\infty+\left(T_w-T_\infty\right)\theta\left(\eta,\tau\right),$$
$$C=C_\infty+\left(C_w-C_\infty\right)\phi\left(\eta,\tau\right),\quad \tau=ct \tag{15}$$

Thus, equations (2)-(4) changed to the following equations:

$$\frac{\partial^3 f}{\partial\eta^3}+f\frac{\partial^2 f}{\partial\eta^2}-\left(\frac{\partial f}{\partial\eta}\right)^2-M\left(\frac{\partial f}{\partial\eta}\right)-\frac{\partial^2 f}{\partial\eta\partial\tau}=0 \tag{16}$$

$$\frac{\partial^2\theta}{\partial\eta}+Pr\left(f\frac{\partial\theta}{\partial\eta}+\lambda\theta-\frac{\partial\theta}{\partial\tau}\right)=0 \tag{17}$$

$$\frac{\partial^2\phi}{\partial\eta^2}+Le\left(f\frac{\partial\phi}{\partial\eta}-\frac{\partial\phi}{\partial\tau}\right)=0 \tag{18}$$

alongside boundary conditions as follows:

$$f\left(0,\tau\right)=S,\ \frac{\partial f}{\partial\eta}\left(0,\tau\right)=\lambda,$$
$$\theta\left(0,\tau\right)=1,\ \phi\left(0,\tau\right)=1$$
$$\frac{\partial f}{\partial\eta}\left(\infty,\tau\right)\to 0,\ \theta\left(\infty,\tau\right)\to 0,\ \phi\left(\infty,\tau\right)\to 0 \tag{19}$$

To test the stability of the solution $f\left(\eta\right)=f_0\left(\eta\right)$, $\theta\left(\eta\right)=\theta_0\left(\eta\right)$ and $\phi\left(\eta\right)=\phi_0\left(\eta\right)$ satisfying the boundary value problem (8)-(11), we write:

$$\begin{aligned}f\left(\eta,\tau\right)&=f_0\left(\eta\right)+e^{-\gamma\tau}F\left(\eta,\tau\right),\\ \theta\left(\eta,\tau\right)&=\theta_0\left(\eta\right)+e^{-\gamma\tau}H\left(\eta,\tau\right),\\ \phi\left(\eta,\tau\right)&=\phi_0\left(\eta\right)+e^{-\gamma\tau}G\left(\eta,\tau\right),\end{aligned} \tag{20}$$

where γ is an unknown eigenvalue. $F\left(\eta\right)$, $H\left(\eta\right)$ and $G\left(\eta\right)$ are small relative to $f_0\left(\eta\right)$, $\theta_0\left(\eta\right)$ and $\phi_0\left(\eta\right)$. Next, differentiate equation (20) and the substitute into equations (16)-(18) to obtain eigenvalue problem as below:

$$F_0'''+f_0F_0''+f_0''F_0-2f_0'F_0'-MF_0'+\gamma F_0'=0, \tag{21}$$

$$H_0''+Pr\left(f_0H_0'+\theta_0'F_0+\Delta H_0+\gamma H_0\right)=0, \tag{22}$$

$$G_0''+Le\left(f_0G_0'+\phi_0'F_0+\gamma G_0\right)=0 \tag{23}$$

together with the new boundary conditions

$$F_0\left(0\right)=0,\ F_0'\left(0\right)=0,\ H_0\left(0\right)=0,\ G_0\left(0\right)=0,$$
$$F_0'\left(\eta\right)\to 0,\ H_0\left(\eta\right)\to 0,\ G_0\left(\eta\right)\to 0 \tag{24}$$

To be noted, the stability of the problem can be determine by the smallest eigenvalue γ. Therefore, the condition $F_0'\left(\eta\right)\to 0$ as $\eta\to\infty$ has been put at rest as suggested by (Harris et al., 2009) and for fixed value of eigenvalue, γ.

IV. Results and Discussion

The system of equations (8)-(10) with the boundary conditions in (11) is solved using an implemented bvp4c package in MATLAB software. If the profiles satify the far field boundary conditions (11) asymptotically, the numerical results obtained are considered correct. Besides, to support the results obtained, we compared our results with those reported by (Zaimi et al., 2014) and (Yasin et al., 2016) as illustrated in Table 1 by considering the values of $\Delta = S = M = Le = 0$ and $\lambda = 1$. The comparison shows very good agreement. The respective results are given to carry out the influences of several kind of parameters on the parametric study such as magnetic parameter, M, heat source or sink parameter, Δ and Lewis number, Le.

Table 1: Comparison of the values of $-\theta'(0)$

Pr	Zaimi et al.	Yasin et al.	Present
0.72	0.463145	0.4631	0.463145
1	0.581977	0.5820	0.581977
3	1.165246	1.1652	1.165246
7	1.895403	-	1.895403

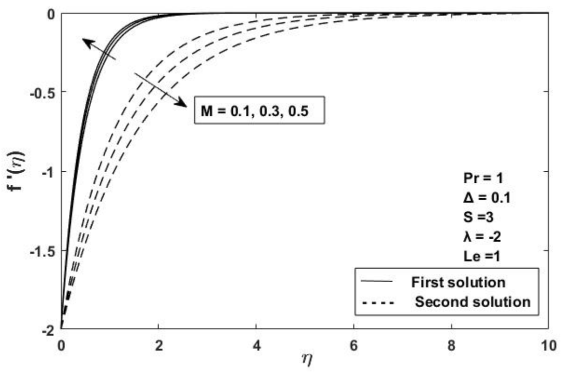

Figure 1: Velocity profile for various values of M

We now consider velocity profiles $f'(\eta)$ as illustrated in Figure 1 for selected values of magnetic parameter, M when we consider the value of $Pr = 1$, $\Delta = 0.1$, $S = 3$, $\lambda = -2$ and $Le = 1$. From Figure 1, as magnetic parameter, M increase, the momentum boundary layer thickness decrease. This is due to the magnetic force acting on the sheet increases as well, causing the boundary layer thickness to become smaller. In addition, Figure 2 shows the various values of heat source/sink parameter, Δ on temperature profile $\theta(\eta)$. Increasing heat generation ($\Delta > 0$) significantly ac-

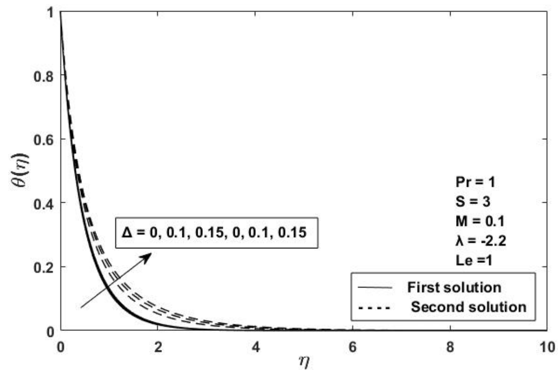

Figure 2: Temperature profile for various values of Δ

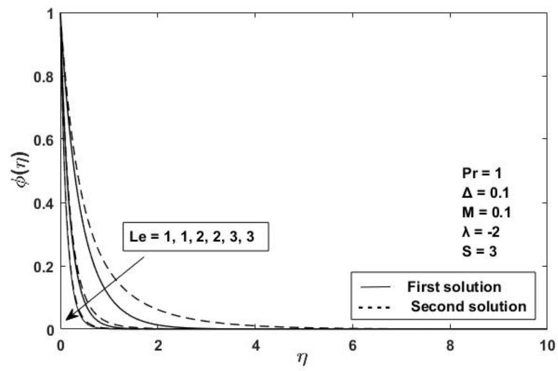

Figure 3: Concentration profile for various values of Le

celerates the flow and also increases temperature magnitudes. Conversely, with a heat sink ($\Delta < 0$) present, the flow is retarded which means that momentum boundary layer thickness is lowered. Then, thermal boundary layer thickness is reduced. Next, as shown in Figure 3, when Lewis number, Le increase, the boundary layer thickness decrease.

Figure 4-6 shows the numerous values of suction parameter, S on velocity profile, temperature profile and concentration profile. All these three figures show that the reduction in boundary layer thickness with the increase of suction parameter, S. This happened because suction reduces drag force in order to avoid boundary layer separation. From Figure 5, we can say that the viscosity of the fluid increases as suction parameter, S increase.

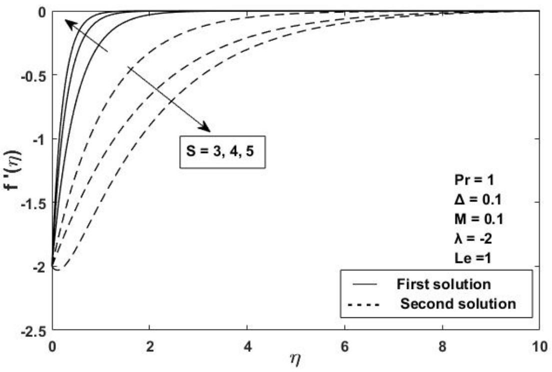

Figure 4: Velocity profile for various values of S

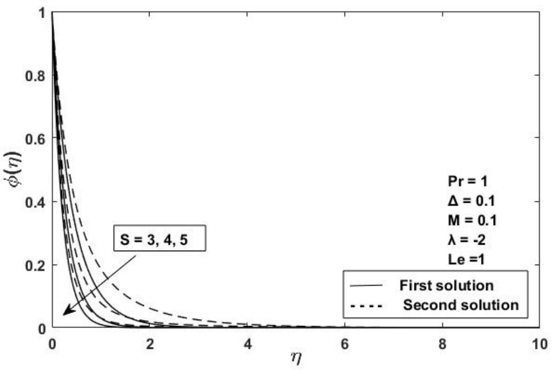

Figure 5: Temperature profile for various values of S

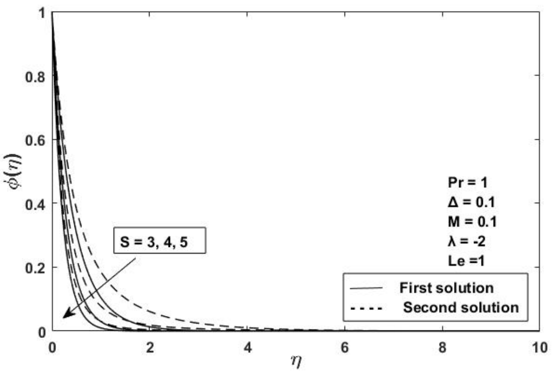

Figure 6: Concentration profile for various values of S

On other hand, the thickness of the thermal boundary layer is decreasing when we increase the value of suction parameter, S. Hence, suction parameter, S with larger value will enhance the heat transfer rate more quickly compared to smaller suction parameter, S.

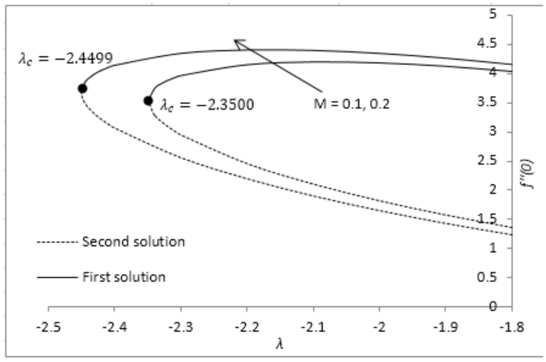

Figure 7: Skin friction coefficient for various values of M

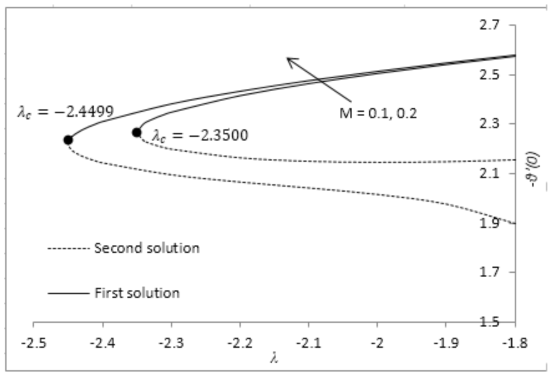

Figure 8: Local Nusselt number for various values of M

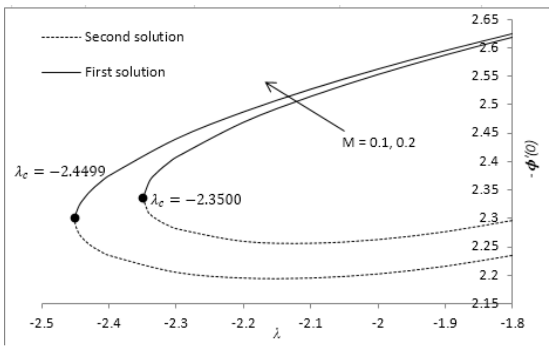

Figure 9: Concentration gradient for various values of M

The changes of skin friction coefficient, local Nusselt number and concentration gradient with magnetic parameter, M are show in Figure 7-9. Based on Figure 7, it is seen that upon increasing of magnetic parameter, M, the skin friction coefficient is increase. In fact, the value of $f''(0)$ is positive when $\lambda < 1$. Physically positive value of $f''(0)$ means the fluid exerts a drag force on the solid boundary. Normally, when $\lambda = 1$, $f''(0) = 0$. This is due to the fact that there is no friction at the friction at the fluid-solid interface when the fluid and the solid boundary move with the same velocity. Besides, from Figure 8, the local Nusselt number increase when the increasing of the magnetic parameter, M. It is also good to know that an increment of magnetic parameter, M leads to a increase in the ratio of thermal conductivity. Next, Figure 9 also shows that concentration gradient increase as the magnetic parameter, M is increase. These Figures admit dual solution when $\lambda > \lambda_c$ while when $\lambda < \lambda_c$, no similarity solutions exist for equations (8)-(10). From Figure 1-9, it is shown that there exist dual solutions for this current problem. Hence, an analysis of stability is performed in order to identify which solution is most stable between two solutions. The results displayed in Table 2 states that first solution is in positive value while second solution in negative value. Hence, we can finally conclude that the first solution is stable and significantly realizable meanwhile the second solution is in opposite manner.

Table 2: Smallest eigenvalues

λ	First solution	Second solution
-2.445	0.1038	-0.0147
-2.44	0.1282	-0.0393
-2.43	0.1625	-0.0740
-2.42	0.1888	-0.1007

V. Conclusion

The study of a stability analysis on MHD boundary layer flow and heat transfer towards shrinking sheet with suction has been numerically analyzed and discussed in detail in this paper. It was found that the involving parameters-specifically magnetic parameter, heat generation parameter, suction parameter and Lewis number significantly affected the flow field. Then, we noticed that there are dual solutions. Hence, we continue further in perform stability analysis to identify which solution is stable. It is found that dual solution exist when $\lambda > \lambda_c$. The $\lambda_c = -2.3500$ when magnetic parameter, $M = 0.1$ while when we consider for $M = 0.2$, the $\lambda_c = -2.4499$. Lastly, we can conclude that the first solution is always in stable state while the second solution is not.

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